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TRIANGULATIONS AND THE STABILITY THEOREM FOR FOLIATIONS

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Let (M, F) be a smooth foliated manifold. We prove that there exists a triangulation of M such that each simplex *is* a distinguished chart for the foliation. This result enables us to give a complete geometric proof of the stability theorem. We also show that the relation between $C^*(M, F)$ and the C^*-algebra of a regular covering, is a stability result.

Introduction.

This paper is devoted to the stability theorem. The first goal is to show the complete geometric nature of this theorem. We also check that the relation between C^*-algebras of foliations and those of their regular coverings is basiquely a stability result.

In [**HS**], M. Hilsum and G. Skandalis proved that the Connes' C^*-algebra $C^*(V, F)$ associated with any $C^{\infty,0}$-foliation F on a Hausdorff σ-compact manifold V, is stable (that is: Isomorphic to its tensor product by the algebra of compact operators on a separable Hilbert space). More precisely, they proved, using the Kasparov's stabilization theorem for Hilbert C^*-modules [**K**], that $C^*(V, F)$ only depends on the *transversal structure* of the foliation. However and because of its geometric nature, the stability theorem should be proved in a complete geometric way. This was the starting point of this work, and we prove here that the use of "good" triangulations enables to readapt the Hilsum-Skandalis method at least when the manifold is compact.

The method uses the Thurston's jiggling lemma [**Th**] and the geometric results obtained are of independent interest. The geometric theorem we prove in section 1 and which allows us to avoid the stabilization, is the following:

Let (V^n, F) be a C^∞-compact $C^{\infty,0}$-foliated manifold of codimension q. Then:

(i) *There exists a triangulation β of V in general position with respect to F (the jiggling lemma);*

(ii) *The foliation equivalence relation is trivial when restricted to each simplex of* β;

(iii) *For each* n*-simplex* σ *of* β*, there exists a* C^0*-open-transversal* T_σ *to* F *such that:*

  (a) $T_\sigma \subset \sigma$;

  (b) σ *is homeomorphic to* $T_\sigma \times D_1^p$*, where* D_1^p *is the* 1*-ball in* R^p $(p = \dim(F))$;

  (c) $\overline{T}_{\sigma_1} \bigcap \overline{T}_{\sigma_2} = \emptyset$ *if* $\sigma_1 \neq \sigma_2$.

We believe that this result and the way it is applied to derive the stability can be extended to treat other problems.

This paper is organized as follows: In the first and second sections, we review some of the basic facts about triangulations and give a detailed proof of the jiggling lemma because of its importance here and in order to convince some people about the validity of the very nice and astute proof given by W. Thurston. In the third section, we treat some geometric results and demonstrate the main result above. In the fourth section, we derive the stability of foliation C^*-algebras in complete geometric terms. In the last section, we show that the link between the foliation C^*-algebra of a regular covering of a foliated manifold and the C^*-algebra of the base (see [**W1**]), is closely related to the stability theorem.

Notations. We shall denote by (V, F) or (M, F) a compact foliated manifold which is smooth in the leaf direction, triangulable (think of a 1-differentiable manifold), and without boundary (see [**HS**], Remark 10). Let p be the dimension of the leaves and q their codimension $(p+q = n)$. As usually, G will be the holonomy groupoïd of (V, F), and $C^*(V, F)$ the Connes' C^*-algebra associated with (V, F) [**C1**]. For an affine simplex σ, we shall denote by $P(\sigma)$ the plane generated by σ and the dimension of σ will be the vector space dimension of $P(\sigma)$. If $k \leq n$, then $G(k, n)$ is the Grassmannian of k-planes in $\mathbf{R}^n$, which has a differentiable manifold structure when identified with the symmetric space $O(n)/(O(k) \times O(n-k))$. We fix some metric on $G(k, n)$ for each k, and denote all these metrics by d since no confusion can occur.

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1. Triangulations and plane fields.

Let us begin by recalling some basic facts from [**Wh**]. Let E be an affine space. The points $p_0, ..., p_r$ are dependent if they belong to an affine subspace of E with dimension $< r$, if not they are independent. The cell whose vertices are $p_0, ..., p_r$ is the closed bounded set of all the barycenters of those points. If the points $p_0, ..., p_r$ are independent, the cell is called a simplex of dimension r or an r-simplex often denoted by $\langle p_0, ..., p_r \rangle$. The k-faces of the r-simplex $\langle p_0, ..., p_r \rangle$ are all the k-simplices $\langle p_{\lambda_0}, ..., p_{\lambda_k} \rangle$ with $0 \leq \lambda_0 < ... < \lambda_k \leq r$. So the 0-faces are the vertices of the simplex. A simplicial complex (resp. finite complex) K is a set (resp. a finite set) S of simplices with the following properties:

(1) If $\sigma \in S$, then each face of σ is a union of simplices in S (all we will need here is that all the faces live in S);

(2) If σ, σ' belong to S with $\dim(\sigma') < \dim(\sigma)$ then $\overset{\circ}{\sigma} \bigcap \sigma' = \emptyset$;

(3) If σ, σ' belong to S then $\sigma \bigcap \sigma'$ is a union of simplices in S.

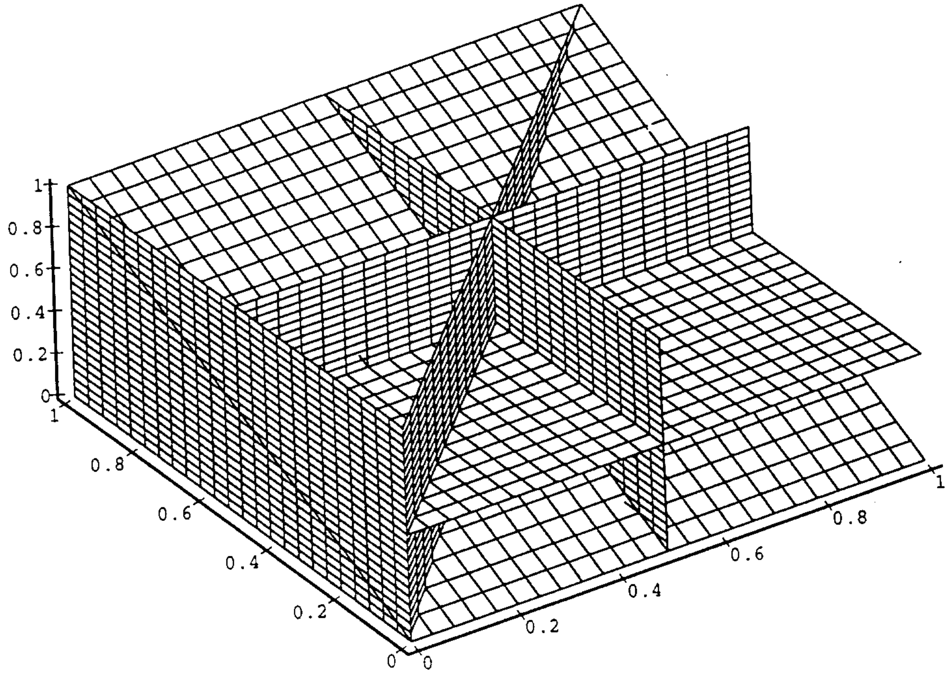


Figure 1.

In E, a simplicial complex K is a triangulation of S and we will also call it a simplicial subdivision of S. If K and K' are two subdivisions of S, and if each simplex of K' is contained in a simplex of K, then K' is a refinement of K and one easily shows that then each simplex of K is a union of simplices of K'. Note that each cell admits a simplicial subdivision [**Wh**]. For example, we can subdivide the 1-cube in $\mathbf{R}^n$ in a regular way into $n!$-simplices, each of them is given by a permutation λ of $\{1, ..., n\}$, and defined by $\sigma_\lambda = \{(x_1, ..., x_n)/0 \leq x_{\lambda(1)} < ... < x_{\lambda(n)} \leq 1\}$.

Now we can define the cristalline subdivision of a triangulation α of a finite complex K in $\mathbf{R}^n$:

We first put an order on the vertices of K. We then linearly imbed each n-simplex $\sigma = \langle p_{i_0}, ..., p_{i_n}\rangle$ in the 1-cube $C = \{(x_1, ..., x_n)/0 \leq x_k \leq 1\}$ of $\mathbf{R}^n$ by bringing p_{i_k} onto $(0, .., 0, 1, ..1)$ with k 0's and $(n-k)$ 1's. For example, when $n = 3$, this imbedding is described in Fig. 1. Now, subdividing C into $l = 4^h$ little cubes and then regularly subdividing each cube into $n!$ simplices, we get the standard subdivision of σ and these triangulations fit together to give the cristalline subdivision of K.

Definition 1.1. Let K be an affine simplicial complex. We shall say that a familly of n-simplices $(\sigma_i)_{i\in I}$ is a model-familly for K if there exists $l > 0$ such that: Each n-simplex of K is a contraction of a factor $\leq l$ of a translation of a simplex of the familly.

Note that we don't accept rotations for example, and one easily gets:

Lemma 1.2. *Let K be an affine simplicial complex* (*not necessarily finite*) *in $\mathbf{R}^n$. Assume that K admits a finite familly of model simplices, then:*

(1) *There exists a fixed finite set of simplices which are model-simplices for all the cristalline subdivisions of K;*

(2) *There exists a fixed integer P such that for each cristalline subdivision of K, each vertex of this subdivision is linked with less than P n-simplices.*

Proof. (1) Since the set of compositions of a contraction with a translation is a group, it suffices to give the proof when K is one affine simplex σ. Using the linear bijection f sending σ to its image in the 1-cube C, we can restrict to the case where σ is the simplex $\beta = f(\sigma)$ whose vertices are the points $(0, .., 0, 1, ..1)$ with j 0's and $(n-j)$ 1's, for $0 \leq j \leq n$.

Now, if we subdivide the cube into little cubes, this induces a subdivision of β into cells. But since β is one of the simplices of the regular subdivision of the cube, it is easy to see that the standard subdivision of any little cube is not disturbed by the intersection with β. Whence, all the simplices of the induced subdivision of β are small copies of the $n!$ simplices of the regular subdivision of the 1-cube.

(2) This is an immediate consequence of (1), for: Given a vertex x of a standard subdivision of K, the number of simplices linked with x is $\leq$ to the fixed number of model simplices described in (1) (since we don't accept rotations). ☐

Remark 1.3. If K is a finitely modelled simplicial complex (say: If there exists a finite model-familly for K), then we can find $\delta > 0$ such that:

If we move the vertices of each simplex of K in balls of radius $\delta \times l$ where l is any factor defined as in 1.1, then the new simplex is still non degenerate. For: By considering the finite number of model simplices, we only need to see that: Given one simplex σ, there exists a number $\delta > 0$ such that moving the vertices of σ in a radius of δ, we get a new non-degenerate simplex. But this is an easy exercise of affine geometry.

Let now V^n be a manifold. An r-simplex σ of V is defined to be the image of an affine r-simplex in $\mathbf{R}^n$ by a homeomorphism π satisfying:
There exists a local chart (U, φ) of V such that $\varphi \circ \pi : \mathbf{R}^n \to \mathbf{R}^n$ is affine when restricted to $\pi^{-1}(\sigma \bigcap U)$.

Now we can recall the exact definition of a triangulation to avoid any confusion:

Definition 1.4. Let V be a manifold of dimension n. By a triangulation of V we mean a simplicial complex K together with a homeomorphism π from K to V such that: For each n-simplex σ of K, there exists a local chart (U, φ) of V defined in a neighborhood of $\pi(\sigma)$ in V with $\varphi \circ \pi$ affine in σ.

For simplicity however, we will not distinguish between a triangulation and the image $\pi(K)$ of the complex K. Let us now recall the starting theorem [**Wh**]:

Theorem 1.5. *Each differentiable manifold V admits a triangulation.*

Now, let V^n be a smooth σ-compact manifold. We can imbed M in some $\mathbf{R}^N$ for N large enough ($N \geq 2n+1$). Let α be a triangulation of M and τ^p a C^0-plane field of dimension p. We will say that τ^p is transverse to α if it is transverse to each l-simplex α_i^l of α when $l \geq q$, and if $\tau^p + T(\alpha_i^l)$ is of dimension $p + l$ everywhere in α_i^l for $l \leq q$. Subdividing α if necessary, we may assume that α is the projection, along the normal fibres of some tubular neighborhood of V in $\mathbf{R}^N$, of a PL-approximation to V [**Th**], and we will always assume here that the triangulations are so. Since the C^0-plane field τ^p induces a local plane field $\pi^*\tau$ on each right n-simplex of the PL- approximation, we can define the notion of general position without ambiguity. τ^p is in general position with respect to the triangulation α if $\pi^*\tau$ is in general position with respect to the PL-approximation (i.e.: For each (affine) n-simplex σ of the PL-approximation, and for each point x of σ, the linear projection defined by $(\pi^*\tau)_x$ takes each q-face to a non-degenerate q-simplex in $\mathbf{R}^q$ [**Th**, p. 219]). This notion is stronger than the transversality since it insures the transversality to the faces at every point of the simplex (even interior points).

A differentiable manifold will be called an FT-manifold (finitely triangulable manifold) if it admits a triangulation which is the projection of a finitely-modelled PL-approximation (say: If there exists an imbedding in a Euclidian space and a finitely-modelled PL-approximation of the manifold in this imbedding). A compact manifold is for example obviously FT and one can prove modulo some additional work that the universal covering of any FT-manifold is still FT, and the same is true for a finite product of FT-manifolds and for the total space of a locally trivial fibration by FT-manifolds over an FT-manifold. All the results below are available in every FT-manifold under the adequate conditions, however and in order to save some unnecessary complications which would take us away from the goal of the paper, we will often restrict ourselves to the case of a compact manifold. We notice that the study of FT-manifolds and their identification in some coarse-geometric context in relation with bounded geometry [**GL**] seems to be an interesting problem.

2. About the jiggling lemma.

For the convenience of the reader and because of its importance here, we recall in detail the proof of the Thurston's jiggling lemma [**Th**]. This lemma asserts the existence of a triangulation in general position with respect to a given plane field in a neighborhood of a compact subset of a differentiable manifold. More precisely:

The jiggling lemma 2.1 [**Th**]. *Let M^n be a smooth manifold (not necessarily compact) and let K be a compact subset of M^n. Let τ^p be a C^0-plane field on M^n and let α be a triangulation of M^n. Then there is a jiggling α' of a subdivision of α which is in general position with respect to τ^p in a neighborhood of K.*

Some preliminary remarks [**Th**]. For the convenience of the reader, we will keep as much as possible the notations of [**Th**, p. 227-9]. We imbed M in $\mathbf{R}^N$ for a large integer N. Given a triangulation of M of the type already described, that is: The projection of a PL-approximation L to M along the normal fibres of a tubular neighborhood of M, any simplicial subdivision L' of L is still a PL-approximation which induces a triangulation of M. Now, using the local plane field induced by τ on the PL-approximation, one easily checks that the proof of 2.1 can be given in the case where $M = \mathbf{R}^N$ and α

is an affine triangulation.

So, assume that $M = \mathbf{R}^N$ and let α be any affine triangulation of M. By a cristalline subdivision of the simplices of α meeting K, we obtain a new triangulation α' of a neighborhood of K parametrized by the integer $l = 4^h$ which is the number of little cubes the 1-cube is subdivided into. Let $x_0, ..., x_v$ be the vertices of the simplices of α' meeting K. By 1.2 and since the subdivision is cristalline, there exists a finite number of model-simplices such that for each l, each simplex meeting K in the new triangulation is a translation of a contraction of factor $1/l$ of a model-simplex. So, by 1.3, we can find $\delta > 0$ such that if the $x_i's$ wobble in a ball of radius δ/l, then all the simplices remain non degenerate, and this is valid for each l. We have also by 1.2 the existence of a bound P for the number of simplices linked with the vertices of all the cristalline subdivisions.

The method is an induction on the $x_i's$ for a good choice of l. So, we shall first try to roughly describe it.

Main ideas of this induction.

The problem is that we have some lost of transversality when growing in dimension. So we must define a way to measure that transversality. If $\tau \in G(n,p)$ and $\beta \in G(n,p')$ then we set:

$$\mu(\tau,\beta) = \mathrm{Inf}\{d(\tau,\tau'), \tau' \text{ not transverse to } \beta\}$$

and

$$\mu(\tau,\beta) = \infty \qquad \text{if } p = 0 \text{ or } p' = 0.$$

Since the Grassmann-manifolds are compact, it is easy to see that:

$$\mu(\tau,\beta) = 0 \quad \text{if and only if } \ (\tau \text{ is not transverse to } \beta).$$

For each dimension q, we then have to define lower bounds $\epsilon(q)$ with $\epsilon(0) = \infty$. But let us first state the following lemma:

Lemma 2.2. *Let $\alpha \in G(N,q)$ be a fixed q-plane $(0 \leq q \leq k)$. Let $\tau_1 \in G(N, n-k)$ be an $(n-k)$-plane which is transverse to α. Then the map*

$$\tau \mapsto \tau \oplus \alpha$$

is well defined and continuous on an open neighborhood of τ_1.

Proof. This is an easy consequence of the Gram-Schmidt-process. □

Now the induction hypothesis in degree p is the following:

We have defined x'_i in the δ/l-ball around x_i for $1 \leq i \leq p$ such that: For each $q \in [0,k]$, for each q-simplex $\beta = \langle x_{i_0}, ..., x_{i_q} \rangle$, with $0 \leq i_0 < ... < i_q \leq p$, the jiggled q-simplex $\beta' = \langle x'_{i_0}, ..., x'_{i_q} \rangle$ satisfies:

$$\mu(\tau_x, \beta') \geq \epsilon(q), \forall x \in N_{x_{i_0}}$$

where $N_{x_{i_0}}$ is the δ/l-neighborhood of the star of x_{i_0}.

One can notice that this hypothesis only concerns the simplices of dimension $\leq k$, but this insures the transversality of the upper dimension simplices as can be immediately checked. Let then x_{p+1} be a new vertex to be treated. We have to find $x'_{p+1} \in B_{\delta/l}(x_{p+1})$ such that:

$$\forall \beta \text{ as above, if } \gamma' = \langle x'_{i_0}, ..., x'_{i_q}, x'_{p+1} \rangle \text{ then } \mu(\tau_x, \gamma') \geq \epsilon(q+1), \forall x \in N_{x_{i_0}}.$$

From the definition of $\mu(\tau_x, \gamma')$, we have to find x'_{p+1} such that

$$\forall x \in N_{x_{i_0}}, d(\tau, \tau_x) \leq \epsilon(q+1) \Rightarrow \forall \beta \text{ as above, } x'_{p+1} \notin \tau \oplus \beta'.$$

So let β be any such a q-simplex with $q \leq k-1$. If we fix a point $x_0 \in N_{x_{i_0}}$, then we have:

$$d(\tau, \tau_{x_0}) \leq \epsilon(q+1) + d(\tau_x, \tau_{x_0}) \leq \epsilon'(q+1)$$

and

$$\mu(\tau, \beta') \geq \mu(\tau_x, \beta') - d(\tau, \tau_x) \geq \epsilon(q) - \epsilon(q+1).$$

So using 2.2, we see that all the technical difficulty is to choose $\epsilon(j)$ and $\epsilon'(j)$ for $0 \leq j \leq k$ so that the induction works. To do this, we will need the following lemma:

Lemma 2.3. *Let σ be an affine simplex in $\mathbf{R}^N$. Let δ be a non-degeneresce-nce number for σ. Then there exists $\epsilon > 0$ such that:*

For each vertex ω of σ, for each $q \leq k-1$, for each q-face $\langle \omega_{j_0}, ..., \omega_{j_q} \rangle$ of σ not incident to ω, for each $\omega'_{j_0}, ..., \omega'_{j_q}$ in the δ-balls around $\omega_{j_0}, ..., \omega_{j_q}$, for each $(q+N-k)$-plane π through $\langle \omega'_{j_0}, ..., \omega'_{j_q} \rangle$, there exists ω' in the ball of radius δ around ω such that ω' does not lie in any $(q+N-k)$-plane π' through $\langle \omega'_{j_0}, ..., \omega'_{j_q} \rangle$ and at a distance $\leq \epsilon$ from π.

Proof. The number of vertices being finite, it suffices to work with one of them. Let then ω be a vertex of σ. The number of faces of σ of dimension $\leq k-1$ not touching ω being finite, it suffices to prove the existence of ϵ

for one fixed q-face $\langle \omega_{j_0}, ..., \omega_{j_q}\rangle$. So let $\omega'_{j_0}, ..., \omega'_{j_q}$ be points in the δ-balls around $\omega_{j_0}, ..., \omega_{j_q}$, let π be a $(q+N-k)$-plane containing $\langle \omega'_{j_0}, ..., \omega'_{j_q}\rangle$. For each $\eta > 0$, we consider the sector $S^{\pi}_{\eta}(\omega'_{j_0}, ..., \omega'_{j_q})$ which is the union of the $(q+N-k)$-planes through $\langle \omega'_{j_0}, ..., \omega'_{j_q}\rangle$ which are at a distance $\leq \eta$ from π.

It is clear that the measure of $S^{\pi}_{\eta}(\omega'_{j_0}, ..., \omega'_{j_q}) \bigcap B_\delta(\omega)$ goes to zero with η. So, there exists $\eta_0 > 0$ such that

$$\text{measure } \left(S_{\eta_0}(\omega'_{j_0}, ..., \omega'_{j_q}) \bigcap B_\delta(\omega)\right) < \text{ measure } (B_\delta(\omega))/3.$$

Now, the set

$$A = \{(\omega'_{j_0}, ..., \omega'_{j_q}; \pi) \in \Pi_{i=0}^{q} B_\delta(\omega_{j_i}) \times G(N, q+N-k) \text{ s.t. } \forall i \in [0,q], \omega'_{j_i} \in \pi\}$$

is compact for the product topology since its complement is open (in a compact). So, if $(\omega''_{j_0}, ..., \omega''_{j_q}; \pi_1) \in A$ is in a little neighborhood of $(\omega'_{j_0}, ..., \omega'_{j_q}; \pi)$, then with the same η_0 we can insure

$$\text{measure } \left(S^{\pi_1}_{\eta_0}(\omega"_{j_0}, ..., \omega"_{j_q}) \bigcap B_\delta(\omega)\right) < 2 \times \text{ measure } (B_\delta(\omega))/3.$$

Thus, ϵ does exist. ☐

Now, we give the Thurston's proof of the jiggling lemma:

Proof of 2.1 [**Th**]. Using the above explanations, we only have to finish the induction. Note first that the induction hypothesis is true for $p = 0$. Let δ be defined to deal with all the model-simplices. Let us take $\epsilon > 0$ as defined by 2.3, but which is valid for the finite collection of model-simplices and such that the intersection-measures are less than $1/P\times$ the measure of the δ-ball. Define the sequence $(\epsilon_j)_{0\leq j\leq k}$ as follows:

$\epsilon_0 = \infty$, $\epsilon_1 = \epsilon$, if ϵ_q has been defined, then we take $\epsilon_{(q+1)}$ satisfying: $\epsilon_{(q+1)} > 0$, $\epsilon_{(q+1)} \leq \epsilon_q/2$ and if τ_1, τ_2 are two $(N-k)$-planes with $d(\tau_1, \tau_2) \leq \epsilon_{(q+1)}$, if α is a q-plane enough transverse to τ_1 say: such that $\mu(\tau_1, \alpha) \geq \epsilon_q/2$, then $d(\tau_1 \oplus \alpha, \tau_2 \oplus \alpha) \leq \epsilon$. This is possible by 2.2.

We also need the plane field not to move too quickly in any N_x, where x is a vertex of the cristalline subdivision α^l of the simplices touching K. So, let l be as large as to insure: $\forall x$ vertex in α^l, $\forall u, v \in N_x : d(\tau_u, \tau_v) \leq \epsilon_k/3$.

Now replace in the induction hypothesis $\epsilon(q)$ by $2\times\epsilon_q/3$. If β is in the link of x_{p+1} and if τ is any plane such that $d(\tau, \tau_x) \leq 2\times\epsilon_q/3$ for some $x \in N_{x_{i_0}}$, we deduce that τ varies with a radius $\leq \epsilon_{q+1}$ from a fixed plane τ_{x_0}, and that $\mu(\tau, \beta') \geq \epsilon_q/3$. So, by the definition of ϵ_{q+1} we see that $d(\tau\oplus\beta', \tau_{x_0}\oplus\beta') \leq \epsilon$. By the choice of ϵ, we are done. ☐

Proposition 2.4. *Let M be an FT-manifold and let δ a non-degenerescence number for the model simplices of an associated PL-approximation α to M.*

Let τ be a plane field on M such that: For each $\epsilon > 0$, there exists a large l satisfying: In the cristalline subdivision α^l of α, in every δ/l-neighborhood of the star around a vertex of $\alpha^l, d(\tau_u, \tau_v) \leq \epsilon$. Then:

There exists a jiggling α' of a subdivision of α which is in general position with respect to τ over M.

Proof. Everything remains the same, and the induction used to prove the Thurston's jiggling lemma is valid here. □

In particular, if M is a compact differentiable manifold, then the condition on the plane field is automatically satisfied for any C^0-plane field. The same is true for the universal covering or any locally trivial fibration by FT-manifolds of a compact differentiable manifold M together with the plane field induced by a C^0-plane field over M.

3. The integrable case.

We will assume from now on that the plane field $\tau = F$ is integrable and that the induced foliation is smooth. The differentiable manifold M^n will be compact, but we will show how the results can be extended to deal with FT-manifolds. So, let (M^n, F^p) be a compact foliated differentiable manifold with $p + k = n$. Let β be a triangulation of M which is in general position with respect to F. Let σ be an n-simplex of β and let $(\sigma_j^{(k)})_{j\in J(\sigma)}$ be the finite collection of k-faces of σ.

Proposition 3.1. *Under the above hypothesises, there exists a familly $(T_j)_{j\in J(\sigma)}$ of open transversals such that:*

(a) $\forall j \neq i; \bar{T}_j \bigcap \bar{T}_i = \partial\sigma_j^{(k)} \bigcap \partial\sigma_i^{(k)}$;

(b) $T_j \subset \sigma, \forall j \in J(\sigma)$;

(c) *Every leaf which intersects $\sigma_j^{(k)}$, intersects T_j once and only once;*

(d) $\bar{T}_j \bigcap \bar{\sigma}_i^{(k)} \subset \partial\sigma_i^{(k)}$*, for $i \neq j$; and* $\bar{T}_j \bigcap \bar{\sigma}_j^{(k)} = \partial\sigma_j^{(k)}$.

Proof. Using the PL-approximation, we only need to prove the proposition for an affine n-simplex σ in $\mathbf{R}^n$. We give here a proof taking in account the variation of F inside σ. However, and since F moves as slight as wanted, and this is to be done in the definition of β (see the proof of 2.2), this proof can be read -the main ideas are then better understood- assuming that the plaques are affine p-planes.

Let $\sigma_1^{(k)}$ be a k-face of the n-simplex σ. Let $G_1^{(k)}$ (resp. G) be the centre of gravity of $\sigma_1^{(k)}$ (resp. σ). Since σ is non-degenerate the line $(G_1^{(k)}G)$ is transverse to $\sigma_1^{(k)}$ and we set $P_1^{(k+1)}$ for the $(k+1)$-plane generated by $\sigma_1^{(k)}$ and $(G_1^{(k)}G)$. Since F_x is transverse to $\sigma_1^{(k)}$ for every $x \in \sigma$, the plaques of F in σ induce a 1-dimensional foliation of $P_1^{(k+1)} \bigcap \sigma$.

Now we set:

$$\epsilon_1^{(k)} = \text{Inf}\left\{d\left((MG_1^{(k)}), T_N(\mathcal{L}_N)\right), \ \text{ for } \ M \in \partial\sigma_1^{(k)} \ \text{ and } \ N \in P_1^{(k+1)} \bigcap \bar{\sigma}\right\}$$

where $\mathcal{L}_N$ is the leaf through N in $P_1^{(k+1)}$.

Let $\mathcal{L}_1^{(k)}$ be the 1-dimensional plaque which is the trace of the leaf through $G_1^{(k)}$ in $P_1^{(k+1)} \bigcap \sigma, (p+k=n)$. To each point $M \in \partial\sigma_1^{(k)}$, we assign a piece of surface S_M^1 which is the natural imbedding of:

$$\left(\mathcal{L}_1^{(k)} \times (MG_1^{(k)})\right) \bigcap \sigma$$

in $P_1^{(k+1)}$.

On the other hand, let D_M be a line through M, inside the plane generated by $T_{G_1^{(k)}}(\mathcal{L}_1^{(k)})$ and M, and which is included in $P_1^{(k+1)}$, such that:

$$\mu\left(D_M, \sigma_1^{(k)}\right) = \frac{b_1^{(k)}}{2} \quad \text{where } \ b_1^{(k)} = \text{Inf}\left\{d\left(\sigma_1^{(k)}, \sigma^{(k)}\right), \ \text{ for } \ \sigma^{(k)} \subset \partial\sigma\right\}.$$

Now, we consider the parabolic part $\mathcal{C}_M^1$ between M and $\mathcal{L}_1^{(k)}$, which is tangent to D_M at M and to a k-plane parallel to $\sigma_1^{(k)}$ at the intersection with $\mathcal{L}_1^{(k)}$. We denote by P_M the intersection-point of $\mathcal{C}_M^1$ with $\mathcal{L}_1^{(k)}$ and we put:

$$d_1^{(k)}(M) = d\left(G_1^{(k)}, P_M\right)$$

$$\alpha_1^{(k)}(P, M) = \text{Sup}_{N \in \mathcal{C}_M^P}\left\{d\left(T_N\left(\mathcal{C}_M^P\right), MG_1^{(k)}\right)\right\} \quad \text{ for } \ P \in \mathcal{L}_1^{(k)} \bigcap \sigma$$

where $\mathcal{C}_M^P$ is the parabolic part between M and P excluding M and including P, inside the plane generated by $G_1^{(k)}, M$ and P, and whose tangent at P is parallel to $\sigma_1^{(k)}$. We have

$$\lim_{P \to G_1^{(k)}} \alpha_1^{(k)}(P, M) = 0,$$

so:

$$\exists e_1^{(k)}(M) > 0 \ \text{ such that } d\left(G_1^{(k)}, P\right) \leq e_1^{(k)}(M) \Rightarrow \alpha_1^{(k)}(P, M) \leq \epsilon_1^{(k)}/2.$$

Set:

$$\delta_1^{(k)}(M) = \text{Min}\left(e_1^{(k)}(M), d_1^{(k)}(M)\right) \ \text{ and } \ \delta_1^{(k)} = \text{Inf}_{M \in \partial\sigma_1^{(k)}} \delta_1^{(k)}(M).$$

To justify rigorously that $\delta_1^{(k)} > 0$, one proves that there exists $e_1^{(k)}(M) > 0$ and $d_1^{(k)}(M) > 0$ also available for M' in an open neighborhood of M in $\partial\sigma_1^{(k)}$, and uses the fact that $\partial\sigma_1^{(k)}$ is compact.

So, if A_1 is a point in $\mathcal{L}_1^{(k)} \bigcap \sigma$, with $A_1 \neq G_1^{(k)}$ such that:

$$d\left(G_1^{(k)}, A_1\right) \leq \delta_1^{(k)}.$$

Then we define $T_1 = \bigcup_{M \in \partial\sigma_1^{(k)}} \mathcal{C}_M^{A_1}$. For every $N_1 \in \sigma, N_2 \in \mathcal{C}_M^{A_1}$, we have:

$$d\left(T_{N_1}(\mathcal{L}_{N_1}), T_{N_2}(\mathcal{C}_M^{A_1})\right) \geq \epsilon_1^{(k)}/2,$$

and T_1 satisfies (b), (c) and (d), for:

(b) and (d) are evident;

(c) If a leaf intersects $\sigma_1^{(k)}$, consider its trace $\mathcal{L}_1^{(k)}$ in $P_1^{(k+1)} \bigcap \sigma$. Since $\mathcal{L}_1^{(k)}$ has to leave the closed region in $P_1^{(k+1)}$ delimited by $\bar{\sigma}_1^{(k)}$ and T_1 and since $\mathcal{L}_1^{(k)}$ cannot return back to cut $\sigma_1^{(k)}$ because of the general position, it is clear that $\mathcal{L}_1^{(k)}$ has to intersect T_1. Now, in the same way:

$$\text{for } \ N_1 \in \mathcal{L}_1^{(k)}, \ N_2 \in \mathcal{C}_M^{A_1}, \ d\left(T_{N_1}(\mathcal{L}_1^{(k)}), \ T_{N_2}(\mathcal{C}_M^{A_1})\right) \geq \epsilon_1^{(k)}/2,$$

so $\mathcal{L}_1^{(k)}$ cannot cut T_1 more than once.

If we assume now that $T_1, ..., T_{j-1}$ have been constructed satisfying (a)-(d) and consider $\sigma_j^{(k)}$, then the above arguments remain valid except that $b_j^{(k)}$ becomes the minimum of two strictly positive numbers:

$$a_j^{(k)} = \operatorname{Inf}_{i<j}\left[\operatorname{Inf}_{M \in \partial\sigma_j^{(k)} \bigcap \bar{T}_i} d\left(T_M(\bar{T}_i), \sigma_j^{(k)}\right)\right];$$

and

$$c_j^{(k)} = \operatorname{Inf}\left\{d\left(\sigma_j^{(k)}, \sigma^{(k)}\right), \quad \text{for} \quad \sigma^{(k)} \subset \partial\sigma\right\}.$$

□

Remark 3.2. If we set $T = \bigcup_{\sigma \in \beta} \bigcup_{j \in J(\sigma)} T_j$, where $J(\sigma)$ is a finite ordered set with fixed cardinal, then T is an open complete transversal and a C^0-submanifold. This is a consequence of the following:

The transversality insures that every leaf goes through the interior of at least one n-simplex. Now, the general position insures that each leaf that cuts the interior of an n-simplex cuts the interior of a k-face of the simplex.

Theorem 3.3. *Let (M, F) be a differentiable compact foliated manifold, smooth in the leaf direction and without boundary. Then there exists a triangulation β of M in general position with respect to F (Lemma 2.4), and for each n-simplex σ of β, there exists a k-submanifold of class C^0, $T_\sigma \subset \sigma$ satisfying:*

(1) *T_σ is transversal to F;*

(2) $\sigma \cong T_\sigma \times D_1^p$ *where* D_1^p *is the* 1*-ball in* $\mathbf{R}^n$*;*

(3) $\bar{T}_{\sigma_1} \bigcap \bar{T}_{\sigma_2} =$ *if* $\sigma_1 \neq \sigma_2$.

Proof. Let $(T_j)_{j \in J(\sigma)}$ be the open transversals constructed in 3.1, with $J(\sigma)$ ordered. We set:

$$S_j = \bar{T}_j \setminus \bigcup_{i<j} r\left(G(\sigma)_{\bar{T}_i}^{\bar{T}_j}\right)$$

where $G(\sigma)$ is the holonomy groupoid of the trivial foliation induced in σ and r is the map which assigns to any path its terminal point.

Suppose, as before and without loss of generality, that σ is an affine n-simplex, more and for simplicity, we will assume that the plaques are affine p-planes (the general case is to be treated in a way similar to the proof of 3.1). $S = \bigcup_j S_j$ is then a complete transversal in $\bar{\sigma}$ and it is clear that each plaque intersecting $\bar{\sigma}$ cuts S once and only once. Let K_1 be the set of extremal points in σ with respect to $F(\sigma)$, ie: points, the plaques of which do not interset the interior of σ. Then, by transversality, K_1 is a complex of dimension $< k$. Put

$$S' = S \setminus K_1 \quad \text{and} \quad S'_j = \bar{T}_j \setminus \bigcup_{i<j} r(G(\sigma)_{\bar{T}_i}^{\bar{T}_j}) \setminus K_1.$$

Now, assume by induction, that for $1 \leq i \leq (l-1), T_{\sigma_i}$ has been constructed. Suppose, on the other hand, that for $1 \leq s \leq (j-1)$, the s^{th} q-face has gone through the following treatment:

We have constructed C^0-k-manifolds T'_s transverse to F with disjoint closures, each other, with the T_{σ_i}'s and with the k-faces of σ_l, for $i = 1, ..., (l-1)$. The T'_s are also assumed to satisfy the following conditions:

(i) Every plaque intersecting S'_s intersects T'_s once and only once;

(ii) $T'_s \bigcap \partial\sigma = \emptyset$.

The method used below obviously applies to the first simplex and to the first k-face of a simplex. Let N_j be a tubular neighborhood of $\partial\sigma_j^{(k)}$ such that:

(a) For every $(k-1)$-face $\bar{\sigma}_j^{(k-1)}$ in $\bar{\sigma}_j^{(k)}$, N_j only touches the adjacent k-faces to $\bar{\sigma}_j^{(k-1)}$ in the affine triangulation β;

(b) The normal fibers contain the plaques of σ;

(c) $G_j^{(k)} \notin N_j$;

(d) N_j does not intersect the T'_m's already constructed (and which then satisfy $T'_m \bigcap \partial\sigma_j^{(k)} = \emptyset$).

Let $M \in \bar{S}'_j \bigcap \partial\sigma_j^{(k)} \subset P_j^{(k+1)}$, where $P_j^{(k+1)}$ is the $(k+1)$-plane corresponding to $S_j^{(k+1)}$ in the proof of 3.1 when we assume that the plaques are affine. We are interested with the 2-plane P_M^j and we put:

$$\mathcal{C}_M^{A_1} \bigcap \partial N_j = \left\{ C_j^M \right\}, \quad \text{see the proof of 3.1.}$$

Let D_j^M be the line in P_M^j through C_j^M and parallel to $(MG_j^{(k)})$. The trace of the leaf $\mathcal{L}_M$ through M, is a line that intersects D_j^M in a point B_j^M. Note that since M is not extremal, the traces of $P_j^{(k+1)}$ and $\mathcal{L}_M$ in a neighborhood of M inside σ generate (linearly) all the directions and meet in a segment, so:

$$B_j^M \in \sigma.$$

If $M \in S'_j$, then we replace $\mathcal{C}_M^{A_j^M} \bigcap N_j$ by the segment $]C_j^M, B_j^M]$. If not, we open the segment and take $]C_j^M, B_j^M[$. We get then a C^0-curve, Γ_j^M, which cuts all the leaves meeting the segment $(MG_j^k]$. Set:

$$T'_j = \bigcup_{M \in \bar{S}'_j \bigcap \partial\sigma_j^{(k)}} \Gamma_j^M.$$

It is now clear that the induction is then complete, and that with $T_\sigma = \bigcup_{j \in J(\sigma)} T'_j$, we are done. $\square$

Remark 3.4. The proof of 3.3 we have given here remains valid for an FT-manifold under the condition of uniform continuity with respect to the triangulation that we have defined before.

4. The stability of foliation C^*-algebras.

As a first application to our geometric result, we will show how to derive the stability theorem for foliations. Recall that the stability theorem states that the C^*-algebra $C^*(V, F)$ associated by A. Connes (see below) to the foliated manifold is isomorphic to its tensor product by the algebra of compact operators on a separable Hilbert space.

Let (V, F) be any foliated manifold and denote as usual by G its holonomy groupoid. If W is a transverse submanifold, then $C_r^*(G_W^W)$ is the C^*-algebra completion of the convolution algebra $C_c(G_W^W)$ with respect to the regular representations in the continuous field of Hilbert spaces $(L^2(G_x^W))_{x \in W}$ over W, and we denote by $C^*(V, F)$ the C^*-algebra corresponding to $W = V$. Recall that $G_A^B = \{\gamma \in G \text{ s.t. } s(\gamma) \in A \text{ and } r(\gamma) \in B\}$ and we refer to [**C1**] for further developments.

In the same way, we define for any transverse submanifolds W_1, W_2 the Hilbert C^*-module $\mathcal{E}_{W_1}^{W_2}$ over $C_r^*(G_{W_1}^{W_1})$ to be the completion of $C_c(G_{W_1}^{W_2})$ with respect the $C_c(G_{W_1}^{W_1})$-valued inner product given by:

$$\begin{aligned}&\text{If } \ \xi_1, \xi_2 \in C_c\left(G_{W_1}^{W_2}\right) \ \text{ and } \ \gamma \in G_{W_1}^{W_1} \ \text{ then:}\\ &\langle \xi_1, \xi_2\rangle(\gamma) = \int_{G_{s(\gamma)}^{W}} \bar{\xi}_1(\gamma_1)\xi_2(\gamma\gamma_1^{-1}).\end{aligned}$$

To be rigorous, we have to replace the functions by half densities so that the integrals be well defined ([**C1**]). The main result which allows us to look for the geometric theorem and to use triangulations is the following lemma proven in [**HS**]:

Lemma 4.1. *Let C be a closed subset of V such that for each leaf $\ell, C \bigcap \ell$ is negligible with respect to the longitudinal Lebesgue class, then*

$$\mathcal{E}_W^V \cong \mathcal{E}_W^{V\setminus C},$$

for any transverse submanifold W.

In fact, to prove 4.1, Hilsum and Skandalis use the fact that locally in G, the norm induced by the above inner product is a “Sup” in the transverse direction of longitudinal L^2-norms. Essentially the same argument implies (see also [**HS**]):

$$\mathcal{E}_W^{T\times D_p^1} \cong \mathcal{E}_W^T \otimes L^2(D_p^1)$$

for any transverse submanifold W, where T is the transversal defined in chapter 3 and D_p^1 is the 1-ball in dimension p, and where $T \times D_p^1$ is imbeded in V.

Now, we can state and demonstrate the stability theorem in the case where V is FT and F satisfies the “uniform continuity” with respect to the triangulation:

Theorem 4.2. *If T' is any C^0-submanifold of V which is supposed strictly transversal to F and cutting all the leaves of (V, F), then $C^*(V, F) \cong C_r^*(G_{T'}^{T'}) \otimes k$, where k is the C^*-algebra of compact operators on a separable Hilbert space.*

Proof. We can apply 3.3 and consider the triangulation β together with the transversal T. Now, with $C = \bigcup_{\sigma\in\beta} \partial\sigma$ and $W = T$ in Lemma 4.1 we deduce:

$$\mathcal{E}_T^V \cong \mathcal{E}_T^{\bigcup_{\sigma\in\beta}\sigma}.$$

But $\bigcup_{\sigma\in\beta}\sigma \cong T\times D_p^1$ by 3.3. Thus,

$$\mathcal{E}_T^V \cong \mathcal{E}_T^T \otimes L^2(D_p^1) \ \text{ and so } \ k\left(\mathcal{E}_T^V\right) \cong k\left(\mathcal{E}_T^T\right)\otimes k(L^2(D_p^1));$$

where the algebra of compact operators on a Hilbert C^*-module is defined according to [**K**]. Now we need the following lemma:

Lemma 4.3. *For every transverse submanifold W, $k(\mathcal{E}_T^W)\cong C_r^*(G_W^W)$.*

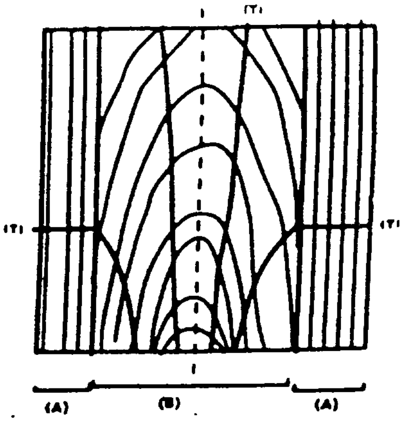
Proof. The proof given in [**HS**] is obviously valid in our context. □

This lemma enables to conclude when the transversal is the one constructed in Chapter 3, and it also shows that $C^*(V,F)$ is stable. To end the proof of Theorem 4.2 we notice that if T' is another transversal, the C^*-algebras $C_r^*(G_{T'}^{T'})$ and $C_r^*(G_V^V)$ are Morita equivalent since $\mathcal{E}_{T'}^V$ is an imprimitivity bimodule between these σ-unital C^*-algebras [**BGR**]. So,

$$C_r^*\left(G_V^V\right)\otimes k \cong C_r^*\left(G_{T'}^{T'}\right)\otimes k.$$

Now, $C^*(V,F)=C_r^*(G_V^V)$ is stable, so the proof is complete. □

Remark 4.4. The stability of $C^*(V,F)$ has been very much useful, especially in the determination of the K-theory groups of foliations whose C^*-algebras are not easy to compute. Some examples where this method is applied can be consulted in [**FW**]. If one takes for instance the 2-torus π^2 with one non orientable Reeb Component and one trivial component, then the following transversal T gives:



and we obtain:

$$C_r^*\left(G_{T\bigcap A}^{T\bigcap A}\right)\cong M_2(C(S^1))\otimes k(l^2(Z)),\quad C_r^*\left(G_{T\bigcap B}^{T\bigcap B}\right)\cong C_0(R).$$

So, using a classical exact sequence argument, we deduce the K-theory of this simple foliation avoiding the calculation of $C^*(V,F)$:

$$K_0(C^*(V,F)) = 0; K_1(C^*(V,F)) = Z \oplus (Z/2).$$

The stability theorem is also very useful when one tries to define additive maps from $K^*(V/F)$ to the scalar field, see for example [**C2**] or [**Be1**, **2**].

5. The foliation C^*-algebra of the universal covering.

As was explained in [**W2**], the relation between the C^*-algebra of a foliated manifold and the one associated with some regular covering is very usefull in the determination of the first algebra. The example of foliated hyperbolic surfaces where the C^*-algebra is not easy to compute shows the power of such a theorem. For simplicity, we will restrict to the most practical case: the universal covering and make the assumptions of [**W1**]. So, let (M,F) be a C^∞-compact foliated manifold and let $\hat{p} : \hat{M} \to M$ be the universal covering of M, we will denote by Γ the first homotopy group of M so that $M \cong \hat{M}/\Gamma$. Then the pull-back $\hat{F}$ of F by $\hat{p}$ is still complete and generates a foliation on $\hat{M}$ whose leaves are exactly the components of the pre-images of the leaves of (M,F).

Let N be the cylinder defined as the quotient of $\hat{M} \times \hat{M}$ by the diagonal action of Γ. $\hat{F} \times T(\hat{M})$ defines a foliation F_1 on $\hat{M} \times \hat{M}$ whose leaves are exactly the products of the leaves of $\hat{F}$ by $\hat{M}$. Now, since $\hat{F}$ is globally Γ-invariant, so is F_1 for the diagonal action. Hence, N is foliated by the images of the leaves of F_1 under the quotient projection π.

On the other hand, N is also a total manifold of a locally trivial fibration over M with fibre $\hat{M}$ and one easily sees that F_N is nothing but the pull back of F under this fibration. To summarize the situation, we have the following commutative diagram:

$$\begin{tikzcd}
\hat{M} \times \hat{M} \arrow[r, "pr_1"] \arrow[d, "\pi"'] & \hat{M} \arrow[d, "\hat{p}"'] \\
N \arrow[r, "p"] & M
\end{tikzcd} .$$

Proposition 5.1.

(1) *The foliation C^*-algebras of (M,F) and (N,F_N) are isomorphic;*

(2) *$\pi(\hat{M} \times \{pt\})$ is a transverse submanifold of (N,F_N) and cuts all the leaves.*

Proof. (1) As was explained in [**HS**, 7] where the result is proved directly without using the stability and is not a corollary of this theorem, there is a quite trivial C^*-bimodule ϵ_p defined as the Hilbert C^*-completion of $C_c(G_p)$ with respect to the $C_c(G)$-valued inner product (see [**CS**]), where G_p is given by:

$$G_p = \{(n, \gamma) \in N \times G(M, F) : p(n) = r(\gamma)\}.$$

Now, the C^*-algebra of compact operators on the C^*-module ϵ_p [**K**] coincides with $C^*(N, F_N)$ because $s_p : G_p \to M$, given by $s_p(n, \gamma) = s(\gamma)$ is retro-connected. On the other hand, we immediately see that $\langle \epsilon_p, \epsilon_p \rangle$ equals the whole of $C^*(M, F)$. So, ϵ_p is an imprimitivity bimodule between the stable σ-unital C^*-algebras $C^*(M, F)$ and $C^*(N, F_N)$. So, the proof of (1) is complete.
(2) It is quite trivial that $\pi(\hat{M} \times \{pt\})$ is a submanifold of N which is transverse to the leaves of F_N. Now, any leaf of F_N is of the form $\pi(\hat{L} \times \hat{M})$ where $\hat{L}$ is a leaf of $(\hat{M}, \hat{F})$. Since $\pi(\hat{L} \times \{pt\}) \bigcap \pi(\hat{M} \times \{pt\}) \neq \emptyset$; we deduce that $\pi(\hat{M} \times \{pt\})$ cuts all the leaves of (N, F_N). □

Proposition 5.2. *$C^*(M, F) \cong C^*(N, F_N) \cong C^*(\hat{M}, \hat{F}) \rtimes \Gamma$;*

Proof. By the definition of the crossed product, we see that the C^*-algebra $C^*(\hat{M}, \hat{F}) \rtimes \Gamma$ is exactly the reduced C^*-algebra associated with the groupoid $\hat{G} \times \Gamma$ where the groupoid law is given by:

$$(\hat{\gamma}_1, g_1) \circ (\hat{\gamma}_2, g_2) = (\hat{\gamma}_1(g_1.\hat{\gamma}_2), g_1.g_2).$$

On the other hand, if $M_1 = \pi(\hat{M} \times \{pt\})$ is the transverse submanifold defined in 5.1, then with our assumptions the groupoid $G(N, F_N)_{M_1}^{M_1}$ is exactly the preceding groupoid $\hat{G} \times \Gamma$. The identification is described below:

If $[(\hat{\gamma}_1, \hat{\gamma}_2)]$ is any path drawn in a leaf of (N, F_N) such that, $\hat{\gamma}_2(1) = h.pt, \hat{\gamma}_2(0) = k.pt$, with $h, k \in \Gamma$ then $k^{-1}\hat{\gamma}_1$ defines the element of $\hat{G}$, and $k^{-1}.h$ determines the element of Γ.

Conversly, if $[\hat{\gamma}_1] \in \hat{G}$ and if $g \in \Gamma$, then take any path $\hat{\gamma}_2(t)$ between pt and $g.pt$, and with the class of $(\hat{\gamma}_1, \hat{\gamma}_2)$ in $G(N, F_N)_{M_1}^{M_1}$, we are done since it is easy to check that the groupoid laws are exactly the same. Whence, Proposition 5.2 is reduced to the Hilsum-Skandalis theorem. □

Remark 5.3.

(1) One can prove that N is FT and since its foliation is the pull-back of the foliation of the compact manifold M, the plane field satisfies the uniform condition and we deduce from 4.2 that $C^*(N, F_N) \cong C^*(\hat{M}, \hat{F}) \rtimes \Gamma$ so that by Section 4, the stabilization is not needed to prove 5.2.

(2) To get the result for any regular covering, there is some foliated condition to put on the covering, and it would be interesting to compare this condition with the hypothesis taken in [**W1**].

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