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REGULARITY OF CANONICAL AND DEFICIENCY MODULES FOR MONOMIAL IDEALS

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We show that the Castelnuovo–Mumford regularity of the canonical or a deficiency module of the quotient of a polynomial ring by a monomial ideal is bounded by its dimension.

1. Introduction

Let $R = \Bbbk[x_1, \dots, x_n]$ be a standard graded polynomial ring over a field $\Bbbk$, and let $\mathfrak{m} = (x_1, \dots, x_n)$ be the homogeneous maximal ideal of R. We study the Castelnuovo–Mumford regularity of the modules $\operatorname{Ext}^i_R(R/I, \omega_R)$ when $I \subset R$ is a monomial ideal; here $\omega_R = R(-n)$ denotes the canonical module of R. The modules

$$\operatorname{Ext}^i_R(R/I, \omega_R) \quad \text{for } i > n - \dim R/I$$

are called the *deficiency modules* of R/I, while

$$\operatorname{Ext}^{n-\dim R/I}_R(R/I, \omega_R)$$

is called the *canonical module* of R/I.

For any homogeneous ideal $I \subseteq R$, the local cohomology modules $\mathrm{H}^i_{\mathfrak{m}}(R/I)$ are important in commutative algebra and algebraic geometry. One is often interested in the vanishing of homogeneous components of $\mathrm{H}^i_{\mathfrak{m}}(R/I)$. While one cannot expect the vanishing of $\mathrm{H}^i_{\mathfrak{m}}(R/I)$ in negative degrees (unless it has finite length), one can, using the local duality theorem of Grothendieck, obtain some information from $\operatorname{Ext}^{n-i}_R(R/I, \omega_R)$. For a finitely generated graded R-module M, its (*Castelnuovo–Mumford*) *regularity* $\operatorname{reg}(M)$ is an invariant that contains information about the stability of homogeneous components in sufficiently large degrees. In light of these, it is desirable to get bounds on $\operatorname{reg}(\operatorname{Ext}^i_R(R/I, \omega_R))$. Such bounds were studied by L. T. Hoa and E. Hyry [2006] and by M. Chardin, D. T. Ha and Hoa [2009]; see also the references in those papers.

Unfortunately, canonical and deficiency modules can have large regularity. For a finitely generated graded R-module M, known bounds for $\operatorname{reg}(\operatorname{Ext}^i_R(M, \omega_R))$

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are large; see, for example, [Hoa and Hyry 2006, Theorems 9 and 14]. On the other hand, more optimal bounds for $\operatorname{reg}(\operatorname{Ext}^i_R(R/I, \omega_R))$ are known to exist for certain classes of graded ideals I; see [Hoa and Hyry 2006, Section 4]. It is an interesting problem to find a class of graded ideals $I \subset R$ with optimal bounds for $\operatorname{reg}(\operatorname{Ext}^i_R(R/I, \omega_R))$. In this paper, we focus on monomial ideals. It follows from the theory of square-free modules, introduced by K. Yanagawa [2000], that if I is a square-free monomial ideal, then $\operatorname{reg}(\operatorname{Ext}^i_R(R/I, \omega_R)) \le \dim \operatorname{Ext}^i_R(R/I, \omega_R)$. This bound is small, since $\dim \operatorname{Ext}^i_R(R/I, \omega_R) \le n - i$; see [Bruns and Herzog 1993, Corollary 3.5.11].

While one cannot apply the theory of square-free modules to all monomial ideals, there are results that show that $\operatorname{reg}(\operatorname{Ext}^i_R(R/I, \omega_R))$ is not large when I is a monomial ideal. For example, we see from [Takayama 2005, Proposition 1, page 333] that if $\operatorname{Ext}^i_R(R/I, \omega_R)$ has finite length, then its regularity is negative or equal to zero. Again, Hoa and Hyry [2006, Proposition 21] showed that if $\mathrm{H}^i_{\mathfrak{m}}(R/I)$ has finite length for $i = 0, 1, \dots, d-1$, where $d = \dim R/I$, then $\operatorname{reg}(\operatorname{Ext}^{n-d}_R(R/I, \omega_R)) \le d$. We generalize these results in the next theorem:

Theorem 1.1. *Let $I \subseteq R$ be a monomial ideal. Then*

$$\operatorname{reg}(\operatorname{Ext}^i_R(R/I, \omega_R)) \le \dim \operatorname{Ext}^i_R(R/I, \omega_R) \quad \textit{for all } 0 \le i \le n.$$

Since $\dim \operatorname{Ext}^i_R(R/I, \omega_R) \le n - i$, we immediately get this:

Corollary 1.2. *Let $I \subseteq R$ be a monomial ideal. Then*

$$\operatorname{reg}(\operatorname{Ext}^i_R(R/I, \omega_R)) \le n - i \quad \textit{for all } 0 \le i \le n.$$

In general, this conclusion need not hold without the assumption that I is a monomial ideal; see [Chardin and D'Cruz 2003, Example 3.5].

Our approach to bounding the regularity of canonical and deficiency modules differs from that of Hoa and Hyry. We show that if I is a monomial ideal, then $\operatorname{Ext}^i_R(R/I, \omega_R)$ has a multigraded filtration, called the *Stanley filtration* and introduced by D. Maclagan and G. G. Smith [2005]; the bound on regularity follows from this filtration.

In the next section, we discuss some preliminaries on Stanley filtrations and local cohomology. In Section 3, we prove our main result.

2. Preliminaries

Hereafter we take R-modules to be graded by $\mathbb{Z}^n$, giving $\deg x_i = \boldsymbol{e}_i$, the i-th unit vector of $\mathbb{Z}^n$. We call this the *multigrading* of R and R-modules.

Notation 2.1. Let $\boldsymbol{a} = (a_1, \dots, a_n) \in \mathbb{Z}^n$. Write

$$\boldsymbol{x}^{\boldsymbol{a}} = \prod_{i=1}^{n} x_i^{a_i} \in \Bbbk[x_1^{\pm 1}, \dots, x_n^{\pm 1}].$$

We say that $\boldsymbol{a}$ is the *degree* of $\boldsymbol{x}^{\boldsymbol{a}}$ and write $\deg \boldsymbol{x}^{\boldsymbol{a}} = \boldsymbol{a}$. Define $\operatorname{Supp}(\boldsymbol{a}) = \{i : a_i \neq 0\}$, and define $\boldsymbol{a}^+, \boldsymbol{a}^- \in \mathbb{N}^n$ by the conditions

$$\boldsymbol{a} = \boldsymbol{a}^+ - \boldsymbol{a}^- \quad \text{and} \quad \operatorname{Supp}(\boldsymbol{a}^+) \cap \operatorname{Supp}(\boldsymbol{a}^-) = \varnothing.$$

Write $\|\boldsymbol{a}\|$ for $\sum_{i=1}^{n} a_i$, the *total degree* of $\boldsymbol{a}$ (and of the monomial $\boldsymbol{x}^{\boldsymbol{a}}$). We will say that $\boldsymbol{a}$ (or equivalently $\boldsymbol{x}^{\boldsymbol{a}}$) is *square-free* if $a_i \in \{0, 1\}$ for all i. Let $[n] = \{1, \dots, n\}$. For $\Lambda \subseteq [n]$, we set $\boldsymbol{e}_\Lambda = \sum_{i \in \Lambda} \boldsymbol{e}_i$ and abbreviate the (square-free) monomial $\boldsymbol{x}^{\boldsymbol{e}_\Lambda}$ as x_Λ. The canonical module of R is $\omega_R = R(-\boldsymbol{e}_{[n]})$.

Let M be a finitely generated multigraded R-module. Let $m \in M$ be a homogeneous element, and let $G \subset \{x_1, \dots, x_n\}$ be a subset such that $um \neq 0$ for all monomials $u \in \Bbbk[G]$. The $\Bbbk$-subspace $\Bbbk[G]m$ of M generated by all the um, where u is a monomial in $\Bbbk[G]$, is called a *Stanley space*. A *Stanley decomposition* of M is a finite set $\mathcal{S}$ of pairs (m, G) of homogeneous elements $m \in M$ and $G \subseteq \{x_1, \dots, x_n\}$ such that $\Bbbk[G]m$ is a Stanley space for all $(m, G) \in \mathcal{S}$ and

$$M =_{\Bbbk} \bigoplus_{(m,G) \in \mathcal{S}} \Bbbk[G]m. \tag{1}$$

(We used "$=_{\Bbbk}$" to emphasize that the decomposition is only as vector spaces.) Properties of such decompositions have been widely studied; we follow the approach of [Maclagan and Smith 2005, Section 3], where Stanley decompositions were used to get bounds for multigraded regularity. Following [Maclagan and Smith 2005, Definition 3.7], we define a *Stanley filtration* to be a Stanley decomposition with an ordering of pairs $\{(m_i, G_i) : 1 \leq i \leq p\}$ such that

$$\Bigl(\sum_{i=1}^{j} Rm_i\Bigr) \Big/ \Bigl(\sum_{i=1}^{j-1} Rm_i\Bigr) = \Bbbk[G_j](-\deg m_j) \quad \text{for } j = 1, 2, \dots, p$$

as R-modules. Note, in this case, that

$$0 \subseteq Rm_1 \subseteq \cdots \subseteq \sum_{i=1}^{j} Rm_i \subseteq \cdots \subseteq \sum_{i=1}^{p} Rm_i = M$$

is a prime filtration of M, as in [Eisenbud 1995, Proposition 3.7, page 93].

Proposition 2.2. *Let M be a multigraded R-module with a Stanley decomposition $\mathcal{S}$ such that $(\deg m)^+$ is square-free and $G = \operatorname{Supp}((\deg m)^+)$ for all $(m, G) \in \mathcal{S}$. Then, $\mathcal{S}$ gives a Stanley filtration. Moreover,* $\operatorname{reg} M \leq \max\{\|\deg m\| : (m, G) \in \mathcal{S}\}$.

Proof. We order $\mathcal{S} = \{(m_1, G_1), \dots, (m_p, G_p)\}$ so that $\|\deg m_1\| \geq \cdots \geq \|\deg m_p\|$. It follows from our hypothesis that

$$\operatorname{span}_{\Bbbk}\{m_1, \dots, m_p\} = \operatorname{span}_{\Bbbk}\{m \in M : \operatorname{Supp}((\deg m)^+) \text{ is square-free}\}, \tag{2}$$

where $\operatorname{span}_{\Bbbk}(V)$ denotes the $\Bbbk$-vector space spanned by the elements in V. We write $M^{(j)}$ for $\sum_{i=1}^{j} Rm_i$. We will now show, inductively on j, that

(a) $M^{(j-1)} :_R m_j = (x_k ; x_k \notin G_j)$, and

(b) the set $\bigcup_{i=1}^{j}\{um_i : u \text{ is a monomial in } \Bbbk[G_i]\}$ is a $\Bbbk$-basis for $M^{(j)}$.

These imply that $\mathcal{S}$ is a Stanley filtration of M.

Let $j = 1$. We will show that $(0 :_R m_1) = (x_k ; x_k \notin G_1)$. We have $um_1 \neq 0$ for all monomials $u \in \Bbbk[G_1]$ from the definition of the decomposition. Therefore we must show that $x_l m_1 = 0$ for any $x_l \notin G_1$. Let $x_l \notin G_1$. Then $(\deg x_l m_1)^+$ is square-free, and $x_l m_1 \in \operatorname{span}_{\Bbbk}\{m_1, \dots, m_p\}$ by (2). However, from the choice of m_1, we see that $x_l m_1 = 0$. Therefore $(0 :_R m_1) = (x_k ; k \notin G_1)$, proving (a). Then (b) follows immediately.

Now, assume that $j > 1$ and that the assertion is known for all $i < j$. We first show (a). Let u be a monomial in $\Bbbk[G_j]$. By statement (b) for $j - 1$, the set $\bigcup_{i=1}^{j-1}\{vm_i : v \text{ is a monomial in } \Bbbk[G_i]\}$ is a $\Bbbk$-basis for $M^{(j-1)}$. Since um_j is an element of the basis of M coming from the Stanley decomposition, um_j is not in the $\Bbbk$-linear span of $\bigcup_{i=1}^{j-1}\{vm_i : v \text{ is a monomial in } \Bbbk[G_i]\}$, that is, $um_j \notin M^{(j-1)}$. It remains to prove that $x_l m_j \in M^{(j-1)}$ for any $x_l \notin G_j$. Let $x_l \notin G_j$. Since $(\deg x_l m_j)^+$ is square-free, it follows from (2) and the ordering of the (m_i, G_i) that

$$x_l m_j \in \operatorname{span}_{\Bbbk}\{m_i : 1 \leq i \leq p,\ \deg m_i > \deg m_j\} \subseteq \operatorname{span}_{\Bbbk}\{m_1, \dots, m_{j-1}\}.$$

Therefore $x_l m_j \in M^{(j-1)}$, proving the statement (a) for j.

From (a), we see that the sequence

$$0 \to M^{(j-1)} \to M^{(j)} \to \Bbbk[G_j]m_j \to 0 \tag{3}$$

is exact. Now, statement (b) for j follows from the induction hypothesis.

Theorem 4.1 of [Maclagan and Smith 2005] essentially gives the assertion about regularity, but we give a quick proof here by showing that

$$\operatorname{reg} M^{(j)} \leq \max\{\|\deg m_i\| : 1 \leq i \leq j\} \quad \text{for all } 1 \leq j \leq p.$$

It holds for $j = 1$. For $j > 1$, it follows from [Eisenbud 1995, Corollary 20.19] and the exact sequence (3) that

$$\operatorname{reg} M^{(j)} \leq \max\{\operatorname{reg} M^{(j-1)}, \|\deg m_j\|\}.$$

Then induction completes the proof. □

Finally, we recall some basics of local cohomology. We follow [Bruns and Herzog 1993, Sections 3.5 and 3.6]. Let $\check{C}^\bullet$ be the Čech complex on $x_1, \dots, x_n$; the term at the i-th cohomological degree is

$$\check{C}^i = \bigoplus_{\Lambda \subseteq [n],\, |\Lambda| = i} R_{x_\Lambda},$$

where R_{x_Λ} denotes inverting the monomial x_Λ. Note that $\check{C}^\bullet$ is a complex of $\mathbb{Z}^n$-graded R-modules, with differentials of degree 0. For a finitely generated R-module M, we set $\check{C}^\bullet(M) = \check{C}^\bullet \otimes_R (M)$. Then $\mathrm{H}^i_{\mathfrak{m}}(M) = \mathrm{H}^i(\check{C}^\bullet(M))$.

Definition 2.3. Let $F \subseteq [n]$. We define $\check{C}^\bullet_F$ to be the subcomplex of $\check{C}^\bullet$ obtained by setting

$$\check{C}^i_F = \begin{cases} 0 & \text{if } i < |F|, \\ \displaystyle\bigoplus_{\substack{F \subseteq \Lambda \subseteq [n] \\ |\Lambda| = i}} R_{x_\Lambda} & \text{otherwise.} \end{cases}$$

Lemma 2.4. *Let I be a monomial ideal and $F \subseteq [n]$. If $\boldsymbol{a} \in \mathbb{Z}^n$ is such that* $\mathrm{Supp}(\boldsymbol{a}^-) = F$, *then* $\mathrm{H}^i_{\mathfrak{m}}(R/I)_{\boldsymbol{a}} = \mathrm{H}^i(\check{C}^\bullet_F \otimes_R (R/I))_{\boldsymbol{a}}$.

Proof. The proof of [Takayama 2005, Theorem 1] uses this argument implicitly. Since $\mathrm{H}^i_{\mathfrak{m}}(R/I)_{\boldsymbol{a}} = \mathrm{H}^i((\check{C}^\bullet(R/I))_{\boldsymbol{a}}$, it suffices to show that

$$(\check{C}^\bullet(R/I))_{\boldsymbol{a}} = (\check{C}^\bullet_F \otimes_R (R/I))_{\boldsymbol{a}}.$$

This, in turn, stems from the fact that $\check{C}^j_F \otimes_R (R/I)$ consists precisely of the direct summands of $\check{C}^j(R/I)$ that are nonzero in multidegree $\boldsymbol{a}$ for all $1 \le j \le n$. □

3. Proof of the main theorem

Lemma 3.1. *Let $I \subset R$ be a monomial ideal. Let $\boldsymbol{a} \in \mathbb{Z}^n$ and $j \in$* $\mathrm{Supp}(\boldsymbol{a}^+)$. *The multiplication map*

$$x_j : \mathrm{Ext}^i_R(R/I, \omega_R)_{\boldsymbol{a}} \to \mathrm{Ext}^i_R(R/I, \omega_R)_{\boldsymbol{a}+\boldsymbol{e}_j}$$

is bijective.

Proof. We first claim that the multiplication map

$$x_j : \mathrm{H}^{n-i}_{\mathfrak{m}}(R/I)_{-\boldsymbol{a}-\boldsymbol{e}_j} \to \mathrm{H}^{n-i}_{\mathfrak{m}}(R/I)_{-\boldsymbol{a}}$$

is bijective. By local duality [Bruns and Herzog 1993, Theorem 3.6.19], this map is the Matlis dual of the multiplication by x_j on $\mathrm{Ext}^i_R(R/I, \omega_R)_{\boldsymbol{a}}$; hence, it suffices to prove the claim above.

Set $F = \mathrm{Supp}(\boldsymbol{a}^+)$. Note that $\mathrm{Supp}(\boldsymbol{a}^+ + \boldsymbol{e}_j) = F$. For all i, the map x_j acts as a unit on $\check{C}^i_F$. Therefore the homomorphism of complexes

$$\check{C}^\bullet_F \otimes_R (R/I) \to \check{C}^\bullet_F \otimes_R (R/I)$$

induced by the multiplication map $x_j : \check{C}^i_F \otimes_R (R/I) \to \check{C}^i_F \otimes_R (R/I)$ is an isomorphism. The claim now follows from Lemma 2.4, which implies that

$$\begin{aligned} \mathrm{H}^i_{\mathfrak{m}}(R/I)_{-\boldsymbol{a}-\boldsymbol{e}_j} &= \mathrm{H}^i(\check{C}^\bullet_F \otimes_R (R/I))_{-\boldsymbol{a}-\boldsymbol{e}_j}, \\ \mathrm{H}^i_{\mathfrak{m}}(R/I)_{-\boldsymbol{a}} &= \mathrm{H}^i(\check{C}^\bullet_F \otimes_R (R/I))_{-\boldsymbol{a}}. \end{aligned} \qquad \square$$

The previous lemma says that, if I is a monomial ideal, then $\mathrm{Ext}^i_R(R/I, \omega_R)$ is a $(1, 1, \ldots, 1)$-determined module in the sense of [Miller 2000, Definition 2.1].

Proof of Theorem 1.1. For $F \subseteq [n]$, let $\mathcal{M}^i_F$ be a multigraded $\Bbbk$-basis for

$$\bigoplus_{\boldsymbol{a} \in \mathbb{N}^n,\, \mathrm{Supp}(\boldsymbol{a}) \cap F = \varnothing} \mathrm{Ext}^i_R(R/I, \omega_R)_{\boldsymbol{e}_F - \boldsymbol{a}}.$$

Let $\mathcal{S}_i = \{(m, F) : F \subseteq [n] \text{ and } m \in \mathcal{M}^i_F\}$. It follows from Lemma 3.1 that $\mathcal{S}_i$ is a Stanley decomposition of $\mathrm{Ext}^i_R(R/I, \omega_R)$. In particular,

$$\dim \mathrm{Ext}^i(R/I, \omega_R) = \max\{|F| : \mathcal{M}^i_F \neq \varnothing\}.$$

By the construction of $\mathcal{M}^i_F$, this Stanley decomposition satisfies the assumption of Proposition 2.2. Therefore

$$\begin{aligned} \mathrm{reg}(\mathrm{Ext}^i_R(R/I, \omega_R)) &\le \max_{F \subseteq [n]} \{\max\{\|\deg m\| : m \in \mathcal{M}^i_F\}\} \\ &\le \max_{F \subseteq [n]} \{|F| : \mathcal{M}^i_F \neq \varnothing\} \\ &= \dim \mathrm{Ext}^i_R(R/I, \omega_R), \end{aligned}$$

as desired. (The second inequality follows since $\|\deg u\| = |F| - \|(\deg u)^-\|$ for any $u \in \mathcal{M}^i_F$.) $\square$

We remark that, by using [Takayama 2005, Theorem 1] and local duality, one can determine whether $\mathcal{M}^i_F \neq \varnothing$ from certain subcomplexes of the Stanley–Reisner complex of the radical $\sqrt{I}$ of I.

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References

[Bruns and Herzog 1993] W. Bruns and J. Herzog, *Cohen–Macaulay rings*, Cambridge Studies in Advanced Mathematics **39**, Cambridge University Press, 1993. MR 95h:13020 Zbl 0788.13005

[Chardin and D'Cruz 2003] M. Chardin and C. D'Cruz, "Castelnuovo–Mumford regularity: Examples of curves and surfaces", *J. Algebra* **270**:1 (2003), 347–360. MR 2004m:13036 Zbl 1056.14065

[Chardin et al. 2009] M. Chardin, D. T. Ha, and L. T. Hoa, "Castelnuovo–Mumford regularity of Ext modules and homological degree", preprint, 2009. arXiv 0903.4535

[Eisenbud 1995] D. Eisenbud, *Commutative algebra*, Graduate Texts in Mathematics **150**, Springer, New York, 1995. MR 97a:13001 Zbl 0819.13001

[Hoa and Hyry 2006] L. T. Hoa and E. Hyry, "Castelnuovo–Mumford regularity of canonical and deficiency modules", *J. Algebra* **305**:2 (2006), 877–900. MR 2007g:13023 Zbl 1108.13017

[Maclagan and Smith 2005] D. Maclagan and G. G. Smith, "Uniform bounds on multigraded regularity", *J. Algebraic Geom.* **14**:1 (2005), 137–164. MR 2005g:14098 Zbl 1070.14006

[Miller 2000] E. Miller, "The Alexander duality functors and local duality with monomial support", *J. Algebra* **231**:1 (2000), 180–234. MR 2001k:13028 Zbl 0968.13009

[Takayama 2005] Y. Takayama, "Combinatorial characterizations of generalized Cohen–Macaulay monomial ideals", *Bull. Math. Soc. Sci. Math. Roumanie* (*N.S.*) **48**:3 (2005), 327–344. MR 2006e:13017 Zbl 1092.13020

[Yanagawa 2000] K. Yanagawa, "Alexander duality for Stanley–Reisner rings and squarefree $\mathbb{N}^n$-graded modules", *J. Algebra* **225**:2 (2000), 630–645. MR 2000m:13036 Zbl 0981.13011

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