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The paper in question included an appendix, titled "A Wasserman-type theorem for the Rabinowitz action functional", where we showed that the Rabinowitz action functional is generically Morse–Bott and the Morse–Bott manifold is the disjoint union of the energy hypersurface itself, representing the constant Reeb orbits, and a circle for each Reeb orbit. The treatment of multiple covered Reeb orbits contained a gap, which is filled in this note.

Appendix B of s devoted to showing that the Rabinowitz action functional is generically Morse–Bott and the corresponding Morse–Bott manifold is the disjoint union of the energy hypersurface itself, representing the constant Reeb orbits, and a circle for each Reeb orbit. Here we fix a gap in the proof, pointed out to us by Will Merry and Gabriel Paternain.

In the Claim in Step 2 of the proof of Theorem B.1 we asserted that $\bar{D}S(H, w)$ is surjective for every $(H, w) \in S^{-1}(0)$ whenever w is not a fixed point of the S^1-action. This assertion is incorrect as stated; it is only true if the underlying Reeb orbit v is simple. The trouble is inequality (70), which a priori only holds in a neighborhood of t_0, and might fail to hold globally on the circle if the Reeb orbit is multiply covered and hence comes back to $v(t_0)$. Therefore the proof of Theorem B.1 as it stands only proves that the Rabinowitz action functional is generically Morse–Bott on the constant and simple Reeb orbits.

To prove the full assertion of Theorem B.1 we need to show in addition that generically no root of unity arises as an eigenvalue of the linearized Reeb flow at a simple periodic orbit. But this fact follows from a classical theorem of C. Robinson [1970, Lemma 19].

Here is how this works. For $T > 0$ and $k \in \mathbb{N}$, denote by $\mathcal{U}(T, k) \subset C_c^\infty(V)$ the subset of Hamiltonians H with the following properties. If $k = 1$, then $\mathcal{U}(T, 1)$ consists of all Hamiltonians such that the Rabinowitz action functional $\mathcal{A}^H$ is Morse–Bott at the constant Reeb orbits and all simple Reeb orbits of period less

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than or equal to T (since the Reeb orbit is allowed to traverse backwards we here actually mean the absolute value of the period). If $k \geq 2$ then $\mathcal{U}(T, k) \subset \mathcal{U}(T, 1)$ consists of all $H \in \mathcal{U}(T, 1)$ with the additional property that the linearized Reeb flow at each simple Reeb orbit of period less than or equal to T has no eigenvalues equal to roots of unity of order less than or equal to k. As it follows from our arguments in the proof of Theorem B.1, for each $T > 0$ the subset $\mathcal{U}(T, 1)$ is open and dense in $C_c^\infty(V)$. If $H \in \mathcal{U}(T, 1)$, we deduce from the Arzelà–Ascoli Theorem that there are only finitely many simple Reeb orbits of period at most T. Hence by Robinson's result for each $k \in \mathbb{N}$ the subset $\mathcal{U}(T, k)$ is dense in $\mathcal{U}(T, 1)$. Again by Arzelà–Ascoli $\mathcal{U}(T, k)$ is also open in $\mathcal{U}(T, 1)$. Hence we conclude that for each $T > 0$ and for each $k \in \mathbb{N}$ the set $U(T, k)$ is open and dense in $C_c^\infty(V)$. Now set

$$\mathcal{U} = \bigcap_{\substack{N \in \mathbb{N} \\ k \in \mathbb{N}}} \mathcal{U}(N, k).$$

The subset $\mathcal{U}$ is obviously of second category in $C_c^\infty(V)$ and if $H \in \mathcal{U}$ then the Rabinowitz action functional $\mathcal{A}^H$ is Morse–Bott at the constants and at all simple Reeb orbits. Moreover, the linearized Reeb flow at each simple Reeb orbit has no root of unity as eigenvalue. Hence $\mathcal{A}^H$ is Morse–Bott at all Reeb orbits and its critical manifold consists of the disjoint union of a copy of the hypersurface and circles for each nontrivial Reeb orbit. This fills up the gap in Appendix B.

References

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