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# CORRECTION TO THE ARTICLE A FLOER HOMOLOGY FOR EXACT CONTACT EMBEDDINGS

KAI CIELIEBAK AND URS ADRIAN FRAUENFELDER

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The paper in question included an appendix, titled "A Wasserman-type theorem for the Rabinowitz action functional", where we showed that the Rabinowitz action functional is generically Morse–Bott and the Morse–Bott manifold is the disjoint union of the energy hypersurface itself, representing the constant Reeb orbits, and a circle for each Reeb orbit. The treatment of multiple covered Reeb orbits contained a gap, which is filled in this note.

Appendix B of s devoted to showing that the Rabinowitz action functional is generically Morse–Bott and the corresponding Morse–Bott manifold is the disjoint union of the energy hypersurface itself, representing the constant Reeb orbits, and a circle for each Reeb orbit. Here we fix a gap in the proof, pointed out to us by Will Merry and Gabriel Paternain.

In the Claim in Step 2 of the proof of Theorem B.1 we asserted that  $\overline{D}S(H, w)$  is surjective for every  $(H, w) \in S^{-1}(0)$  whenever w is not a fixed point of the  $S^1$ -action. This assertion is incorrect as stated; it is only true if the underlying Reeb orbit v is simple. The trouble is inequality (70), which a priori only holds in a neighborhood of  $t_0$ , and might fail to hold globally on the circle if the Reeb orbit is multiply covered and hence comes back to  $v(t_0)$ . Therefore the proof of Theorem B.1 as it stands only proves that the Rabinowitz action functional is generically Morse–Bott on the constant and simple Reeb orbits.

To prove the full assertion of Theorem B.1 we need to show in addition that generically no root of unity arises as an eigenvalue of the linearized Reeb flow at a simple periodic orbit. But this fact follows from a classical theorem of C. Robinson [1970, Lemma 19].

Here is how this works. For T > 0 and  $k \in \mathbb{N}$ , denote by  $\mathfrak{U}(T, k) \subset C_c^{\infty}(V)$  the subset of Hamiltonians H with the following properties. If k = 1, then  $\mathfrak{U}(T, 1)$  consists of all Hamiltonians such that the Rabinowitz action functional  $\mathcal{A}^H$  is Morse–Bott at the constant Reeb orbits and all simple Reeb orbits of period less

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than or equal to *T* (since the Reeb orbit is allowed to traverse backwards we here actually mean the absolute value of the period). If  $k \ge 2$  then  $\mathfrak{U}(T, k) \subset \mathfrak{U}(T, 1)$ consists of all  $H \in \mathfrak{U}(T, 1)$  with the additional property that the linearized Reeb flow at each simple Reeb orbit of period less than or equal to *T* has no eigenvalues equal to roots of unity of order less than or equal to *k*. As it follows from our arguments in the proof of Theorem B.1, for each T > 0 the subset  $\mathfrak{U}(T, 1)$  is open and dense in  $C_c^{\infty}(V)$ . If  $H \in \mathfrak{U}(T, 1)$ , we deduce from the Arzelà–Ascoli Theorem that there are only finitely many simple Reeb orbits of period at most *T*. Hence by Robinson's result for each  $k \in \mathbb{N}$  the subset  $\mathfrak{U}(T, k)$  is dense in  $\mathfrak{U}(T, 1)$ . Again by Arzelà–Ascoli  $\mathfrak{U}(T, k)$  is also open in  $\mathfrak{U}(T, 1)$ . Hence we conclude that for each T > 0 and for each  $k \in \mathbb{N}$  the set U(T, k) is open and dense in  $C_c^{\infty}(V)$ . Now set

$$\mathfrak{U} = \bigcap_{\substack{N \in \mathbb{N} \\ k \in \mathbb{N}}} \mathfrak{U}(N, k).$$

The subset  $\mathcal{U}$  is obviously of second category in  $C_c^{\infty}(V)$  and if  $H \in \mathcal{U}$  then the Rabinowitz action functional  $\mathcal{A}^H$  is Morse–Bott at the constants and at all simple Reeb orbits. Moreover, the linearized Reeb flow at each simple Reeb orbit has no root of unity as eigenvalue. Hence  $\mathcal{A}^H$  is Morse–Bott at all Reeb orbits and its critical manifold consists of the disjoint union of a copy of the hypersurface and circles for each nontrivial Reeb orbit. This fills up the gap in Appendix B.

#### References

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KAI CIELIEBAK DEPARTMENT OF MATHEMATICS LUDWIG-MAXIMILIAN UNIVERSITY THERESIENSTRASSE 39 D-80333 MUNICH GERMANY kai@mathematik.uni-muenchen.de

URS ADRIAN FRAUENFELDER DEPARTMENT OF MATHEMATICS AND RESEARCH INSTITUTE OF MATHEMATICS SEOUL NATIONAL UNIVERSITY SEOUL 151-747 SOUTH KOREA frauenf@snu.ac.kr

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#### EDITORS

V. S. Varadarajan (Managing Editor) Department of Mathematics University of California Los Angeles, CA 90095-1555 pacific@math.ucla.edu

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Vyjayanthi Chari Department of Mathematics University of California Riverside, CA 92521-0135 chari@math.ucr.edu

Robert Finn Department of Mathematics Stanford University Stanford, CA 94305-2125 finn@math.stanford.edu

Kefeng Liu Department of Mathematics University of California Los Angeles, CA 90095-1555 liu@math.ucla.edu Sorin Popa Department of Mathematics University of California Los Angeles, CA 90095-1555 popa@math.ucla.edu

Jie Qing Department of Mathematics University of California Santa Cruz, CA 95064 qing@cats.ucsc.edu

Jonathan Rogawski Department of Mathematics University of California Los Angeles, CA 90095-1555 jonr@math.ucla.edu

# **PACIFIC JOURNAL OF MATHEMATICS**

Volume 249 No. 2 February 2011

A gluing construction for prescribed mean curvature	257
Adrian Butscher	
Large eigenvalues and concentration	271
BRUNO COLBOIS and ALESSANDRO SAVO	
Sur les conditions d'existence des faisceaux semi-stables sur les courbes multiples primitives	291
JEAN-MARC DRÉZET	
A quantitative estimate for quasiintegral points in orbits	321
LIANG-CHUNG HSIA and JOSEPH H. SILVERMAN	
Möbius isoparametric hypersurfaces with three distinct principal curvatures, II ZEJUN HU and SHUJIE ZHAI	343
Discrete Morse theory and Hopf bundles DMITRY N. KOZLOV	371
Regularity of canonical and deficiency modules for monomial ideals MANOJ KUMMINI and SATOSHI MURAI	377
SL <sub>2</sub> (C)-character variety of a hyperbolic link and regulator WEIPING LI and QINGXUE WANG	385
Hypergeometric evaluation identities and supercongruences LING LONG	405
Necessary and sufficient conditions for unit graphs to be Hamiltonian H. R. MAIMANI, M. R. POURNAKI and S. YASSEMI	419
Instability of the geodesic flow for the energy functional DOMENICO PERRONE	431
String structures and canonical 3-forms CORBETT REDDEN	447
Dual pairs and contragredients of irreducible representations BINYONG SUN	485
On the number of pairs of positive integers $x_1, x_2 \le H$ such that $x_1x_2$ is a k-th power DOYCHIN I. TOLEV	495
Correction to the article A Floer homology for exact contact embeddings KAI CIELIEBAK and URS ADRIAN FRAUENFELDER	509