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**TWO-DIMENSIONAL DISJOINT
MINIMAL GRAPHS**

LINFENG ZHOU

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TWO-DIMENSIONAL DISJOINT MINIMAL GRAPHS

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Under the assumption of Gauss curvature vanishing at infinity, we prove Meeks' conjecture: the number of disjointly supported minimal graphs in \mathbb{R}^3 is at most two.

1. Introduction

Let Ω be an open subset in \mathbb{R}^2 and denote its boundary by $\partial\Omega$. As we know, if a function $u(x)$ defined on Ω satisfies the equation

$$(1) \quad \operatorname{div} \frac{\nabla u}{\sqrt{1 + |\nabla u|^2}} = 0,$$

$G = \{(x, u(x)) : x \in \Omega\}$ is called a minimal graph in \mathbb{R}^3 . We say the minimal graph G is supported on Ω if $u|_{\partial\Omega} = 0$ and $u \geq 0$.

Meeks [2005] has conjectured that the number of disjointly supported minimal graphs with zero boundary values over an open subset in \mathbb{R}^2 is at most 2. In fact, for arbitrary dimension, Meeks and Rosenberg [2005] proved if a set of disjointly supported minimal graphs have bounded gradient, then the number of the graphs must be finite. Later, Li and Wang [2001] gave an upper bound of the number of the graphs without any assumption on the growth rate of each graph. As a corollary, when minimal graphs are two dimensional in \mathbb{R}^3 , they obtained the number is at most 24. At the same time, Spruck [2002] proved that there are at most two admissible sublinear growth solution pairs of Equation (1) defined over disjoint domains. Recently, by using angular density, Tkachev [2009] showed the number of two dimensional disjointly supported minimal graphs is less than or equals 3.

Observing the similarity between disjoint d -massive sets and disjointly supported minimal graphs, we can apply the method for proving the finiteness theorem of disjoint d -massive sets in \mathbb{R}^2 [Li and Wang 1999] to study disjoint minimal graphs. We obtain the following theorem:

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Theorem 1.1. *Suppose $\{G_i = (\Omega_i, u_i)\}_{i=1}^k$ is a set of disjointly supported minimal graphs in \mathbb{R}^3 , where each Ω_i is an open subset in \mathbb{R}^2 . If the Gauss curvature $K_i(x)$ of each graph satisfies*

$$K_i(x) \rightarrow 0 \quad \text{as } |x| \rightarrow \infty,$$

then the number k is at most two.

By choosing a different region of integration, one obtains an improvement on a theorem of Spruck [2002]:

Corollary 1.2. *Suppose $\{G_i = (\Omega_i, u_i)\}_{i=1}^k$ is a set of disjointly supported minimal graphs in \mathbb{R}^3 , where each Ω_i is an open subset in \mathbb{R}^2 . If each graph has sublinear growth, then k is at most two.*

2. Proof of Theorem 1.1

We denote the 3-dimensional ball of radius R centered at the origin of \mathbb{R}^3 by $B^3(R)$ and the 2-dimensional sphere of radius R by $S^2(R)$. The key is to estimate the sum of all curves' length $\ell(G_i \cap S^2(R))$ when R is sufficiently large.

Theorem 2.1. *Suppose $\{G_i = (\Omega_i, u_i)\}_{i=1}^k$ is a set of disjointly supported minimal graphs in \mathbb{R}^3 , where the Gauss curvature $K_i(x)$ of each G_i satisfies*

$$K_i(x) \rightarrow 0 \quad \text{as } |x| \rightarrow \infty.$$

For a sufficiently large radius R , we have the bound

$$\sum_{i=1}^k \ell(G_i \cap S^2(R)) \leq \pi^2 R + o(1)R.$$

In the particular case when $k = 3$, we have the better estimate

$$\sum_{i=1}^3 \ell(G_i \cap S^2(R)) \leq 2\sqrt{2}\pi R + o(1)R.$$

Before proving this, we introduce a lemma.

Lemma 2.2. *Let $B_+^3(R)$ be a 3-dimensional upper half-ball with radius R and let $S_+^2(R)$ be a 2-dimensional upper half-sphere. Suppose $\pi_i : G_i \rightarrow \mathbb{R}^2$ is the natural projection map. If $\Sigma_1, \Sigma_2, \dots, \Sigma_s$ are planes in \mathbb{R}^3 such that the interiors of $\pi_i(\Sigma_i \cap B_+^3(R))$ are pairwise disjoint for sufficiently large R , we have*

$$\sum_{i=1}^s \ell(\Sigma_i \cap S_+^2(R)) \leq \pi^2 R.$$

Moreover, when $s = 3$, we have the better estimate

$$\sum_{i=1}^3 \ell(\Sigma_i \cap S_+^2(R)) \leq 2\sqrt{2}\pi R.$$

Proof. Suppose $D(R) = \{(x_1, x_2, 0) : x_1^2 + x_2^2 \leq R^2\}$ is a disk in \mathbb{R}^3 . Since each Σ_i is a plane, $\Sigma_i \cap D(R)$ is a chord; let θ_i be the corresponding central angle. Here we only need to consider the case that the union $\bigcup_{i=1}^s (\Sigma_i \cap D(R))$ is a polygon; otherwise, one can add more planes still satisfying the required conditions and such that the union of chords becomes a polygon.

If the center of the disk $D(R)$ is in the interior of the polygon or on one of the edges of the polygon, each central angle θ_i satisfies $0 < \theta_i \leq \pi$. Since the interiors of the $\pi_i(\Sigma_i \cap B_+^3(R))$ are pairwise disjoint, a simple computation yields the bound

$$\ell(\Sigma_i \cap S_+^2(R)) \leq \pi R \sin \frac{\theta_i}{2}.$$

on the length of the arc $\ell(\Sigma_i \cap S_+^2(R))$. The right-hand side achieves the maximum if and only if Σ_i is perpendicular to the disk $D(R)$. Thus

$$(2) \quad \begin{aligned} \sum_{i=1}^s \ell(\Sigma_i \cap S_+^2(R)) &\leq \sum_{i=1}^s \pi R \sin \frac{\theta_i}{2} \leq \pi R s \sin \left(\frac{1}{s} \sum_{i=1}^s \frac{\theta_i}{2} \right) \\ &\leq \pi R s \sin \frac{\pi}{s} \leq \pi^2 R. \end{aligned}$$

In the second inequality, we have used the concave property of the sine function on the interval $[0, \pi]$.

For the special case when $s = 3$, one gets from (2)

$$(3) \quad \sum_{i=1}^3 \ell(\Sigma_i \cap S_+^2(R)) \leq 3\pi R \sin \frac{\pi}{3} = \frac{3\sqrt{3}}{2} \pi R.$$

If the center of the disk $D(R)$ is outside the polygon, there exists an i_0 such that $\theta_{i_0} > \pi$. For simplicity, let us assume $i_0 = s$. A similar computation leads to

$$\ell(\Sigma_i \cap S_+^2(R)) \leq \begin{cases} \pi R \sin \frac{\theta_i}{2} & \text{for } 1 \leq i \leq s-1, \\ R\theta_s & \text{for } i = s. \end{cases}$$

In the first case, equality holds if and only if Σ_i is perpendicular to the disk, and in the second, if and only if Σ_s is in the same plane of the disk $D(R)$. Hence

$$(4) \quad \begin{aligned} \sum_{i=1}^s \ell(\Sigma_i \cap S_+^2(R)) &\leq \sum_{i=1}^{s-1} \pi R \sin \frac{\theta_i}{2} + R\theta_s \leq \sum_{i=1}^{s-1} \pi R \sin \frac{\theta_i}{2} + 2\pi R \sin \frac{\theta_s}{4} \\ &\leq \pi R(s+1) \sin \frac{\pi}{s+1} \leq \pi^2 R. \end{aligned}$$

If $s = 3$, by (4) we obtain that

$$(5) \quad \sum_{i=1}^3 \ell(\Sigma_i \cap S_+^2(R)) \leq 4\pi R \sin \frac{\pi}{4} = 2\sqrt{2}\pi R$$

The conclusion is derived from (2), (4) and (3), (5). □

Proof of Theorem 2.1. For each minimal graph G_i , since the Gauss curvature $K_i = 0$ at infinity, it means G_i is asymptotic to a flat plane. Therefore, we can use the intersection of a plane Σ_i and $S_+^2(R)$ to approximate the curve $G_i \cap S^2(R)$. By Lemma 2.2, one has

$$\ell(G_i \cap S^2(R)) \leq \ell(\Sigma_i \cap S_+^2(R)) + o(1)R.$$

Therefore

$$\sum_{i=1}^k \ell(G_i \cap S^2(R)) \leq \sum_{i=1}^k \ell(\Sigma_i \cap S_+^2(R)) + o(1)R \leq \pi^2 R + o(1)R. \quad \square$$

The following area growth estimate of a minimal graph is proved using a well-known argument; one can see [Li and Wang 2001] for the details.

Lemma 2.3. *If $G = (\Omega, u)$ is a minimal graph in \mathbb{R}^3 , the area of $G \cap B^3(R)$ satisfies*

$$A(G \cap B^3(R)) \leq 3\pi R^2.$$

Proof of Theorem 1.1. Let $B^3(R)$ be the ball of radius R in \mathbb{R}^3 . Since

$$\int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2 \leq \int_{G_i \cap \partial B^3(R)} u_i \left(\tilde{\nabla} u_i \cdot \frac{\partial}{\partial r} \right),$$

where $\tilde{\nabla}$ means the gradient operator on G_i , one has

$$\begin{aligned} 2\lambda_1^{1/2}(G_i \cap \partial B^3(R)) \int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2 &\leq 2\lambda_1^{1/2} \int_{G_i \cap \partial B^3(R)} u_i \cdot \frac{\partial u_i}{\partial r} \\ &\leq \lambda_1 \int_{G_i \cap \partial B^3(R)} u_i^2 + \int_{G_i \cap \partial B^3(R)} \left(\frac{\partial u_i}{\partial r} \right)^2 \\ &\leq \int_{G_i \cap \partial B^3(R)} |\bar{\nabla} u_i|^2 + \int_{G_i \cap \partial B^3(R)} \left(\frac{\partial u_i}{\partial r} \right)^2 = \int_{G_i \cap \partial B^3(R)} |\tilde{\nabla} u_i|^2. \end{aligned}$$

Here $\lambda_1^{1/2}(G_i \cap \partial B^3(R))$ denotes the first Dirichlet eigenvalue on $G_i \cap \partial B^3(R)$. We know that

$$\lambda_1^{1/2}(G_i \cap \partial B^3(R)) \geq \frac{\pi^2}{\ell^2(G_i \cap \partial B^3(R))}$$

in \mathbb{R}^3 . Therefore

$$\frac{\int_{G_i \cap \partial B^3(R)} |\tilde{\nabla} u_i|^2}{\int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2} \geq 2\lambda_1^{1/2} \geq \frac{2\pi}{\ell(\Gamma_i)},$$

where $\Gamma_i := G_i \cap \partial B^3(R)$. Thus we obtain

$$\sum_{i=1}^k \frac{\int_{G_i \cap \partial B^3(R)} |\tilde{\nabla} u_i|^2}{\int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2} \geq \sum_{i=1}^k \frac{2\pi}{\ell(\Gamma_i)}.$$

Notice that

$$k^2 \leq \left(\sum_{i=1}^k \ell(\Gamma_i) \right) \left(\sum_{i=1}^k \frac{1}{\ell(\Gamma_i)} \right).$$

According to Theorem 2.1, one has

$$\sum_{i=1}^k \ell(\Gamma_i) \leq \pi^2 R + o(1)R$$

for a sufficiently large radius R . Then it can be concluded that

$$(6) \quad \sum_{i=1}^k \frac{\int_{G_i \cap \partial B^3(R)} |\tilde{\nabla} u_i|^2}{\int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2} \geq \frac{2\pi k^2}{R(\pi^2 + o(1))}.$$

Observing that

$$(7) \quad \int_{G_i \cap \partial B^3(r)} |\tilde{\nabla} u_i|^2 = \frac{\partial}{\partial r} \int_{G_i \cap B^3(r)} |\tilde{\nabla} u_i|^2,$$

we obtain from (6) that

$$(8) \quad \ln \prod_{i=1}^k \frac{\int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2}{\int_{G_i \cap B^3(R_0)} |\tilde{\nabla} u_i|^2} \geq \frac{2\pi k^2}{\pi^2 + o(1)} \ln \frac{R}{R_0}.$$

Let $(x, y, u_i(x, y))$ be a parametrization of G_i , so the induced metric on G_i is

$$ds^2 = (1 + (u_i)_x)^2 dx^2 + 2(u_i)_x (u_i)_y dx dy + (1 + (u_i)_y)^2 dy^2.$$

We then have

$$|\tilde{\nabla} u_i| = \sqrt{u_{x^i} u_{x^j} g^{ij}} = \sqrt{\frac{|\nabla u_i|^2}{1 + |\nabla u_i|^2}} \leq 1,$$

from which one can deduce

$$(9) \quad \prod_{i=1}^k \int_{G_i \cap B^3(R)} |\tilde{\nabla} u_i|^2 \leq A^k (G_i \cap B^3(R)) \leq (3\pi R^2)^k.$$

Combining (8) and (9) implies

$$\frac{2\pi k^2}{\pi^2 + o(1)}(\ln R - \ln R_0) \leq 2k \ln R + c_1.$$

Letting $R \rightarrow +\infty$ we see that $k \leq \pi$; in particular, $k \leq 3$.

If $k = 3$, an analogous argument using the refined length estimate in Theorem 2.1 leads to $k \leq 2\sqrt{2}$, which is a contradiction. Thus k has to be at most 2. \square

Remark. Tkachev [2009] has already proved the number of two dimensional disjointly supported minimal graphs is at most 3. Here a different approach can lead to a better estimate if assuming the Gauss curvature vanishes at infinity.

3. Proof of Corollary 1.2

Let $\pi_i : G_i \rightarrow \mathbb{R}^2$ be the natural projective map and $B^2(R)$ be the ball of radius R in \mathbb{R}^2 . By employing the same method in the proof of Theorem 1.1 except for using a different region of integration $\pi_i^{-1}(\Omega_i \cap B^2(R))$, one can conclude

Theorem 3.1. *Suppose $\{(\Omega_i, u_i)\}_{i=1}^k$ is a set of disjointly supported minimal graphs in \mathbb{R}^3 where each Ω_i is an open subset in \mathbb{R}^2 . If the gradient of each u_i is bounded, say $|\nabla u_i| \leq c$, then k satisfies $k \leq 2\sqrt{1 + c^2}$.*

Proof. By a similar argument, one can obtain that

$$\sum_{i=1}^k \frac{\int_{\pi_i^{-1}(\Omega_i \cap \partial B^2(R))} |\tilde{\nabla} u_i|^2}{\int_{\pi_i^{-1}(\Omega_i \cap B^2(R))} |\tilde{\nabla} u_i|^2} \geq \frac{2\pi k^2}{\sum_{i=1}^k \ell(\Gamma_i)}.$$

where $\Gamma_i := \pi_i^{-1}(\Omega_i \cap \partial B^2(R))$. If one chooses the parametrization

$$(R \cos \theta, R \sin \theta, u_i(R \cos \theta, R \sin \theta))$$

for the curve Γ_i and assume $|\nabla u_i| \leq c$, then

$$\begin{aligned} \ell(\Gamma_i) &= \int_{\theta_0}^{\theta_1} \sqrt{R^2 + (-(u_i)_x R \sin(\theta) + (u_i)_y R \cos(\theta))^2} d\theta \\ &\leq \int_{\theta_0}^{\theta_1} \sqrt{R^2 + ((u_i)_x^2 + (u_i)_y^2)(R^2 \sin^2(\theta) + R^2 \cos^2(\theta))} d\theta \\ &\leq (\theta_1 - \theta_0) R \sqrt{1 + c^2}. \end{aligned}$$

Since the minimal graphs are disjoint, we get

$$\sum_{i=1}^k \ell(\Gamma_i) \leq 2\pi R \sqrt{1 + c^2}.$$

Then it can be concluded that

$$(10) \quad \sum_{i=1}^k \frac{\int_{\pi_i^{-1}(\Omega_i \cap \partial B^2(R))} |\tilde{\nabla} u_i|^2}{\int_{\pi_i^{-1}(\Omega_i \cap B^2(R))} |\tilde{\nabla} u_i|^2} \geq \frac{k^2}{R\sqrt{1+c^2}}.$$

Integrating (10), one obtains

$$(11) \quad \ln \prod_{i=1}^k \frac{\int_{\pi_i^{-1}(\Omega_i \cap \partial B^2(R))} |\tilde{\nabla} u_i|^2}{\int_{\pi_i^{-1}(\Omega_i \cap B^2(R_0))} |\tilde{\nabla} u_i|^2} \geq \frac{k^2}{\sqrt{1+c^2}} \ln \frac{R}{R_0}.$$

On the other hand,

$$(12) \quad \prod_{i=1}^k \int_{\pi_i^{-1}(\Omega_i \cap B^2(R))} |\tilde{\nabla} u_i|^2 \leq A^k (\pi_i^{-1}(\Omega_i \cap B^2(R))) \\ = \left(\int_{\Omega_i \cap B^2(R)} \sqrt{1+|\nabla u|^2} \right)^k \leq (\sqrt{1+c^2} \pi R^2)^k.$$

Combining (11) and (12), we have

$$\frac{k^2}{\sqrt{1+c^2}} (\ln R - \ln R_0) \leq 2k \ln R + c_1.$$

Letting $R \rightarrow +\infty$ yields

$$k \leq 2\sqrt{1+c^2}. \quad \square$$

Obviously, Corollary 1.2 follows from above theorem when each graph satisfies

$$|\nabla u_i| \rightarrow 0 \quad (|x| \rightarrow +\infty).$$

Remark. J. Spruck [2002] proved Corollary 1.2 under the assumption of a certain decay rate at infinity for the Gauss curvature. However, here we do not need any restrictions on the Gauss curvature.

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