Pacific Journal of Mathematics

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Volume 259 No. 1 September 2012

PACIFIC JOURNAL OF MATHEMATICS
Vol. 259, No. 1, 2012
dx.doi.org/10.2140/pjm.2012.259.109

NONLOCAL UNIFORM ALGEBRAS ON THREE-MANIFOLDS

ALEXANDER J. IZZO

The existence of nonlocal uniform algebras was first proven by Eva Kallin in 1963. Here we prove that on every compact C^∞-manifold of dimension greater than or equal to three, there exists a nonlocal uniform algebra generated by C^∞-smooth functions.

Dedicated to the memory of Walter Rudin

1. Introduction

Let A be a uniform algebra on a compact Hausdorff space X. A function f in $C(X)$ is said to *belong locally on X to A* if for each point x in X, there are a neighborhood N of x and a function g in A coinciding with f on N. The algebra A is said to be *local on X* if every function that belongs locally on X to A is in A. The algebra A is said to be *local* if it is local on its maximal ideal space $\mathfrak{M}_A$. It was conjectured for some time that every uniform algebra is local in this sense. This conjecture was disproved by Kallin [1963], who gave an example of a compact set X in $\mathbb{C}^4$ such that $P(X)$ (the uniform closure on X of the polynomials in $z_1, \ldots, z_4$) is nonlocal.

In [Izzo 2010], we studied localization for uniform algebras generated by smooth functions on two-manifolds. The results there suggest that perhaps these uniform algebras are always local. However, it is also shown there that not every uniform algebra generated by smooth functions on a manifold is local. Specifically, on every compact manifold of dimension greater than or equal to four, there exists a nonlocal uniform algebra generated by smooth functions. The question thus arises whether there exist nonlocal uniform algebras generated by smooth functions on manifolds of lower dimension. Here we show that such algebras exist on every three-manifold. No such uniform algebras exist on one-manifolds because if J is a compact one-manifold (possibly with boundary) and A is a uniform algebra on J generated by smooth functions and with maximal ideal space J, then $A = C(J)$. (Proof: By the Stone–Weierstrass theorem, it suffices to show the real-valued functions in A separate points. Given $p \neq q$ in J, choose a smooth function f in A whose real part separates p from q. Then $f(J)$ has two-dimensional Lebesgue measure zero in $\mathbb{C}$. Hence, by the Hartogs–Rosenthal theorem [Gamelin 1984,

MSC2010: 46J10, 46J15, 30H50, 30J15.
Keywords: nonlocal uniform algebra, three-manifold.

II.8.4], the real coordinate function x is a uniform limit on $f(J)$ of rational functions with poles off $f(J)$. Because $\mathfrak{M}_A = J$, it follows that the real part of f is in A.) The question whether a uniform algebra generated by smooth functions and whose maximal ideal space is a two-manifold must be local remains open.

The space on which Kallin's nonlocal uniform algebra is defined cannot be embedded in a three-manifold, so the approach used to construct nonlocal uniform algebras on four-manifolds in [Izzo 2010] does not work for three-manifolds. Instead we use an approach to nonlocal uniform algebras due to Sidney [1968]. Our proof is also related to the Beurling–Rudin theorem on the closed ideals in the disc algebra [Rudin 1957] (or see [Hoffman 1962, pp. 82–89]).

It is with a mixture of joy and sorrow that I dedicate this paper to the memory of Walter Rudin. Sorrow, of course, that he is no longer with us; joy that, through his work and his influence on others, he will in some sense always be with us. The breadth of Rudin's research contributions has been a great inspiration to me and many other mathematicians. In addition, Rudin had a profound impact on my mathematical development and view of the subject. My first exposure to analysis was with his *Principles of mathematical analysis* [Rudin 1976]; I learned real and complex analysis from his *Real and complex analysis* [Rudin 1974] and functional analysis from his *Functional analysis* [Rudin 1973]; and it was his book *Function theory in the unit ball of* $\mathbb{C}^n$ [Rudin 1980] that first interested me in uniform algebras.

2. The theorem and its proof

Theorem 2.1. *On every compact C^∞-manifold M of dimension greater than or equal to* 3, *there exists a nonlocal uniform algebra with maximal ideal space M generated by C^∞-smooth functions.*

Before proving the theorem, we establish some technical lemmas that are used to prove the smoothness assertion. Throughout the paper D will denote the open unit disc $\{z \in \mathbb{C} : |z| < 1\}$ and L will denote the open annulus $\{z \in \mathbb{C} : 3 < |z| < 4\}$.

Lemma 2.2. *There exists a C^∞-smooth map $\Phi : \mathbb{R}^2 \to \mathbb{R}^2$ that takes the closed unit disc $\overline{D}$ one-to-one onto itself and satisfies $\Phi(x, 0) = (1, 0)$ for all $x \geq 1$.*

Proof. Let $U = \{(x, y) \in \mathbb{R}^2 : \frac{1}{2} < x \text{ and } -\frac{1}{2} < y < \frac{1}{2}\}$, and define $\varphi : U \to \mathbb{R}^2$ by $\varphi(x, y) = (x - 1 + \sqrt{1 - y^2}\,, y)$. Then φ is a diffeomorphism of U onto the neighborhood $\varphi(U)$ of the point $\varphi(1, 0) = (1, 0)$.

Let $\alpha : \mathbb{R} \to \mathbb{R}$ be a C^∞-function that is strictly increasing on the interval $(-\infty, 1]$ and that satisfies

$$\alpha(x) = \begin{cases} x & \text{for } x \leq \frac{3}{4}, \\ 1 & \text{for } x \geq 1. \end{cases}$$

Let $\beta : \mathbb{R} \to [0, 1]$ be a C^∞-function such that

$$\beta(y) = \begin{cases} 0 & \text{for } y = 0, \\ 1 & \text{for } |y| \geq \frac{1}{4}. \end{cases}$$

Define $\sigma : U \to \mathbb{R}$ by $\sigma(x, y) = (1 - \beta(y))\alpha(x) + \beta(y)x$. Then the reader can verify that the map $\Psi : U \to U$ given by

$$\Psi(x, y) = (\sigma(x, y), y)$$

is well-defined (that is, it takes U into U) and satisfies

(1) $\Psi(x, 0) = (1, 0)$ for all $x \geq 1$,

(2) $\Psi(\frac{1}{2}, y) = (\frac{1}{2}, y)$ for all y,

(3) $\Psi(1, y) = (1, y)$ for all y, and

(4) $\Psi(x, y) = (x, y)$ whenever either $x \leq \frac{3}{4}$ or $|y| \geq \frac{1}{4}$.

Regarded as a function of the first variable, σ is strictly increasing on the interval $\left[\frac{1}{2}, 1\right]$ for each fixed y, so (2) and (3) imply that Ψ maps the square $\left[\frac{1}{2}, 1\right] \times \left[-\frac{1}{2}, \frac{1}{2}\right]$ one-to-one onto itself.

Define $\Phi : \varphi(U) \to \mathbb{R}^2$ by $\Phi = \varphi \circ \Psi \circ \varphi^{-1}$. Then by (1), Φ takes the constant value $(1, 0)$ on $\{(x, y) : x \geq 1, y = 0\}$. Also, Φ maps $\varphi(U) \cap \overline{D} = \varphi\left(\left[\frac{1}{2}, 1\right] \times \left[-\frac{1}{2}, \frac{1}{2}\right]\right)$ one-to-one onto itself. Of course Φ is of class C^∞. By (4), outside of the closed subset $\varphi\left(\left[\frac{3}{4}, \infty\right) \times \left[-\frac{1}{4}, \frac{1}{4}\right]\right)$ of $\varphi(U)$, the map Φ is the identity. Hence, Φ can be extended to a C^∞-map on all of $\mathbb{R}^2$ by making Φ the identity outside of $\varphi(U)$. Then Φ takes $\overline{D}$ one-to-one onto itself and satisfies $\Phi(x, 0) = (1, 0)$ for all $x \geq 1$. □

Lemma 2.3. *There exists a C^∞-smooth map $F : \mathbb{R}^2 \to \mathbb{R}^2$ that takes the closed disc $\{(x, y) : (x-2)^2 + y^2 \leq 1\}$ homeomorphically onto $\overline{D}$, takes the closed disc $\{(x, y) : (x-5)^2 + y^2 \leq 1\}$ homeomorphically onto $\overline{D}$ as well, and takes the constant value $(1, 0)$ on the set $\{(x, y) : 3 \leq x \leq 4, y = 0\}$.*

Proof. Let Φ be the map in Lemma 2.2. Define $F_1, F_2 : \mathbb{R}^2 \to \mathbb{R}^2$ by $F_1(x, y) = \Phi(x-2, y)$ and $F_2(x, y) = \Phi(5-x, y)$. Let $\{\varphi_1, \varphi_2\}$ be a C^∞-partition of unity on $\mathbb{R}^2$ subordinate to the cover consisting of $\{(x, y) : x < 3 + \frac{2}{3}\}$ and $\{(x, y) : x > 3 + \frac{1}{3}\}$, and define $F : \mathbb{R}^2 \to \mathbb{R}^2$ by $F = \varphi_1 F_1 + \varphi_2 F_2$. Then F has all the required properties. □

Proof of Theorem 2.1. We divide the proof into steps.

Step 1. We define a certain uniform algebra $\mathfrak{U}$ from which the desired uniform algebra will be obtained, and we determine the maximal ideal space $\mathfrak{M}_{\mathfrak{U}}$ and Gelfand transform for $\mathfrak{U}$.

Let $A(D)$ denote the disc algebra on the disc (the algebra of continuous functions on $\overline{D}$ that are holomorphic on D), and let $R_b(L)$ denote the annulus algebra on ∂L

(the algebra of continuous functions ∂L that have holomorphic extension to the annulus L). Let S be the singular inner function given by

$$S(z) = \exp\Big(\frac{z+1}{z-1}\Big),$$

and let

$$I = \{Sg : g \in A(D) \text{ and } g(1) = 0\}.$$

Then I is a proper closed ideal in $A(D)$. (See [Hoffman 1962, pp. 83–84].)

For uniform algebras F and G on spaces X and Y respectively, we take the tensor product $F \otimes G$ to be the linear span of the functions of the form

$$(f \otimes g)(x, y) = f(x)g(y),$$

with $f \in F$ and $g \in G$. Now let $\mathcal{U}$ be the uniform algebra on $\partial L \times \overline{D}$ generated by $C(\partial L) \otimes I$ and $R_b(L) \otimes A(D)$, or, equivalently, set

$$\mathcal{U} = \overline{(C(\partial L) \otimes I) + (R_b(L) \otimes A(D))}.$$

We now apply the material from [Sidney 1968, p. 135] with $A' = C(\partial L)$, $B = A(D)$, $X = \mathfrak{M}_{A'} = \partial L$, $Y = \mathfrak{M}_B = \overline{D}$, and $A = R_b(L)$. As in that reference, let $\tau : A \to \mathcal{U}$ and $\eta : B \to \mathcal{U}$ be the isometric isomorphisms $\tau(a) = a \otimes 1$ and $\eta(b) = 1 \otimes b$, and define $\pi : \mathfrak{M}_{\mathcal{U}} \to \mathfrak{M}_A \times \mathfrak{M}_B$ by $\pi(\varphi) = (\tau_*(\varphi), \eta_*(\varphi))$, where $\tau_* : \mathfrak{M}_{\mathcal{U}} \to \mathfrak{M}_A$ and $\eta_* : \mathfrak{M}_{\mathcal{U}} \to \mathfrak{M}_B$ are the dual maps. Then by [Sidney 1968, Theorem 3.3], π maps $\mathfrak{M}_{\mathcal{U}}$ homeomorphically onto $(\mathfrak{M}_A \times \text{hull}(I)) \cup (X \times Y)$. Because the function $(z-1)S$ is in I and vanishes at no point of $\overline{D}$ other than 1, the hull of I is $\{1\}$. Thus, since $\mathfrak{M}_A = \overline{L}$, we get that the maximal ideal space of $\mathcal{U}$ can be identified with

$$(\overline{L} \times \{1\}) \cup ((\partial L) \times \overline{D}) \subset \mathbb{C}^2.$$

Under the identification, each point of $(\partial L) \times \overline{D}$ is identified with the corresponding point evaluation functional, so the Gelfand transform of a function $f \in \mathcal{U}$ gives an extension of f to the annulus $L \times \{1\}$. The reader can check that this extension is the holomorphic extension of the annulus algebra function $x \mapsto f(x, 1)$ to L.

Step 2. We show that $\mathcal{U}$ is nonlocal by showing that the function

$$h \in C\big((\overline{L} \times \{1\}) \cup ((\partial L) \times \overline{D})\big),$$

given by

$$h(w, z) = \begin{cases} 0 & \text{for } (w, z) \in (\overline{L} \times \{1\}) \cup (\{|w| = 4\} \times \overline{D}), \\ z - 1 & \text{for } (w, z) \in (\overline{L} \times \{1\}) \cup (\{|w| = 3\} \times \overline{D}), \end{cases}$$

is locally in $\mathcal{U}$ but not in $\mathcal{U}$.

That h is locally in $\mathcal{U}$ is clear because the interiors (relative to $\mathfrak{M}_{\mathcal{U}}$) of the sets $(\overline{L} \times \{1\}) \cup (\{|w| = 4\} \times \overline{D})$ and $(\overline{L} \times \{1\}) \cup (\{|w| = 3\} \times \overline{D})$ cover $\mathfrak{M}_{\mathcal{U}}$ and the functions 0 and $z-1$ each lie in $\mathfrak{A}$. To show that h is not in $\mathfrak{A}$, we exhibit a measure on $(\partial L)\times\overline{D}$ that annihilates $\mathfrak{A}$ but does not annihilate h. Because the function that is 1 on the circle $\{|w|=3\}$ and 0 on the circle $\{|w|=4\}$ is not in $R_b(L)$, there is a measure μ on ∂L that annihilates $R_b(L)$ but does not annihilate this function. Also, because the function $z-1$ is not in the ideal I, there is a measure ν on $\overline{D}$ that annihilates I but such that $\int_{\overline{D}}(z-1)\,d\nu(z)=1$. Now for $f\in C(\partial L)$ and $g\in I$,

$$\int_{(\partial L)\times\overline{D}} f(w)g(z)\,d(\mu\times\nu)(w,z)=\int_{\partial R} f\,d\mu\cdot\int_{\overline{D}} g\,d\nu=0,$$

and the same equation holds also for $f\in R_b(L)$ and $g\in A(D)$. Consequently, $\mu\times\nu$ annihilates $\overline{(C(\partial L)\otimes I)+(R_b(L)\otimes A(D))}=\mathfrak{A}$. However,

$$\begin{aligned}\int_{(\partial L)\times\overline{D}} & h(w,z)\,d(\mu\times\nu)(w,z)\\ &=\int_{\{|w|=3\}\times\overline{D}} h(w,z)\,d(\mu\times\nu)(w,z)+\int_{\{|w|=4\}\times\overline{D}} h(w,z)\,d(\mu\times\nu)(w,z)\\ &=\int_{\{|w|=3\}}\int_{\overline{D}}(z-1)\,d\nu(z)\,d\mu(w)+\int_{\{|w|=4\}}\int_{\overline{D}}0\,d\nu(z)\,d\mu(w)\\ &=\int_{\{|w|=3\}}1\,d\mu+\int_{\{|w|=4\}}0\,d\mu\neq 0.\end{aligned}$$

Thus h is not in $\mathfrak{A}$.

Step 3. We show that there is a dense set of functions in I that extend to C^∞-functions on $\mathbb{C}$.

Let $A^\infty(D)$ denote the algebra of functions in $A(D)$ whose complex derivatives to all orders also lie in $A(D)$. The boundary function of each function in $A^\infty(D)$ belongs to $C^\infty(\partial D)$, and $A^\infty(D)$ is a topological algebra with the topology induced by $C^\infty(\partial D)$. Each function in $A^\infty(D)$ extends to a C^∞-function on $\mathbb{C}$. (This follows from Whitney's extension theorem [Boggess 1991, Theorem 2 in Section 5.3].)

Let $J=\{Sg : g\in A^\infty(D) \text{ and } Sg\in A^\infty(D)\}$. Note that J is an ideal in $A^\infty(D)$. Consider the closure $\bar{J}$ of J in the disc algebra $A(D)$. One easily checks that $\bar{J}$ is a (closed) ideal in $A(D)$. By [Taylor and Williams 1970, Theorem 3.3], there is an outer function h in $A^\infty(D)$ with $h^{(n)}(1)=0$ for all $n=0,1,2,\ldots$ (that is, vanishing to infinite order at 1) with no other zeros on $\overline{D}$. Then by [Taylor and Williams 1970, Theorem 4.7], the function Sh is in J. Because Sh vanishes only at $z=1$, we conclude from the Beurling–Rudin theorem [Rudin 1957] (or see [Hoffman 1962, pp. 82–89] that

$$\bar{J}=\left\{g\exp\!\left(r\,\frac{z+1}{z-1}\right) : g\in A(D) \text{ and } g(1)=0\right\}$$

for some $r \geq 0$. Obviously

$$\bar{J} \subset \left\{ g \exp\left(\frac{z+1}{z-1}\right) : g \in A(D) \text{ and } g(1) = 0 \right\},$$

so $r \geq 1$. Because h is outer, Sh is not of the form $g \exp[r(z+1)/(z-1)]$ for any $r > 1$. Thus $r = 1$, that is, $\bar{J} = I$. Because the functions in J extend to C^∞-functions on $\mathbb{C}$, this completes Step 3.

Step 4. We show that there is a dense set of functions in $\mathcal{U}$, regarded as a uniform algebra on $\mathfrak{M}_{\mathcal{U}} = (\overline{L} \times \{1\}) \cup ((\partial L) \times \overline{D})$, that extend to C^∞-functions on $\mathbb{C}^2$.

Recall that $\mathcal{U}$, as a uniform algebra on $(\partial L) \times \overline{D}$, is the closed linear span of the functions of the form $(f \otimes g)(x, y) = f(x)g(y)$, where either $f \in R_b(L)$ and $g \in A(D)$, or else $f \in C(\partial L)$ and $g \in I$. Thus, it suffices to show that each of these functions $f \otimes g$ can be uniformly approximated on $(\partial L) \times \overline{D}$ by functions C^∞ on $\mathbb{C}^2$ that lie in $\mathcal{U}$ on $\mathfrak{M}_{\mathcal{U}}$. For $f \in R_b(L)$ and $g \in A(D)$, we trivially obtain sequences (f_n) and (g_n) of functions C^∞ on $\mathbb{C}$ with $f_n|\overline{L} \in R(\overline{L})$ and $g_n|\overline{D} \in A(D)$ such that $f_n \to f$ and $g_n \to g$ uniformly on ∂L and $\overline{D}$ respectively. Then $f_n \otimes g_n$ gives the required approximation of $f \otimes g$. For $f \in C(\partial L)$, there is a sequence (f_n) of functions C^∞ on $\mathbb{C}$ with $f_n \to f$ uniformly on ∂L, and for $g \in I$, there is, by Step 3, a sequence (g_n) of functions C^∞ on $\mathbb{C}$ with $g_n|\overline{D} \in I$ and $g_n \to g$ uniformly on $\overline{D}$. Then $f_n \otimes g_n$ is of course C^∞ on $\mathbb{C}$ and in $\mathcal{U}$ on $(\partial L) \times \overline{D}$, and $f_n \otimes g_n \to f \otimes g$ uniformly there. Also, because $g_n(1) = 0$, we have $f_n \otimes g_n = 0$ on $\overline{L} \times \{1\}$, so $f_n \otimes g_n$ is in $\mathcal{U}$ on $\mathfrak{M}_{\mathcal{U}}$.

Step 5. We show that there is a C^∞-map $G : \mathbb{R}^3 \setminus \{x_1 = x_2 = 0\} \to \mathbb{C}^2$ that takes some subset K of $\mathbb{R}^3$ homeomorphically onto $\mathfrak{M}_{\mathcal{U}} = (\overline{L} \times \{1\}) \cup ((\partial L) \times \overline{D})$.

Let $K = K_L \cup K_1 \cup K_2$, where

$$K_L = \left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 : 3 \leq \sqrt{x_1^2 + x_2^2} \leq 4,\ x_3 = 0 \right\},$$
$$K_1 = \left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 : \left(\sqrt{x_1^2 + x_2^2} - 2\right)^2 + x_3^2 \leq 1 \right\}, \text{ and}$$
$$K_2 = \left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 : \left(\sqrt{x_1^2 + x_2^2} - 5\right)^2 + x_3^2 \leq 1 \right\}.$$

Let $\rho : \mathbb{R} \to \mathbb{R}$ be a C^∞-function such that $\rho([1, 3]) = \{3\}$, on the interval $[3, 4]$ the function ρ strictly increases from 3 to 4, and $\rho([4, 6]) = \{4\}$. Let F be the map in Lemma 2.3 and define $G : \mathbb{R}^3 \setminus \{x_1 = x_2 = 0\} \to \mathbb{C}^2$ by

$$G(x_1, x_2, x_3) = \left(\rho\left(\sqrt{x_1^2 + x_2^2}\right) \frac{(x_1 + x_2 i)}{\sqrt{x_1^2 + x_2^2}},\ F\left(\sqrt{x_1^2 + x_2^2}, x_3\right) \right).$$

Clearly, G is of class C^∞. One checks easily that G takes K_L one-to-one onto $\overline{L} \times \{1\}$, takes K_1 one-to-one onto $\{|z| = 3\} \times \overline{D}$, and takes K_2 one-to-one onto $\{|z| = 4\} \times \overline{D}$. Thus, G takes K onto $\mathfrak{M}_{\mathcal{U}}$, and a little more thought shows that G

is one-to-one on K. Since K is compact, we conclude that G takes K homeomorphically onto $\mathfrak{M}_{\mathcal{U}}$.

Step 6. We complete the proof of the theorem.

The uniform algebra $B = \{f \circ G : f \in \mathcal{U}\}$ on K is clearly isomorphic to $\mathcal{U}$. By Step 2, B is nonlocal, and by Step 4, B is generated by functions that extend to be C^∞ on $\mathbb{R}^3 \setminus \{x_1 = x_2 = 0\}$.

Given an arbitrary compact C^∞-manifold M of dimension greater than or equal to 3, choose an embedding of a neighborhood of K into M, identify K with its image in M, and define a uniform algebra A on M by taking all continuous functions on M whose restrictions to K lie in B. Then the maximal ideal space of A is M by [Bear 1959, Theorem 4], A is nonlocal by [Izzo 2010, Lemma 2.5], and A is generated by C^∞-smooth functions by [Izzo 2009, Lemma 2.1]. □

Acknowledgement

I would like to thank Stuart Sidney for explaining to me his approach to nonlocal uniform algebras, without which I would not have been able to construct the examples given here.

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Received August 7, 2011. Revised November 29, 2011.

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PACIFIC JOURNAL OF MATHEMATICS

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Founded in 1951 by
E. F. Beckenbach (1906–1982) and F. Wolf (1904–1989)

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The Pacific Journal of Mathematics (ISSN 0030-8730) at the University of California, c/o Department of Mathematics, 969 Evans Hall, Berkeley, CA 94720-3840, is published monthly except July and August. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163.

PJM peer review and production are managed by EditFlow™ from Mathematical Sciences Publishers.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS
at the University of California, Berkeley 94720-3840
A NON-PROFIT CORPORATION
Typeset in LaTeX

PACIFIC JOURNAL OF MATHEMATICS

Volume 259 No. 1 September 2012

0030-8730(201209)259:1;1-1