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For a fibered link of tunnel number one in S^3, with fiber F and unknotting tunnel τ, we show that τ can be isotoped to lie in F.

1. Introduction and motivation

The study of fibered knots and links is as important today as ever. Giroux's correspondence [2002] between open book decompositions and contact structures mingles classical fibered links with more modern contact geometry. Sutured manifold theory continues to reveal information about fibrations (see, for instance, [Ni 2009; Scharlemann and Thompson 2009]). And fibered links are related to the newest advances in Floer homology, as knot Floer homology detects fibered links [Ni 2007] and sutured Floer homology intersects both contact geometry and sutured manifold theory.

Tunnel number one links are among the most studied links. Much of the work on tunnel number one links revolves around trying to isotope the tunnel to sit nicely with respect to some additional structure in the 3-manifold, including a hyperbolic metric [Adams 1995; Adams and Reid 1996; Akiyoshi et al. 1997; Cooper et al. 2010], polyhedral decompositions [Sakuma and Weeks 1995; Heath and Song 2005], bridge decompositions [Goda et al. 2000; Lackenby 2005], Seifert surfaces [Scharlemann and Thompson 2003], and fibrations [Sakuma 1996]. These studies, and others, have led to the classification of tunnels for many classes of knots and links, including torus knots [Boileau et al. 1988], satellite knots [Morimoto and Sakuma 1991], nonsimple links [Eudave Muñoz and Uchida 1996], 2-bridge knots [Morimoto and Sakuma 1991; Kobayashi 1999], and 2-bridge links [Morimoto 1994; Jones 1995].

Further, the Berge conjecture states that if a knot admits a lens space Dehn surgery, then it is in one of the families of knots classified by John Berge. Many are working on this long-standing conjecture, with recent progress contributed by Ozsváth and Szabó [2005], Hedden [2007], Baker, Grigsby, and Hedden [2008],

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Saito [2007], and Williams [2007], among others. Yi Ni [2007] recently proved that if a knot admits a lens space surgery, then it is a fibered knot. Additionally, all Berge knots are both fibered and tunnel number one, so further understanding of tunnel one, fibered knots could have profound impacts on the conjecture.

Jesse Johnson [2008] investigated genus-2 Heegaard splittings of closed surface bundles over the circle. This paper looks at the relationship between unknotting tunnels and fibrations for link complements.

Theorem 1.1. *Let K be an oriented, fibered, tunnel number one link in S^3, with fiber F, and unknotting tunnel τ. Then τ can be isotoped to lie in F.*

2. Background and definitions

3-*manifolds.*

Notation 1. Let A be subset of a 3-manifold M. We fix some notation. Let $n(A)$ denote a small open neighborhood of A in M. If F is a properly embedded surface in M, let $M|F = M \setminus n(F)$. If S is the boundary of M, we will refer to $S|\partial F = S \setminus n(\partial F)$. For convenience, we will also sometimes refer to this as $S|F$.

Definition 2.1. Let F be a surface properly embedded in a 3-manifold M. Then F is said to be *compressible* if there exists a disk D embedded in M with $\partial D = D \cap F$ an essential curve in F, and D is called a *compressing disk* for F. If F is not compressible, and is not a 2-sphere, then it is called *incompressible*. The surface F is said to be *boundary compressible* if there exists a disk D embedded in M with $D \cap F = \alpha \subset \partial D$, $D \cap \partial M = \beta \subset \partial D$, where α is an essential arc in F, $\alpha \cap \beta = \partial\alpha = \partial\beta$, and $\alpha \cup \beta = \partial D$. In this case, D is called a *boundary compression disk*. If F is not boundary compressible, it is called *boundary incompressible*.

Definition 2.2. A *compression body* V is the result of taking the product of a surface with [0, 1], attaching 2-handles along $S \times \{0\}$, and then attaching 3-handles along any resulting 2-sphere components. The surface $S \times \{1\}$ is called $\partial_+ V$, and $\partial V \setminus \partial_+ V$ is called $\partial_- V$. A *handlebody* is a compression body where $\partial_- V = \varnothing$. A *Heegaard splitting* is a triple (S, V, W), where S is a surface, V and W are compression bodies, $\partial_+ V = \partial_+ W = S$, and $M = V \cup_S W$.

Definition 2.3. Let K be a knot in a 3-manifold M, and let λ be an essential closed curve in $\partial n(K)$. Let M' be the manifold obtained from M by removing $n(K)$, and attaching a solid torus $S^1 \times D^2$ to $M \setminus n(K)$ via a homeomorphism of the boundaries such that $\{\text{pt.}\} \times \partial D^2$ is identified with the curve λ. Then M' is said to be the result of λ-*sloped Dehn surgery* on M.

Tunnels.

Definition 2.4. A link L in S^3 is called a *tunnel number one* link if there exists an arc τ properly embedded in $S^3 \setminus n(L)$ such that $S^3 \setminus n(L \cup \tau)$ is a handlebody. Then τ is called a *tunnel* for L.

Observe that the complement of a tunnel number one link has a genus-2 Heegaard splitting. Also, note that a tunnel one link has at most two components, and if it has two components, then any tunnel must have one endpoint on each component.

More generally, a knot is *tunnel number n* if n is the smallest number such that there exists a collection of arcs $\{\tau_1, \dots, \tau_n\}$ such that $S^3 \setminus n(L \cup \tau_1 \cup \cdots \cup \tau_n)$ is a handlebody.

Fibered links.

Definition 2.5. Let $L \subset S^3$ be a link. A *Seifert surface* for L is a compact, orientable surface F embedded in S^3 with no closed components such that $\partial F = L$.

Definition 2.6. A map $f : E \to B$ is a *fibration* with *fiber* F if for every point $p \in B$, there is a neighborhood U of p and a homeomorphism $h : f^{-1}(U) \to U \times F$ such that $f = \pi_1 \circ h$, where $\pi_1 : U \times F \to U$ is projection to the first factor. The space E is called the *total* space, and B is called the *base* space. Each set $f^{-1}(b)$ is called a *fiber*, and is homeomorphic to F.

Definition 2.7. A link $L \subset S^3$ is said to be *fibered* if there is a fibration of $S^3 \setminus n(L)$ over S^1, and the fibration is well-behaved near L. That is, each component L_i of L has a neighborhood $S^1 \times D^2$, with $L_i \cong S^1 \times \{0\}$ such that $f\big|_{S^1 \times (D^2 \setminus \{0\})}$ is given by $(x, y) \to y/|y|$.

Each fiber of a fibered link is a Seifert surface for the link. The complement of a fibered link is foliated by copies of this Seifert surface. Cutting along one of these Seifert surfaces produces a surface cross the interval.

Definition 2.8. Let K be a fibered link in S^3. Then $S^3 \setminus n(K)$ can be obtained from $F \times I$, with F a fiber, by identification $(x, 0) \sim (h(x), 1)$, for $x \in F$, where $h : F \to F$ is an orientation-preserving homeomorphism which is the identity on ∂F. We call h a *monodromy* map.

Theorems. Our starting point is the following theorem.

Theorem 2.9 [Scharlemann and Thompson 2003]. *Suppose K is a knot in S^3, and τ an unknotting tunnel for K. Then τ may be slid and isotoped until it is disjoint from some minimal-genus Seifert surface for K.*

The proof consists of arranging K, τ, and a compressing disk for $S^3 \setminus n(K \cup \tau)$ in some minimal fashion, and showing that if $K \cap \tau \neq \varnothing$, this would lead to a contradiction with those minimality assumptions. The result still holds for two-component fibered links.

Theorem 2.10. *Suppose K is an oriented, fibered link, and τ is an unknotting tunnel for K. Then τ may be slid and isotoped until it is disjoint from a fiber of K.*

Our proof will largely mimic [Scharlemann and Thompson 2003].

Proof. By Theorem 2.9, if K has just one component, then an unknotting tunnel can be isotoped and slid to be disjoint from a minimal-genus Seifert surface. But in a fibered knot complement, a fiber is the unique minimal-genus Seifert surface, so the result follows. Henceforth, let us assume that K is a two-component link, and let the two components of K be K_1 and K_2. Observe that τ has one endpoint on each of the components of K. Choose a fiber F, and slide and isotope τ, so as to minimize the number of intersections between τ and F. Our goal will be to prove that $\tau \cap F = \varnothing$.

Suppose, to the contrary, that after the slides and isotopies above, $\tau \cap F$ is nonempty. Let E be an essential disk in the handlebody $S^3 \setminus n(K \cup \tau)$, chosen to minimize the number $|E \cap F|$ of components in $E \cap F$. If $|E \cap F| = 0$, then the incompressible F would lie in a solid torus, namely (a component of) $S^3 \setminus n(K \cup \tau \cup E)$, and so be an annulus. The only fibered link with fiber an annulus is the Hopf link, in which case the result holds. So we may assume that $|E \cap F| > 0$. Furthermore, since F is incompressible, we may assume that $E \cap F$ consists entirely of arcs.

Let e be an outermost arc of $E \cap F$ in E, cutting off a subdisk E_0 from E. If e were inessential in $F \setminus \tau$, then we could surger E along the trivial subdisk cut off by e. The result would be two disks, at least one of which is also essential in $S^3 \setminus n(K \cup \tau)$, but with one fewer intersection with F, contradicting our assumption of minimality. Thus, the arc e is essential in $F \setminus \tau$. Let $f = \partial(E_0) \setminus e$, an arc in $\partial n(K \cup \tau)$ with each end either on a longitude $\partial F \subset \partial n(K)$ or a meridian disk of τ corresponding to a point of $\tau \cap F$.

Now, either no meridian of τ is incident to an end of f, a meridian of τ is incident to exactly one end of f, or there is a meridian which is incident to both ends of f.

(1) If no meridian of τ is incident to an end of f, then both ends of f lie on $\partial F \subset \partial n(K)$. If the interior of f runs over τ, we have finished, for f is disjoint from F. Otherwise, the interior of f lies entirely in $\partial n(K)$, and e is either essential in F, or it is inessential.

   (a) If e is essential in F, then E_0 would be a boundary compression disk for F, contradicting the minimality of the genus of F.

(b) If e is inessential in F, then the disk D_0 that it cuts off from F necessarily contains points of τ (since e is essential in $F \setminus \tau$). But then we could replace D_0 by E_0, and the loop formed by f and $\partial D_0 \setminus e$ is either a trivial loop on one of the torus components of $\partial n(K)$, or it is an essential loop.

(i) If the loop formed by f and $\partial D_0 \setminus e$ is a trivial loop on the torus, say, $\partial n(K_1)$, then the new surface would, again, be a Seifert surface for K, consistent with the orientation of K (and so be a fiber), but with fewer points of intersection $F \cap \tau$.

(ii) If the loop formed by f and $\partial D_0 \setminus e$ is essential in $\partial n(K_1)$, then the original disk E_0 could be slid across D_0 to show that K_1 is unknotted. But the interior of the disk D_0 is disjoint from K_2, so K must be a split link, and split links do not fiber.

(2) If a meridian of τ is incident to exactly one end of f, then we can use E_0 to describe a simple isotopy of τ by sliding τ along E_0 which reduces the number of intersections between τ and F.

(3) If both ends of f lie on the same meridian of τ, then e forms a loop in F, and the ends of f adjacent to e both run along the same subarc τ_0 of τ. Since f is disjoint from F, τ_0 terminates on, say, $\partial n(K_1)$.

Then since the interior of f is disjoint from F, f must intersect $\partial n(K_1)$ either in an inessential arc in the torus or in a longitudinal arc. That is, if $\tau_0 \cap \partial n(K_1)$ were collapsed to a point p, then f would either represent a trivial loop in $\pi_1(\partial n(K_1), p)$, or a nontrivial element. The former case is impossible, because the trivial disk cut off by f cannot contain the other end of τ (since the other end of τ is on $\partial n(K_2)$). Thus, the disk could be isotoped away, reducing $|E \cap F|$. It follows that f intersects the torus $\partial n(K_1)$ in a longitudinal arc. Then, $n(\tau_0 \cup E_0)$ is a thickened annulus A, defining a parallelism in S^3 between K_1 and the loop e on F. Now, the boundary component of A on $\partial n(K_1)$ can be slid across $\partial n(K_1)$, away from e, onto F, parallel to ∂F in F. Since K is a fibered link, the image of A, call it A', is a product annulus in $S^3 \setminus n(K \cup F) \cong F \times I$. But then this demonstrates that e itself is parallel to ∂F in F. Then, substituting A for the annulus between e and ∂F in F would create a Seifert surface of the same genus, still consistent with the orientation of K, and thus a fiber, but with fewer intersections with τ.

In all cases, we obtain contradictions, and conclude that τ and F can be arranged to be disjoint. □

Another theorem that we will find useful is also given by Scharlemann and Thompson. Ni [2009] proves a more general result, though we will not need it here.

Theorem 2.11 [Scharlemann and Thompson 2009]. *Suppose F is a compact orientable surface, L is a knot in $F \times I$, and $(F \times I)_{\text{surg}}$ is the 3-manifold obtained by some nontrivial surgery on L. If $F \times \{0\}$ compresses in $(F \times I)_{\text{surg}}$, then L is parallel to an essential simple closed curve in $F \times \{0\}$. Moreover, the annulus that describes the parallelism determines the slope of the surgery.*

The proof relies on sutured manifold theory, and a theorem of Gabai [1989]. Gabai proves the result for an annulus cross the interval. The idea of Scharlemann and Thompson's proof is to find product disks or annuli in $(F \times I)$ disjoint from the knot, and cut along these product pieces to reduce the complexity of the surface in question. This, with some additional work, allows them to apply the results of Gabai.

3. Pushing a tunnel into a fiber

Proof of Theorem 1.1. By Theorem 2.10, τ can be isotoped and slid to be disjoint from a fiber. Let $F = F' \setminus n(K)$. Cut $S^3 \setminus n(K)$ along F, to obtain $N \cong F \times I$, a handlebody. Then $\tau \subset N$.

Now, as τ is an unknotting tunnel, there exists a compressing disk for $\partial n(K \cup \tau)$ in $S^3 \setminus n(K \cup \tau)$, say D'. Note that $D' \cap F \neq \varnothing$, for otherwise F would be an essential surface in the solid torus $(S^3 \setminus n(K \cup \tau))|D'$, and thus a disk.

Consider $D' \cap F$. Since F is incompressible and N is irreducible, by standard innermost disk arguments we may assume there are no simple closed curves of intersection. Let α be an arc of intersection which is outermost in D', cutting off a subdisk D. Then, D is a disk in N with boundary consisting of three types of arcs: a single essential arc in $F = F \times \{0\}$, α; (several) arcs in $\partial n(K)$, call them ν_i; and (several) arcs in $\partial n(\tau)$, λ_j. We may assume that every arc of $D \cap \partial n(\tau)$ is an essential spanning arc of the annulus $\partial n(\tau)$, for trivial arcs can be removed by isotopy.

Now, consider the double of N, along the vertical boundary $\partial F \times I$. In other words, let $\widehat{N}$ be the result of gluing two copies of N together by the identity along $\partial F \times I$. Similarly, let $\widehat{\tau}$ be the result of gluing two copies of τ, one in each copy of N, along the boundary points; let $\widehat{D}$ come from two copies of D, one in each copy of N, glued along the ν_i; and let $\widehat{\alpha}$ come from two copies of α in the same way.

Then $\widehat{D}$ is a planar surface with one boundary component corresponding to $\widehat{\alpha}$, and several components coming from $\widehat{\lambda}_j$, the doubles of λ_j (see Figure 1).

Then, $\bigcup_j \widehat{\lambda}_j$ is a collection of (parallel) simple closed curves on the torus $\partial n(\widehat{\tau})$. Call the slope determined by these curves λ. If we perform λ-surgery on $\widehat{\tau}$, the result is to cap off $\widehat{D}$ with disks. Since α was essential in F, $\widehat{\alpha}$ is essential in $\widehat{F}$, so our capped-off surface is a compression disk for $\widehat{F}$ in $\widehat{N} \cong \widehat{F} \times I$.

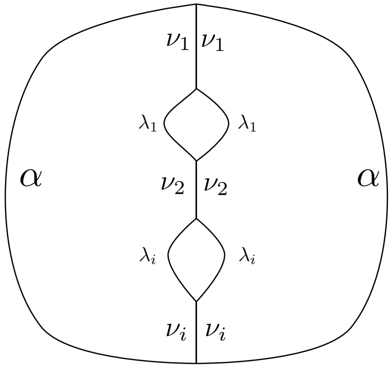

Figure 1. $\widehat{D}$.

By Theorem 2.11, $\widehat{\tau}$ is parallel to an essential closed curve in $\widehat{F} \times \{0\}$. That is, there exists an annulus A properly embedded in $\widehat{F} \times I$ with one boundary component on $\widehat{F} \times \{0\}$, say ψ, and the other boundary component on $\partial n(\widehat{\tau})$, parallel to $\widehat{\tau}$, say ϕ.

Since ϕ is parallel to $\widehat{\tau}$, it must be a longitude of $\partial n(\widehat{\tau})$, and in particular, $|\phi \cap (\partial F \times I)| = 2$. So there are only two possibilities for arcs of intersection between A and $\partial F \times I$ incident to ϕ. Either there is one arc of intersection which is trivial in A, or there are two arcs of intersection, both of which are essential in A (see Figure 2). The former case is impossible, because then the subdisk of A cut off by the arc would show that τ was parallel into $\partial n(K)$, which would imply that K was trivial. Therefore, there are exactly two arcs of $A \cap (\partial F \times I)$, both of which are essential in A.

If there were trivial arcs incident to ψ, then an outermost such arc in A would give rise to a boundary compression for $F \times \{0\}$ in $S^3 \setminus n(K)$. This is impossible as well, so $\partial F \times I$ intersects A in precisely two essential arcs, with no trivial arcs. Cutting A along these arcs provides a parallelism between τ and the arc $\psi \cap (F \times \{0\}) \subset \widehat{F} \times \{0\}$. Thus, τ can be isotoped to lie in the fiber. □

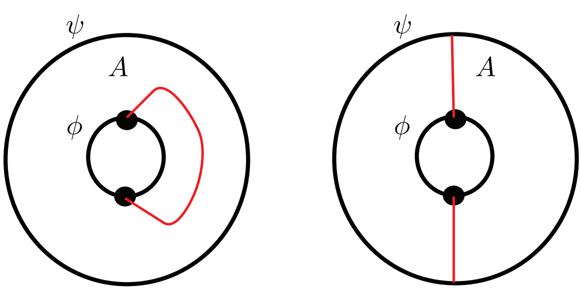

Figure 2. Arcs of $A \cap \partial F \times I$ incident to ϕ.

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