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IDEAL DECOMPOSITIONS OF A TERNARY RING OF OPERATORS WITH PREDUAL

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We show that any TRO (ternary ring of operators) with predual can be decomposed into the direct sum of a two-sided ideal, a left ideal, and a right ideal in some von Neumann algebra using an extreme point of the unit ball of the TRO.

Recall that an operator space X is called a *triple system* or a *ternary ring of operators* (*TRO* for short) if there exists a complete isometry ι from X into a C^*-algebra such that $\iota(x)\iota(y)^*\iota(z) \in \iota(X)$ for all $x, y, z \in X$. Our main result is that any TRO with predual can be decomposed into the direct sum of a two-sided ideal, a left ideal, and a right ideal in some von Neumann algebra:

Theorem. *Let X be a TRO which is also a dual Banach space. Then X can be decomposed into the direct sum of TROs X_T, X_L, and X_R,*

$$X = X_T \overset{\infty}{\oplus} X_L \overset{\infty}{\oplus} X_R,$$

so that there is a complete isometry ι from X into a von Neumann algebra in which $\iota(X_T)$, $\iota(X_L)$, and $\iota(X_R)$ are a weak-closed two-sided, left, and right ideal, respectively, and*

$$\iota(X) = \iota(X_T) \overset{\infty}{\oplus} \iota(X_L) \overset{\infty}{\oplus} \iota(X_R).$$

In the special case that the TRO is finite-dimensional, the decomposition is into a direct sum of rectangular matrices, as first proved essentially by R. R. Smith [2000]. In the Appendix we give a short proof of that result. The following lemma is a version of Kadison's theorem [1951, Theorem 1] as found in [Pedersen 1979, Proposition 1.4.8] or [Sakai 1971, Proposition 1.6.5]. Together with the idea of embedding an off-diagonal corner into a diagonal corner developed in [Blecher and Kaneda 2004, Section 2] (see also [Kaneda 2003, Section 2.2]), it plays a key role in the proof of our theorem.

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Lemma (Kadison's theorem). *Let $\mathcal{A}$ be a C^*-algebra, and let p, q be orthogonal projections in $\mathcal{A}$. Then an element $x \in p\mathcal{A}q$ is an extreme point of* $\operatorname{Ball}(p\mathcal{A}q)$ *if and only if $(p - xx^*)\mathcal{A}(q - x^*x) = \{0\}$. In this case, x is a partial isometry.*

Proof of the Theorem. By [Effros et al. 2001, Theorem 2.6], we may regard X as a weak*-closed subspace of $\mathbb{B}(\mathcal{K}, \mathcal{H})$ for some Hilbert spaces $\mathcal{H}$ and $\mathcal{K}$ such that $XX^*X \subset X$. We may assume that $[X\mathcal{K}] = \mathcal{H}$ and $[X^*\mathcal{H}] = \mathcal{K}$. We also identify $\mathbb{B}(\mathcal{K}, \mathcal{H})$ with the $(1, 2)$-corner of $\mathbb{B}(\mathcal{H} \oplus \mathcal{K})$, and let $1_{\mathcal{H}} \in \mathbb{B}(\mathcal{H} \oplus \mathcal{K})$ and $1_{\mathcal{K}} \in \mathbb{B}(\mathcal{H} \oplus \mathcal{K})$ denote the orthogonal projections on $\mathcal{H}$ and $\mathcal{K}$. Then

$$\mathcal{L}(X) := \begin{bmatrix} \overline{XX^*}^{\mathrm{w}^*} & X \\ X^* & \overline{X^*X}^{\mathrm{w}^*} \end{bmatrix}$$

is the linking von Neumann algebra, $1_{\mathcal{H}}, 1_{\mathcal{K}} \in \mathcal{L}(X)$, and $X = 1_{\mathcal{H}}\mathcal{L}(X)1_{\mathcal{K}}$. Since $\operatorname{Ball}(X)$ is weak*-closed in $\mathbb{B}(\mathcal{K}, \mathcal{H})$, there is an extreme point $e \in \operatorname{Ball}(X)$. By Kadison's theorem above,

$$(1_{\mathcal{H}} - ee^*)X(1_{\mathcal{K}} - e^*e) = \{0\}, \tag{1}$$

and e is a partial isometry. Let $p \in \overline{X(1_{\mathcal{K}} - e^*e)X^*}^{\mathrm{w}^*}$ and $q \in \overline{X^*(1_{\mathcal{H}} - ee^*)X}^{\mathrm{w}^*}$ be the identities of these two von Neumann algebras. Then by the adjoint of (1), it follows that

$$pXq = \{0\}, \tag{2}$$

$$p = pee^* = ee^*p = pee^*p \quad \text{and} \quad q = e^*eq = qe^*e = qe^*eq. \tag{3}$$

Noting that $pxy^* \in \overline{X(1_{\mathcal{K}} - e^*e)X^*}^{\mathrm{w}^*}$ and $qx^*y \in \overline{X^*(1_{\mathcal{H}} - ee^*)X}^{\mathrm{w}^*}$, we also get

$$pxy^* = pxy^*p = xy^*p \quad \text{and} \quad qx^*y = qx^*yq = x^*yq \quad \text{for all } x, y \in X. \tag{4}$$

Put

$$q_1 := e^*(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q) \quad \text{and} \quad q_2 := 1_{\mathcal{K}} - q - q_1.$$

We claim that q_1 and q_2 are orthogonal projections. Indeed, (4) and the fact that $pe \in X$ yield

$$\begin{aligned} q_1^* &= (1_{\mathcal{K}} - q)e^*(1_{\mathcal{H}} - p)e = e^*e - e^*pe - qe^*e + qe^*pe \\ &= e^*e - e^*pe - e^*eq + e^*peq = q_1 \end{aligned}$$

and

$$\begin{aligned} q_1^2 &= e^*(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q)e^*(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q) = e^*(1_{\mathcal{H}} - p)eq_1^*(1_{\mathcal{K}} - q) \\ &= e^*(1_{\mathcal{H}} - p)eq_1(1_{\mathcal{K}} - q) = e^*(1_{\mathcal{H}} - p)ee^*(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q)(1_{\mathcal{K}} - q) \\ &= e^*ee^*(1_{\mathcal{H}} - p)(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q)(1_{\mathcal{K}} - q) = e^*(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q) \\ &= q_1. \end{aligned}$$

Noting that $q_1 q = 0$, we have $q_2^2 = q_2 = q_2^*$.

To see that

$$(1_{\mathcal{H}} - p)X(1_{\mathcal{K}} - e^*e) = \{0\}, \tag{5}$$

let $\{u_\alpha\}$ be an approximate identity of the C^*-algebra X^*X. Then for each $x \in X$, $px(1_{\mathcal{K}} - e^*e)u_\alpha = x(1_{\mathcal{K}} - e^*e)u_\alpha$. Taking the limit $\alpha \to \infty$ yields that

$$px(1_{\mathcal{K}} - e^*e) = x(1_{\mathcal{K}} - e^*e)$$

for $x \in X$, and hence (5) holds. Similarly,

$$(1_{\mathcal{H}} - ee^*)X(1_{\mathcal{K}} - q) = \{0\} \tag{6}$$

also holds.

Let $x, y \in X$. Then

$$\begin{aligned} q_1 x^* y &= e^*(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q)x^* y \\ &= e^*(1_{\mathcal{H}} - p)ex^* y(1_{\mathcal{K}} - q) && \text{by (4)} \\ &= e^*ex^*(1_{\mathcal{H}} - p)y(1_{\mathcal{K}} - q) && \text{by (4)} \\ &= x^*(1_{\mathcal{H}} - p)y(1_{\mathcal{K}} - q) && \text{by the adjoint of (5)} \\ &= x^*(1_{\mathcal{H}} - p)ye^*e(1_{\mathcal{K}} - q) && \text{by (5)} \\ &= x^* y e^*(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q) && \text{by (4)} \\ &= x^* y q_1, \end{aligned}$$

and so we have

$$q_1 x^* y = x^* y q_1 = q_1 x^* y q_1 \quad \text{for all } x, y \in X. \tag{7}$$

Put $X_T := Xq_1$, $X_L := Xq$, and $X_R := Xq_2$. Then these are weak*-closed TROs, and $X = X_T \oplus X_L \oplus X_R$. Using (4) and (7) and noting that q_1, q, and q_2 are mutually disjoint, we have

$$X_T^* X_L = X_T^* X_R = X_L^* X_T = X_L^* X_R = X_R^* X_T = X_R^* X_L = \{0\}$$

and

$$X^* X = X_T^* X_T \overset{\infty}{\oplus} X_L^* X_L \overset{\infty}{\oplus} X_R^* X_R.$$

This proves that $X = X_T \overset{\infty}{\oplus} X_L \overset{\infty}{\oplus} X_R$.

Define

$$\iota : X \to \overline{XX^*}^{\mathrm{w}^*} \overset{\infty}{\oplus} \overline{X^*X}^{\mathrm{w}^*}$$

by

$$\iota(x) := (x_T + x_L)e^* \oplus e^* x_R,$$

where $x = x_T + x_L + x_R$ is the unique decomposition of $x \in X$ such that $x_T \in X_T$, $x_L \in X_L$, and $x_R \in X_R$. First note that $\iota(X_T) \cap \iota(X_L) = \{0\}$. Indeed, assume that $\iota(x_T) + \iota(x_L) = 0$, that is, $xq_1e^* + xqe^* = 0$. Then by multiplying both sides by e on the right and using (3) and (7), we obtain that $xe^*eq_1 + xq = 0$. Multiplying both sides by q on the right noting that $q_1q = 0$ yields that $xq = 0$, and hence $xq_1e^* = xqe^* = 0$, that is, $\iota(x_T) = \iota(x_L) = 0$. Since $\iota(X_T)^*\iota(X_L) = eX_T^*X_Le^* = \{0\}$ and $\iota(X_L)^*\iota(X_T) = eX_L^*X_Te^* = \{0\}$, we obtain

$$(\iota(X_T) \oplus \iota(X_L))^*(\iota(X_T) \oplus \iota(X_L)) = \iota(X_T)^*\iota(X_T) \overset{\infty}{\oplus} \iota(X_L)^*\iota(X_L)$$

noting that $\iota(X_T)^*\iota(X_T) = q_1X_T^*X_Tq_1$ and $\iota(X_L)^*\iota(X_L) = qX_L^*X_Lq$. Thus $\iota(X) = \iota(X_T) \overset{\infty}{\oplus} \iota(X_L) \overset{\infty}{\oplus} \iota(X_R)$. To show that ι is a complete isometry, it suffices to show that each of $\iota_{|X_T}$, $\iota_{|X_L}$, and $\iota_{|X_R}$ is a complete isometry. Since $e^*eq_1 = q_1$,

$$\|\iota(x_T)\|^2 = \|\iota(x_T)\iota(x_T)^*\| = \|xq_1e^*eq_1x^*\| = \|xq_1x^*\| = \|xq_1\|^2 = \|x_T\|^2.$$

A similar calculation works at the matrix level, which concludes that $\iota_{|X_T}$ is a complete isometry. Similarly, (3) yields that $\iota_{|X_L}$ is a complete isometry.

$$\begin{aligned}\|\iota(x_R)\|^2 &= \|\iota(x_R)^*\iota(x_R)\| = \|q_2x^*ee^*xq_2\| = \|q_2x^*ee^*x(1_{\mathcal{K}} - q - q_1)\| \\ &= \|q_2x^*x(1_{\mathcal{K}} - q)\| = \|q_2x^*x(1_{\mathcal{K}} - q - q_1)\| = \|q_2x^*xq_2\| = \|x_R\|^2,\end{aligned}$$

where we used (6) and (7) as well as the fact that $q_2q_1 = 0$ in the fourth equality, and (7) together with the fact that $q_2q_1 = 0$ in the fifth equality. A similar calculation works at the matrix level, which concludes that $\iota_{|X_R}$ is a complete isometry.

By [Blecher 2001, Lemma 1.5(3)] or [Blecher and Le Merdy 2004, Theorem A.2.5(3)] for example, $\iota(X_T)$, $\iota(X_L)$, and $\iota(X_R)$ are weak*-closed. Clearly, $\iota(X_T)$ and $\iota(X_L)$ are left ideals and $\iota(X_R)$ is a right ideal in the von Neumann algebra $\overline{XX^*}^{\mathrm{w}^*} \overset{\infty}{\oplus} \overline{X^*X}^{\mathrm{w}^*}$. To see that $\iota(X_T)$ is a right ideal as well, it suffices to show that $\iota(X_T)^* \subset \iota(X_T)$, in which case necessarily $\iota(X_T)^* = \iota(X_T)$. To show this, first note that it follows from the adjoint of (6) that

$$q_1x^* = e^*(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q)x^* = e^*(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q)x^*ee^* = q_1x^*ee^* \quad \text{for all } x \in X.$$

Therefore, together with (7), we obtain

$$\iota(x_T)^* = eq_1x^* = eq_1x^*ee^* = ex^*eq_1e^* \in Xq_1e^* = \iota(X_T) \quad \text{for all } x \in X. \qquad \square$$

Definition. We call the decomposition $X = X_T \overset{\infty}{\oplus} X_L \overset{\infty}{\oplus} X_R$ obtained in the proof of Theorem the *ideal decomposition* of the TRO X with predual with respect to an extreme point e of Ball(X).

Remarks. (A) The reader should distinguish ideal decompositions from Peirce decompositions in the literature of Jordan triples. In fact, a TRO can be regarded as a Jordan triple with the canonical symmetrization of the triple product. However, an ideal decomposition and a Peirce decomposition give totally different decompositions.

(B) It is also possible to define $\iota : X \to \overline{XX^*}^{\mathrm{w}^*} \overset{\infty}{\oplus} \overline{X^*X}^{\mathrm{w}^*}$ by

$$\iota(x) := x_L e^* \oplus e^*(x_R + x_T) \quad \text{for } x \in X.$$

(C) Simpler expressions for X_T and X_R are $X_T = \{x - px - xq \mid x \in X\}$ and $X_R = pX$, which would be more helpful in understanding what is going on in the decomposition. To see the equivalences of expressions, let $x \in X$. Then, using (4), (5), and (2), we have

$$\begin{aligned} x_T :&= xq_1 = xe^*(1_{\mathcal{H}} - p)e(1_{\mathcal{K}} - q) = (1_{\mathcal{H}} - p)xe^*e(1_{\mathcal{K}} - q) \\ &= (1_{\mathcal{H}} - p)x(1_{\mathcal{K}} - q) = x - px - xq. \end{aligned}$$

Accordingly, it follows that

$$x_R := xq_2 = x(1_{\mathcal{K}} - q - q_1) = x(1_{\mathcal{K}} - q) - xq_1 = x(1_{\mathcal{K}} - q) - (x - px - xq) = px.$$

(D) The ideal decomposition highly depends on the extreme point chosen. Indeed, let X be a von Neumann algebra, $u \in X$ be a unitary element, and $w \in X$ be an isometry which is not unitary. Then the ideal decomposition with respect to u is just $X = X_T$, while the one with respect to w is $X = X_T \overset{\infty}{\oplus} X_L$.

Appendix: A short proof of Smith's result

The following theorem was proved in [Smith 2000] (also see [Effros and Ruan 2000, Lemma 6.1.7 and Corollary 6.1.8]). We observed it independently in 2000, together with Corollary A.2. Since these results are a special case of this paper's Theorem, and our proof is short enough to understand the essence of the results transparently, it seems worthwhile to present them here. The key to the shortness of the proof is the obvious fact that if a TRO X is finite-dimensional, then so are the C^*-algebras XX^* and X^*X.

Theorem A.1 [Smith 2000]. *If X is a finite-dimensional TRO, then there exist a finite-dimensional C^*-algebra $\mathcal{A}$ and an orthogonal projection $p \in \mathcal{A}$ such that $X \cong p\mathcal{A}p^\perp$ completely isometrically.*

Proof. Let $X \subset \mathbb{B}(\mathcal{K}, \mathcal{H})$ be a finite-dimensional TRO and $\{x_1, \dots, x_n\} \subset X$ be its base. We may assume that $[X\mathcal{K}] = \mathcal{H}$ and $[X^*\mathcal{H}] = \mathcal{K}$. Then the C^*-algebra $XX^* := \mathrm{span}\{xy^* \mid x, y \in X\}$ is equal to the set $\mathrm{span}\{x_i x_j^* \mid 1 \le i, j \le n\}$, and the latter is obviously a finite-dimensional vector space. Similarly, $X^*X := \mathrm{span}\{x^*y \mid$

$x, y \in X\}$ is a finite-dimensional C^*-algebra. Let $\mathcal{L}(X)$ be the linking C^*-algebra for X, that is,

$$\mathcal{L}(X) := \begin{bmatrix} XX^* & X \\ X^* & X^*X \end{bmatrix} (\subset B(\mathcal{H} \oplus \mathcal{K}))\,.$$

Let e, f be the identities of the C^*-algebras XX^* and X^*X, respectively, and let

$$p := \begin{bmatrix} e & 0 \\ 0 & 0 \end{bmatrix} \in \mathcal{L}(X).$$

Then

$$p^\perp = \begin{bmatrix} 0 & 0 \\ 0 & f \end{bmatrix}$$

and $X \cong p\mathcal{L}(X)p^\perp$ completely isometrically. □

Corollary A.2. *A finite-dimensional TRO is completely isometric to the direct sum of rectangular matrices*: $\mathbb{M}_{l_1,k_1}(\mathbb{C}) \overset{\infty}{\oplus} \cdots \overset{\infty}{\oplus} \mathbb{M}_{l_m,k_m}(\mathbb{C})$.

Proof. Let X be a finite-dimensional TRO. By Theorem A.1, we may assume that $X = p\left(\bigoplus_{i=1}^m \mathbb{M}_{n_i}(\mathbb{C})\right)p^\perp$, where p is an orthogonal projection in $\bigoplus_{i=1}^m \mathbb{M}_{n_i}(\mathbb{C})$. For each $1 \leq i \leq m$, let us denote by 1_i the identity of $\mathbb{M}_{n_i}(\mathbb{C})$ which is identified with an element of $\bigoplus_{i=1}^m \mathbb{M}_{n_i}(\mathbb{C})$ in the obvious way, and let $p_i := p1_i$. Then $X = \bigoplus_{i=1}^m p_i \mathbb{M}_{n_i}(\mathbb{C}) p_i^\perp$. By a unitary transform which is a complete isometry, we may assume that

$$p_i = \operatorname{diag}\{\overbrace{1, \ldots, 1}^{l_i \text{ times}}, \overbrace{0, \ldots, 0}^{(n_i - l_i) \text{ times}}\} \quad \text{and} \quad p_i^\perp = \operatorname{diag}\{\overbrace{0, \ldots, 0}^{l_i \text{ times}}, \overbrace{1, \ldots, 1}^{(n_i - l_i) \text{ times}}\}$$

for each $1 \leq i \leq m$. □

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