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**A STUDY OF REAL HYPERSURFACES  
WITH RICCI OPERATORS  
IN 2-DIMENSIONAL COMPLEX SPACE FORMS**

**DONG HO LIM, WOON HA SOHN AND HYUNJUNG SONG**

# A STUDY OF REAL HYPERSURFACES WITH RICCI OPERATORS IN 2-DIMENSIONAL COMPLEX SPACE FORMS

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**We prove that a real hypersurface  $M$  in complex projective space  $P_2(\mathbb{C})$  or complex hyperbolic space  $H_2(\mathbb{C})$ , whose Ricci operator is  $\eta$ -parallel and commutes with the structure tensor on the holomorphic distribution, is a Hopf hypersurface. We also give a characterization of this hypersurface.**

## 1. Introduction

A complex  $n$ -dimensional Kählerian manifold of constant holomorphic sectional curvature  $c$  is called a *complex space form*, which is denoted by  $M_n(c)$ . As is well known, a complete and simply connected complex space form is complex analytically isometric to a complex projective space  $P_n(\mathbb{C})$ , a complex Euclidean space  $\mathbb{C}^n$  or a complex hyperbolic space  $H_n(\mathbb{C})$ , according to  $c > 0$ ,  $c = 0$  or  $c < 0$ .

In this paper we consider a real hypersurface  $M$  in a complex space form  $M_2(c)$ ,  $c \neq 0$ . Then  $M$  has an almost contact metric structure  $(\phi, g, \xi, \eta)$  induced from the Kähler metric and complex structure  $J$  on  $M_n(c)$ . The structure vector field  $\xi$  is said to be *principal* if  $A\xi = \alpha\xi$  is satisfied, where  $A$  is the shape operator of  $M$  and  $\alpha = \eta(A\xi)$ . In this case, it is known that  $\alpha$  is locally constant [Ki and Suh 1990] and that  $M$  is called a *Hopf hypersurface*.

Takagi [1973] classified homogeneous real hypersurfaces in  $P_n(\mathbb{C})$  into six model spaces  $A_1, A_2, B, C, D$  and  $E$  of Hopf hypersurfaces with constant principal curvatures. Berndt [1989] classified all homogeneous Hopf hypersurfaces in  $H_n(\mathbb{C})$  as four model spaces, which are said to be  $A_0, A_1, A_2$  and  $B$ . A real hypersurface  $M$  of type  $A_1$  or  $A_2$  in  $P_n(\mathbb{C})$  or type  $A_0, A_1$  or  $A_2$  in  $H_n(\mathbb{C})$  is said to be of *type A* for simplicity.

As a typical characterization of real hypersurfaces of type  $A$ , the following is due to Okumura [1975] for  $c > 0$ , and Montiel and Romero [1986] for  $c < 0$ .

**Theorem A [Montiel and Romero 1986; Okumura 1975].** *Let  $M$  be a real hypersurface of  $M_n(c)$ ,  $c \neq 0$ ,  $n \geq 2$ . It satisfies  $A\phi - \phi A = 0$  on  $M$  if and only if  $M$  is locally congruent to one of the model spaces of type  $A$ .*

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The Ricci operator of  $M$  will be denoted by  $S$ , and the shape operator or the second fundamental tensor field of  $M$  by  $A$ . The holomorphic distribution  $T_0$  of a real hypersurface  $M$  in  $M_n(c)$  is defined by

$$(1-1) \quad T_0(p) = \{X \in T_p(M) \mid g(X, \xi)_p = 0\},$$

where  $T_p(M)$  is the tangent space of  $M$  at  $p \in M$ . The Ricci operator  $S$  is said to be  $\eta$ -parallel if

$$(1-2) \quad g((\nabla_X S)Y, Z) = 0$$

for any vector fields  $X, Y$  and  $Z$  in  $T_0$ .

**Theorem B** [Kimura and Maeda 1989; Suh 1990]. *Let  $M$  be a real hypersurface in a complex space form  $M_n(c)$ ,  $c \neq 0$ . Then the Ricci operator of  $M$  is  $\eta$ -parallel and the structure vector field  $\xi$  is a principal if and only if  $M$  is locally congruent to one of the model spaces of type A or type B.*

I.-B. Kim, K. H. Kim and one of the present authors [Kim et al. 2006; 2007] studied real hypersurfaces with certain conditions related to the Ricci operator and the structure tensor field  $\phi$  in  $M_n(c)$ . As for the Ricci operator and structure tensor field  $\phi$ , one of the present authors proved the following.

**Theorem C** [Sohn 2007]. *Let  $M$  be a real hypersurface with  $\eta$ -parallel Ricci operator in a complex space form  $M_n(c)$ ,  $c \neq 0$ ,  $n \geq 3$ . If  $M$  satisfies*

$$(1-3) \quad g((S\phi - \phi S)X, Y) = 0$$

*for any  $X$  and  $Y$  in  $T_0$ , then  $M$  is locally congruent to one of the model spaces of type A or type B.*

The purpose of this paper is to complete the results of [Sohn 2007] and characterize real hypersurfaces with  $\eta$ -parallel Ricci operator such that the Ricci operator and structure tensor field commute in a complex space form  $M_n(c)$ ,  $c \neq 0$ ,  $n \geq 2$ . Namely, we prove:

**Theorem.** *A real hypersurface in a complex space form  $M_2(c)$ ,  $c \neq 0$  satisfies (1-2) and (1-3) if and only if it is pseudo-Einstein.*

The pseudo-Einstein hypersurfaces are classified by Kim and Ryan [2008] and Ivey and Ryan [2009] and are described in detail in these papers. In view of their results, we can state the following.

**Corollary.** *Let  $M$  be a real hypersurface with an  $\eta$ -parallel Ricci operator in a complex space form  $M_2(c)$ ,  $c \neq 0$ . If  $M$  satisfies (1-3) then  $M$  is locally congruent to either a Hopf hypersurface with  $A\xi = 0$  or one of the model spaces of type A.*

## 2. Preliminaries

Let  $M$  be a real hypersurface immersed in a complex space form  $M_2(c)$ , and  $N$  be a unit normal vector field of  $M$ . By  $\tilde{\nabla}$  we denote the Levi-Civita connection with respect to the Fubini–Study metric tensor  $\tilde{g}$  of  $M_2(c)$ . Then the Gauss and Weingarten formulas are given respectively by

$$\tilde{\nabla}_X Y = \nabla_X Y + g(AX, Y)N \quad \text{and} \quad \tilde{\nabla}_X N = -AX$$

for any vector fields  $X$  and  $Y$  tangent to  $M$ , where  $g$  denotes the Riemannian metric tensor of  $M$  induced from  $\tilde{g}$ , and  $A$  is the shape operator of  $M$  in  $M_2(c)$ .

For any vector field  $X$  on  $M$  we put

$$JX = \phi X + \eta(X)N, \quad JN = -\xi,$$

where  $J$  is the almost complex structure of  $M_2(c)$ . Then we see that  $M$  induces an almost contact metric structure  $(\phi, g, \xi, \eta)$ , that is,

$$\begin{aligned} \phi^2 X &= -X + \eta(X)\xi, & \phi\xi &= 0, & \eta(\xi) &= 1, \\ g(\phi X, \phi Y) &= g(X, Y) - \eta(X)\eta(Y), & \eta(X) &= g(X, \xi) \end{aligned}$$

for any vector fields  $X$  and  $Y$  on  $M$ . Since the almost complex structure  $J$  is parallel, we can verify from the Gauss formula that

$$(2-1) \quad \nabla_X \xi = \phi AX.$$

Since the ambient manifold is of constant holomorphic sectional curvature  $c$ , we have the Gauss equation

$$\begin{aligned} (2-2) \quad R(X, Y)Z &= \frac{c}{4} (g(Y, Z)X - g(X, Z)Y + g(\phi Y, Z)\phi X - g(\phi X, Z)\phi Y - 2g(\phi X, Y)\phi Z) \\ &\quad + g(AY, Z)AX - g(AX, Z)AY \end{aligned}$$

for any vector fields  $X, Y$  and  $Z$  on  $M$ , where  $R$  denotes the Riemannian curvature tensor of  $M$ .

From (1-3) the Ricci operator  $S$  of  $M$  is expressed by

$$(2-3) \quad SX = \frac{c}{4} ((2n + 1)X - 3\eta(X)\xi) + mAX - A^2X,$$

where  $m = \text{trace } A$  is the mean curvature of  $M$ , and the covariant derivative of (2-3) is given by

$$\begin{aligned} (\nabla_X S)Y &= -\frac{3c}{4} (g(\phi AX, Y)\xi + \eta(Y)\phi AX) \\ &\quad + (Xm)AY + m(\nabla_X A)Y - (\nabla_X A)AY - A(\nabla_X A)Y. \end{aligned}$$

Let  $U$  be a unit vector field on  $M$  with the same direction of the vector field  $-\phi\nabla_\xi\xi$ , and let  $\beta$  be the length of the vector field  $-\phi\nabla_\xi\xi$  if it does not vanish. It is not possible to define  $U$  without specifying that  $\beta \neq 0$ . Then it is easily seen from (2-1) that

$$(2-4) \quad A\xi = \alpha\xi + \beta U,$$

where  $\alpha = \eta(A\xi)$ . We notice here that  $U$  is orthogonal to  $\xi$ .

We put

$$\Omega = \{p \in M \mid \beta(p) \neq 0\}.$$

Then  $\Omega$  is an open subset of  $M$ .

### 3. $\eta$ -parallel Ricci operators

In this section, we assume that  $\Omega$  is not empty. Then there are scalar fields  $\gamma$ ,  $\varepsilon$  and  $\delta$  and a unit vector field  $U$  and  $\phi U$  orthogonal to  $\xi$  such that

$$(3-1) \quad AU = \beta\xi + \gamma U + \varepsilon\phi U, \quad A\phi U = \varepsilon U + \delta\phi U$$

and

$$(3-2) \quad m = \text{trace } A = \alpha + \gamma + \delta$$

in  $M_2(c)$ .

We shall prove the following lemmas.

**Lemma 3.1.** *Let  $M$  be a real hypersurface in a complex space form  $M_2(c)$ ,  $c \neq 0$ . If  $M$  satisfies (1-3), then we have  $AU = \beta\xi + \gamma U$ ,  $A\phi U = \delta\phi U$  and  $\beta^2 = \alpha(\gamma - \delta)$ .*

*Proof.* If we put  $X = \xi$  into (2-3), we have

$$(3-3) \quad S\xi = \left(\frac{c}{2} + \alpha\gamma + \alpha\delta - \beta^2\right)\xi + \beta\delta U - \beta\varepsilon\phi U.$$

Putting  $X = U$  into (2-3) and taking account of (3-1) yields

$$(3-4) \quad SU = \beta\delta\xi + \left(\frac{5c}{4} + \alpha\gamma + \gamma\delta - \beta^2 - \varepsilon^2\right) + \alpha\varepsilon\phi U.$$

Putting  $X = \phi U$  into (2-3) and using (3-1), we obtain

$$(3-5) \quad S\phi U = -\beta\varepsilon\xi + \alpha\varepsilon U + \left(\frac{5c}{4} + \alpha\delta + \gamma\delta - \varepsilon^2\right)\phi U.$$

If we apply  $\phi$  to (3-4), then we have

$$(3-6) \quad (S\phi - \phi S)U = -\beta\varepsilon\xi + 2\alpha\varepsilon U + (\alpha\delta - \alpha\gamma + \beta^2)\phi U.$$

From condition (1-3), we have, for all  $X \in T_0$ ,

$$(3-7) \quad (S\phi - \phi S)X = -\beta g(\varepsilon U + \delta\phi U, X)\xi$$

If we substitute  $X = U$  into (3-7), then we obtain

$$(3-8) \quad (S\phi - \phi S)U = -\beta\varepsilon\xi.$$

Comparing (3-6) and (3-8), we get  $\varepsilon = 0$  and  $\beta^2 = \alpha(\gamma - \delta)$ . It follows that  $AU$  is expressed in terms of  $\xi$  and  $U$  only and  $A\phi U$  is given by  $\phi U$ .  $\square$

It follows from (2-3) and (3-1) that

$$(3-9) \quad S\xi = \left(\frac{c}{2} + 2\alpha\delta\right)\xi + \beta\delta U,$$

$$(3-10) \quad SU = \beta\delta\xi + \left(\frac{5c}{4} + \gamma\delta + \alpha\delta\right)U,$$

$$(3-11) \quad S\phi U = \left(\frac{5c}{4} + \gamma\delta + \alpha\delta\right)\phi U.$$

**Lemma 3.2.** *Under the assumptions of Lemma 3.1, if  $M$  has the  $\eta$ -parallel Ricci operator  $S$ , then we have  $AU = \beta\xi + \gamma U$ ,  $A\phi U = 0$  and  $\beta^2 = \alpha\gamma$ .*

*Proof.* Differentiating (3-10) covariantly along vector field  $X$  in  $T_0$ , we obtain

$$(\nabla_X S)U = \left(\left(\frac{5c}{4} + \gamma\delta + \alpha\delta\right)I - S\right)\nabla_X U + \beta\delta\phi AX + X(\beta\delta)\xi + X\left(\frac{5c}{4} + \gamma\delta + \alpha\delta\right)U.$$

Taking the inner product of this equation with  $U$  and  $\phi U$  and making use of (3-9)–(3-11) and Lemma 3.1, we obtain

$$(3-12) \quad (\alpha + \gamma)\nabla\delta + \delta(\nabla\gamma + \nabla\alpha) = 2\beta\delta^2\phi U$$

and

$$\delta\gamma = 0.$$

If we differentiate this along the vector field  $X$  in  $T_0$ , then (3-12) is reduced to

$$(3-13) \quad \alpha\nabla\delta + \delta\nabla\alpha = 2\beta\delta^2\phi U.$$

Differentiating (3-11) covariantly along vector field  $X$  in  $T_0$ , we obtain

$$(3-14) \quad (\nabla_X S)\phi U = \left(\left(\frac{5c}{4} + \gamma\delta + \alpha\delta\right)I - S\right)\nabla_X \phi U + \left(X\left(\frac{5c}{4} + \gamma\delta + \alpha\delta\right)\right)\phi U.$$

If we take the inner product of (3-14) with  $\phi U$  and use (3-9)–(3-11), then we have

$$(3-15) \quad \alpha\nabla\delta + \delta\nabla\alpha = 0.$$

Comparing (3-13) and (3-15), we obtain  $\delta = 0$  and  $\beta^2 = \alpha\gamma$  from Lemma 3.1. From this and Lemma 3.1 we conclude that  $AU$  is expressed in terms of  $\xi$  and  $U$  only and  $A\phi U = 0$ .  $\square$

#### 4. Proof of the main theorem

Assume that  $M$  satisfies (1-2) and (1-3). We first show that  $M$  is Hopf. If the open set  $\Omega$  is not empty, then Lemma 3.2 yields  $\delta = 0$ . Thus the Ricci operator, as expressed in (3-9)–(3-11), has the property that  $\xi$ ,  $U$  and  $\phi U$  are eigenvectors and that  $U$  and  $\phi U$  have the same eigenvalue. That is,  $M$  is pseudo-Einstein with

$$SX = \frac{5c}{4}X - \frac{3c}{4}g(X, \xi)\xi.$$

This contradicts a result from [Kim and Ryan 2008]. Thus we conclude that any hypersurface satisfying (1-2) and (1-3) must be Hopf.

Since  $M$  is Hopf, condition (1-3) yields  $\alpha(\gamma - \delta) = 0$  and that the criteria for Proposition 2.21 in [Kim and Ryan 2008] are satisfied. Thus  $M$  is pseudo-Einstein.

Conversely, if  $M$  is pseudo-Einstein, observe that (1-2) and (1-3) must be satisfied. □

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