

*Pacific
Journal of
Mathematics*

**A COUNTEREXAMPLE TO THE SIMPLE
LOOP CONJECTURE FOR $\mathrm{PSL}(2, \mathbb{R})$**

KATHRYN MANN

A COUNTEREXAMPLE TO THE SIMPLE LOOP CONJECTURE FOR $\mathrm{PSL}(2, \mathbb{R})$

KATHRYN MANN

In this note, we give an explicit counterexample to the simple loop conjecture for representations of surface groups into $\mathrm{PSL}(2, \mathbb{R})$. Specifically, we use a construction of DeBlois and Kent to show that for any orientable surface with negative Euler characteristic and genus at least 1, there are uncountably many nonconjugate, noninjective homomorphisms of its fundamental group into $\mathrm{PSL}(2, \mathbb{R})$ that kill no simple closed curve (nor any power of a simple closed curve). This result is not new — work of Louder and Calegari for representations of surface groups into $\mathrm{SL}(2, \mathbb{C})$ applies to the $\mathrm{PSL}(2, \mathbb{R})$ case, but our approach here is explicit and elementary.

1. Introduction

The simple loop conjecture, proved by Gabai [1985], states that any noninjective homomorphism from a closed surface group to another closed surface group has an element represented by a simple closed curve in the kernel. It has been conjectured that the result still holds if the target is replaced by the fundamental group of an orientable 3-manifold (see the problem list in [Kirby 1997]). Although special cases have been proved (e.g., [Hass 1987; Rubinstein and Wang 1998]), the general hyperbolic case is still open.

Minsky [2000] asked whether the conjecture holds if the target group is instead $\mathrm{SL}(2, \mathbb{C})$. This was answered in the negative with the following theorem.

Theorem 1.1 [Cooper and Manning 2011]. *Let Σ be a closed orientable surface of genus $g \geq 4$. Then there is a homomorphism $\rho : \pi_1(\Sigma) \rightarrow \mathrm{SL}(2, \mathbb{C})$ such that:*

- (1) ρ is not injective.
- (2) If $\rho(\alpha) = \pm I$, then α is not represented by a simple closed curve.
- (3) If $\rho(\alpha)$ has finite order, then $\rho(\alpha) = I$.

The third condition implies in particular that no power of a simple closed curve lies in the kernel.

MSC2010: primary 57M05; secondary 57N16.

Keywords: simple loop conjecture, surface group, character variety, representation, representation space, $\mathrm{PSL}(2, \mathbb{R})$.

Inspired by this, we ask whether a similar result holds for $\mathrm{PSL}(2, \mathbb{R})$, this being an intermediate case between Gabai's result for surface groups and Cooper and Manning's for $\mathrm{SL}(2, \mathbb{C})$. Techniques of Cooper and Manning's proof do not seem to carry over directly to the $\mathrm{PSL}(2, \mathbb{R})$ case — their work involves both a dimension count on the $\mathrm{SL}(2, \mathbb{C})$ character variety and a proof showing that a specific subvariety is irreducible and smooth on a dense subset, and complex varieties and their real points generally behave quite differently. However, we will show here with different methods that an analog to Theorem 1.1 does hold for $\mathrm{PSL}(2, \mathbb{R})$.

While this note was in progress, we learned of work of Louder and Calegari (independently in [Louder 2011] and [Calegari 2013]) that can also be applied to answer our question in the affirmative. Louder shows the simple loop conjecture is false for representations into limit groups, and Calegari gives a practical way of verifying no simple closed curves lie in the kernel of a noninjective representation using stable commutator length and the Gromov norm.

The difference here is that our construction is entirely elementary. We use an explicit representation found in [DeBlois and Kent 2006] (which uses work from [Goldman 1988] and [Shalen 1979]), and we verify by elementary means that this representation is noninjective and kills no simple closed curve. Our end result parallels that of Cooper and Manning but also include surfaces with boundary and all genera at least 1:

Theorem 1.2. *Let Σ be an orientable surface of negative Euler characteristic and of genus $g \geq 1$, possibly with boundary. Then there is a homomorphism $\rho : \pi_1(\Sigma) \rightarrow \mathrm{SL}(2, \mathbb{R})$ such that:*

- (1) ρ is not injective.
- (2) If $\rho(\alpha) = \pm I$, then α is not represented by a simple closed curve.
- (3) In fact, if α is represented by a simple closed curve, then $\rho(\alpha^k) \neq I$ for any $k \in \mathbb{Z}$.

Moreover, there are uncountably many nonconjugate representations satisfying (1) through (3).

In the case of a nonorientable surface, the appropriate target group is $\mathrm{PGL}(2, \mathbb{R})$, as the fundamental group of a nonorientable hyperbolic surface can be represented as a lattice in $\mathrm{PGL}(2, \mathbb{R})$. This again gives an intermediate case between the simple loop conjecture for representations into surface groups and into $\mathrm{PSL}(2, \mathbb{C})$. We have the following direct generalization of Theorem 1.2, with essentially the same proof.

Theorem 1.3. *Let Σ be a nonorientable surface of negative Euler characteristic and of nonorientable genus $g \geq 2$ that is not the punctured Klein bottle nor the closed nonorientable genus-3 surface. Then there are uncountably many representations $\rho : \pi_1(\Sigma) \rightarrow \mathrm{PGL}(2, \mathbb{R})$ satisfying conditions (1) through (3) of Theorem 1.2.*

See Section 3 for a comment on the exceptional cases of the punctured Klein bottle and the closed, nonorientable genus-3 surface.

2. Proof of Theorem 1.2

We describe a family of (noninjective) representations constructed in [DeBlois and Kent 2006] based on a construction from [Goldman 1988]. We will then show that this family contains infinitely many nonconjugate representations with no simple closed curve in the kernel.

Let Σ be an orientable surface of genus $g \geq 1$ and negative Euler characteristic, possibly with boundary. Assume for the moment that Σ is not the once-punctured torus — Theorem 1.2 for this case will follow easily later on.

Let $c \subset \Sigma$ be a simple closed curve separating Σ into a genus-1 subsurface with single boundary component c and a genus- $(g - 1)$ subsurface with one or more boundary components. Let Σ_A denote the genus- $(g - 1)$ subsurface and Σ_B the genus-1 subsurface. Finally, we let $A = \pi_1(\Sigma_A)$ and $B = \pi_1(\Sigma_B)$, so that $\pi_1(\Sigma) = A *_C B$, where C is the infinite cyclic subgroup generated by the element $[c]$ represented by the curve c . We assume that the basepoint for $\pi_1(\Sigma)$ lies on c .

Let $x \in B$ and $y \in B$ be generators such that $B = \langle x, y \rangle$, and that the curve c represents the commutator $[x, y]$. See Figure 1.

Fix α and β in $\mathbb{R} \setminus \{0, \pm 1\}$, and following [DeBlois and Kent 2006] define $\phi_B : B \rightarrow \text{SL}(2, \mathbb{R})$ by

$$\phi_B(x) = \begin{pmatrix} \alpha & 0 \\ 0 & \alpha^{-1} \end{pmatrix}, \quad \phi_B(y) = \begin{pmatrix} \beta & 1 \\ 0 & \beta^{-1} \end{pmatrix}.$$

We have then

$$\phi_B([x, y]) = \begin{pmatrix} 1 & \beta(\alpha^2 - 1) \\ 0 & 1 \end{pmatrix},$$

so $\phi_B([x, y])$ is invariant under conjugation by the matrix $\lambda_t := \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$. Projecting these matrices to $\text{PSL}(2, \mathbb{R})$ gives a representation $B \rightarrow \text{PSL}(2, \mathbb{R})$ that is upper

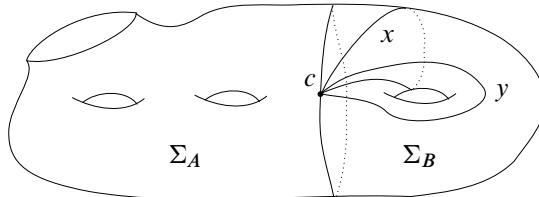


Figure 1. Decomposition of Σ and curves representing generators x and y for B .

triangular, hence solvable, and therefore noninjective. Abusing notation, we let ϕ_B denote this representation.

Now let $\phi_A : A \rightarrow \text{PSL}(2, \mathbb{R})$ be Fuchsian such and that the image of the boundary curve c under ϕ_A agrees with $\phi_B([x, y])$. That such a representation exists is standard — Σ_A has negative Euler characteristic and therefore admits a complete hyperbolic structure. The image of $[c]$ under the corresponding Fuchsian representation is a parabolic element of $\text{PSL}(2, \mathbb{R})$, so after conjugation we may assume that it is equal to $\phi_B([x, y])$, since $\beta(\alpha^2 - 1) \neq 0$.

Finally, we combine ϕ_A with conjugates of ϕ_B to get a one-parameter family of representations $\phi_t : \pi_1(\Sigma) \rightarrow \text{PSL}(2, \mathbb{R})$ as follows. For $t \in \mathbb{R}$ and $g \in \pi_1(\Sigma) = A *_C B$, let

$$\phi_t(g) = \begin{cases} \phi_A(g) & \text{if } g \in A, \\ \lambda_t \circ \phi_B(g) \circ (\lambda_t)^{-1} & \text{if } g \in B. \end{cases}$$

This representation is well-defined because $\phi_B([x, y]) = \phi_A([x, y])$, and is invariant under conjugation by λ_t .

Our next goal is to show that for appropriate choice of α , β , and t , the representation ϕ_t satisfies the criteria in Theorem 1.2. The main difficulty will be checking that no element representing a simple closed curve is of finite order. To do so, we employ a stronger form of Lemma 2 from [DeBlois and Kent 2006]. This trick originally comes from the proof of Proposition 1.3 in [Shalen 1979].

Lemma 2.1. *Suppose $w \in A *_C B$ is a word of the form $w = a_1 b_1 a_2 b_2 \cdots a_l b_l$, with $a_i \in A$ and $b_i \in B$ for $1 \leq i \leq l$. Assume that for each i , the matrix $\phi_0(a_i)$ has a nonzero $(2, 1)$ -entry and $\phi_0(b_i)$ is hyperbolic. If t is transcendental over the entry field of $\phi_0(A *_C B)$, then $\phi_t(w)$ is not of finite order.*

By the *entry field* of a group Γ of matrices, we mean the field generated over \mathbb{Q} by the collection of all entries of matrices in Γ .

Remark 2.2. Lemma 2 of [DeBlois and Kent 2006] is a proof that $\phi_t(w)$ is not the *identity*, under the assumptions of Lemma 2.1. We use some of their work in our proof.

Proof of Lemma 2.1. DeBlois and Kent show by a straightforward induction (we leave it as an exercise) that under the hypotheses of Lemma 2.1, the entries of $\phi_t(w)$ are polynomials in t such that the degree of the $(2, 2)$ -entry is l , the degree of the $(1, 2)$ -entry is at most l , and the other entries have degree at most $l - 1$. Now, suppose that $\phi_t(w)$ is finite order. Then it is conjugate to a matrix of the form $\begin{pmatrix} u & v \\ -v & u \end{pmatrix}$, where $u = \cos \theta$ and $v = \sin \theta$ for θ a rational multiple of π . In particular, it follows from the de Moivre formula for sine and cosine that u and v are algebraic.

Now suppose that the matrix conjugating $\phi_t(w)$ to $\begin{pmatrix} u & v \\ -v & u \end{pmatrix}$ has entries a_{ij} . Then we have

$$\phi_t(w) = \begin{pmatrix} u - (a_{12}a_{22} - a_{11}a_{21})v & (a_{12}^2 a_{11}^2)v \\ -(a_{22}^2 a_{21}^2)v & u + (a_{12}a_{22} + a_{11}a_{21})v \end{pmatrix}.$$

Looking at the $(2, 2)$ -entry we see that $a_{12}a_{22} + a_{11}a_{21}$ must be a polynomial in t of degree l . But this means that the $(1, 1)$ -entry is also a polynomial in t of degree l , contradicting DeBlois and Kent’s calculation. This proves the lemma. □

To complete our construction, choose any $t \in \mathbb{R}$ that is transcendental over the entry field of $\phi_0(A *_C B)$. We want to show that no power of an element representing a simple closed curve lies in the kernel of ϕ_t . To this end, consider any word w in $A *_C B$ that has a simple closed curve as a representative. There are three cases to check.

Case i: w is a word in A alone. In this case $\phi_t(w)$ is not finite order, since $\phi_t(A)$ is Fuchsian and therefore injective.

Case ii: w is a word in B alone. Theorem 5.1 of [Birman and Series 1984] states that w can be represented by a simple closed curve only if it has one of the following forms after cyclic reduction:

1. $w = x^{\pm 1}$ or $w = y^{\pm 1}$.
2. $w = [x^{\pm 1}, y^{\pm 1}]$.
3. Up to replacing x with x^{-1} , y with y^{-1} , and interchanging x and y , there is some $n \in \mathbb{Z}^+$ such that $w = x^{n_1} y x^{n_2} y \cdots x^{n_s} y$, where $n_i \in \{n, n + 1\}$.

The heuristic for Case 3 of the Birman–Series theorem is shown in Figure 2 — if w is represented by a simple closed curve and terminates with $x^{n_s} y$, this forces the

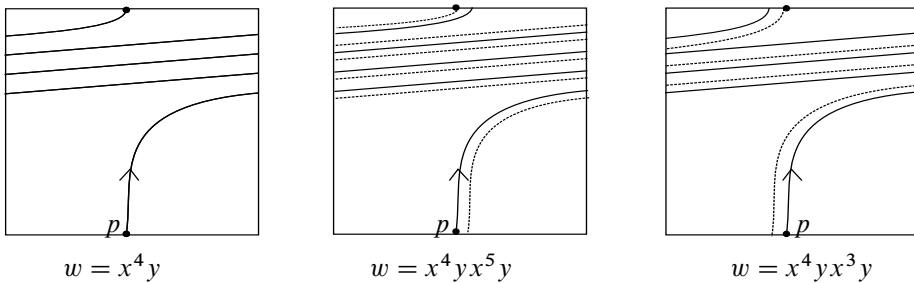


Figure 2. Simple closed curves on the once punctured torus. Assume the puncture is at the vertex, x is represented by a horizontal loop oriented from left to right, and y is a vertical loop oriented from bottom to top.

rest of the curve representing w to wind around the punctured torus in a set pattern. The figure shows the behavior for $n_s = 4$.

By construction, no word of type 1, 2 or 3 above is finite order, provided that $\alpha^s \beta^k \neq 1$ for any integers s and k other than zero — indeed, we only need to check words of type 3, and these necessarily have trace equal to $\alpha^s \beta^k + \alpha^{-s} \beta^{-k}$ for some $s, k \neq 0$. Since cyclic reduction corresponds to conjugation, no word in B has finite order image.

Note also that, in particular, under the condition that $\alpha^s \beta^k \neq 1$ for $s, k \neq 0$, all type 3 words are hyperbolic. We will use this fact again later on.

Case iii: general case. If w is a word including both A and B , we claim that it can be written in a form where Lemma 2.1 applies. To write it this way, take a simple curve γ on Σ that represents w and has a minimal number of (geometric) intersections with c . We can write γ as a concatenation of simple arcs $\gamma = \gamma_1 \delta_1 \gamma_2 \delta_2 \cdots \gamma_n \delta_n$, with $\gamma_i \subset \Sigma_A$ and $\delta_i \subset \Sigma_B$. Since we chose γ to have a minimal number of intersections with c , no arc γ_i (or δ_i) is homotopic in Σ_A (respectively in Σ_B) to a segment of c — if it were, we could apply an isotopy of Σ supported in a neighborhood of the disc bounded by the arc and the segment of c to push the arc across c and reduce the total number of intersections.

Now choose a proper segment c' of c that contains the basepoint p and all endpoints of all γ_i and δ_i , and close each of the arcs γ_i and δ_i into a simple loop by attaching a segment of c' . If $a_i \in A$ and $b_i \in B$ are represented by the loops γ_i and δ_i , then $a_1 b_1 a_2 b_2 \cdots a_n b_n = w$ in $\pi_1(\Sigma)$.

Since no arc γ_i or δ_i was homotopic to a segment of c , no a_i or b_i is represented by a power of $[c]$ in $\pi_1(\Sigma)$. We claim that in this case $a_1 b_1 a_2 b_2 \cdots a_n b_n$ satisfies the hypotheses of Lemma 2.1. Indeed, since ϕ_A is Fuchsian, the only elements with a nonzero $(2, 1)$ -entry are powers of $[c]$, and the Birman–Series classification of simple closed curves on Σ_b implies that the only simple closed curves which are *not* hyperbolic represent $[c]$ or $[c]^{-1}$.

It remains only to remark that the representation ϕ_t is noninjective and that, by choosing appropriate parameters, we can produce uncountably many nonconjugate representations. Noninjectivity follows immediately since $\phi_t(B)$ is solvable, so the restriction of ϕ_t to B is noninjective. Now, for any fixed α and β (satisfying the requirement that $\alpha^s \beta^k \neq 1$ for all integers s, k), varying t among transcendentals over the entry field of $\phi_0(A *_C B)$ produces uncountably many nonconjugate representations that are all noninjective, but have no power of a simple closed curve in the kernel. This concludes the proof of Theorem 1.2, assuming that the surface was not the punctured torus.

The punctured torus case is now immediate: any representation of the form of ϕ_B where $\alpha^s \beta^k \neq 1$ for any integers s and k is noninjective and our work above

shows that no element represented by a simple closed curve has finite order. Fixing α and varying β produces uncountably many nonconjugate representations.

3. Nonorientable surfaces

Recall that the *genus* of a nonorientable surface Σ is defined to be the number of \mathbb{RP}^2 -summands in a decomposition of the surface as $\Sigma = \mathbb{RP}^2 \# \mathbb{RP}^2 \# \dots \# \mathbb{RP}^2$. A closed, nonorientable genus- g surface has Euler characteristic $\chi = 2 - g$.

Let Σ be a nonorientable surface of negative Euler characteristic and nonorientable genus $g \geq 2$ that is not the punctured Klein bottle nor the closed nonorientable genus-3 surface. The same strategy as in the orientable case can then be used to produce uncountably many noninjective representations $\pi_1(\Sigma) \rightarrow \mathrm{PGL}(2, \mathbb{R})$ such that no power of a simple closed curve lies in the kernel. In detail, our assumptions on Σ imply that we may decompose Σ along a (2-sided) curve c into a genus-1 orientable surface Σ_B with one boundary component and a nonorientable surface Σ_A of negative Euler characteristic.

We define ϕ_B exactly as in the orientable case, but now consider the matrices as elements of $\mathrm{PGL}(2, \mathbb{R})$ rather than $\mathrm{PSL}(2, \mathbb{R})$. We let $\phi_A : \pi_1(\Sigma_A) \rightarrow \mathrm{PGL}(2, \mathbb{R})$ be a discrete, faithful representation such that $\phi_A([c]) = \phi_B([c])$. As in the case of the orientable surface, we may take this to be a representation corresponding to a complete hyperbolic structure on Σ . Define $\phi_t : \pi_1(\Sigma) \rightarrow \mathrm{PGL}(2, \mathbb{R})$ by “gluing together” ϕ_A with a conjugate of ϕ_B by λ_t exactly as in the orientable case. The proof now carries through verbatim, for none of the topological arguments that we used required orientability of Σ_A . We also reassure the reader (who may be unfamiliar with lattices in $\mathrm{PGL}(2, \mathbb{R})$) that powers of $\phi_A([c])$ are indeed the only elements of the image of ϕ_A with 0 as the (2, 1)-entry.

This strategy does not cover the case of the punctured Klein bottle, which cannot be decomposed with a T^2 -summand, nor the closed nonorientable genus-3 surface, which decomposes as $T^2 \# \mathbb{RP}^2$. It would be interesting to try to cover this case in a manner analogous to the punctured torus case of Theorem 1.2 by providing a classification of simple closed curves on these surfaces. Indeed (as the referee has pointed out) the punctured Klein bottle case is not too difficult. The closed, nonorientable genus-3 surface case appears to be more challenging.

Acknowledgements

I am grateful to the referee for bringing the nonorientable case to my attention, for suggesting improvements to Case iii of the main proof, and for providing additional context and references. I am also indebted to Benson Farb, who introduced me to the simple loop conjecture and much of the background material that went into this paper.

References

- [Birman and Series 1984] J. S. Birman and C. Series, “An algorithm for simple curves on surfaces”, *J. London Math. Soc.* (2) **29**:2 (1984), 331–342. MR 85m:57002 Zbl 0507.57006
- [Calegari 2013] D. Calegari, “Certifying incompressibility of noninjective surfaces with scl”, *Pacific J. Math.* **262**:2 (2013), 257–262. MR 3069061 Zbl 06189535
- [Cooper and Manning 2011] D. Cooper and J. F. Manning, “Non-faithful representations of surface groups into $SL(2, \mathbb{C})$ which kill no simple closed curve”, preprint, 2011. To appear in *Geom. Dedicata*. arXiv 1104.4492v1
- [DeBlois and Kent 2006] J. DeBlois and I. Kent, Richard P., “Surface groups are frequently faithful”, *Duke Math. J.* **131**:2 (2006), 351–362. MR 2007b:57001 Zbl 1109.57002
- [Gabai 1985] D. Gabai, “The simple loop conjecture”, *J. Differential Geom.* **21**:1 (1985), 143–149. MR 86m:57013 Zbl 0556.57007
- [Goldman 1988] W. M. Goldman, “Topological components of spaces of representations”, *Invent. Math.* **93**:3 (1988), 557–607. MR 89m:57001 Zbl 0655.57019
- [Hass 1987] J. Hass, “Minimal surfaces in manifolds with S^1 actions and the simple loop conjecture for Seifert fibered spaces”, *Proc. Amer. Math. Soc.* **99**:2 (1987), 383–388. MR 88e:57015 Zbl 0627.57008
- [Kirby 1997] R. Kirby, ed., “Problems in low-dimensional topology”, pp. 35–473 in *Geometric topology* (Athens, GA, 1993), vol. 2, edited by W. H. Kazez, AMS/IP Stud. Adv. Math. **2**, Amer. Math. Soc., Providence, RI, 1997. MR 1470751 Zbl 0888.57014
- [Louder 2011] L. Louder, “Simple loop conjecture for limit groups”, preprint, 2011. To appear in *Israel J. Math.* arXiv 1106.1350v1
- [Minsky 2000] Y. N. Minsky, “Short geodesics and end invariants”, *Sūrikaiseikikenkyūsho Kōkyūroku* 1153 (2000), 1–19. MR 1805224 Zbl 0968.57500 arXiv math/0006002
- [Rubinstein and Wang 1998] J. H. Rubinstein and S. Wang, “ π_1 -injective surfaces in graph manifolds”, *Comment. Math. Helv.* **73**:4 (1998), 499–515. MR 99h:57039 Zbl 0916.57001
- [Shalen 1979] P. B. Shalen, “Linear representations of certain amalgamated products”, *J. Pure Appl. Algebra* **15**:2 (1979), 187–197. MR 80e:20011 Zbl 0401.20024

Received November 27, 2012. Revised September 4, 2013.

KATHRYN MANN
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF CHICAGO
5734 SOUTH UNIVERSITY AVENUE
CHICAGO, IL 60637
UNITED STATES
mann@math.uchicago.edu

PACIFIC JOURNAL OF MATHEMATICS

msp.org/pjm

Founded in 1951 by E. F. Beckenbach (1906–1982) and F. Wolf (1904–1989)

EDITORS

Don Blasius (Managing Editor)
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
blasius@math.ucla.edu

Paul Balmer
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
balmer@math.ucla.edu

Robert Finn
Department of Mathematics
Stanford University
Stanford, CA 94305-2125
finn@math.stanford.edu

Sorin Popa
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
popa@math.ucla.edu

Vyjayanthi Chari
Department of Mathematics
University of California
Riverside, CA 92521-0135
chari@math.ucr.edu

Kefeng Liu
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
liu@math.ucla.edu

Jie Qing
Department of Mathematics
University of California
Santa Cruz, CA 95064
qing@cats.ucsc.edu

Daryl Cooper
Department of Mathematics
University of California
Santa Barbara, CA 93106-3080
cooper@math.ucsb.edu

Jiang-Hua Lu
Department of Mathematics
The University of Hong Kong
Pokfulam Rd., Hong Kong
jhlu@maths.hku.hk

Paul Yang
Department of Mathematics
Princeton University
Princeton NJ 08544-1000
yang@math.princeton.edu

PRODUCTION

Silvio Levy, Scientific Editor, production@msp.org

SUPPORTING INSTITUTIONS

ACADEMIA SINICA, TAIPEI
CALIFORNIA INST. OF TECHNOLOGY
INST. DE MATEMÁTICA PURA E APLICADA
KEIO UNIVERSITY
MATH. SCIENCES RESEARCH INSTITUTE
NEW MEXICO STATE UNIV.
OREGON STATE UNIV.

STANFORD UNIVERSITY
UNIV. OF BRITISH COLUMBIA
UNIV. OF CALIFORNIA, BERKELEY
UNIV. OF CALIFORNIA, DAVIS
UNIV. OF CALIFORNIA, LOS ANGELES
UNIV. OF CALIFORNIA, RIVERSIDE
UNIV. OF CALIFORNIA, SAN DIEGO
UNIV. OF CALIF., SANTA BARBARA

UNIV. OF CALIF., SANTA CRUZ
UNIV. OF MONTANA
UNIV. OF OREGON
UNIV. OF SOUTHERN CALIFORNIA
UNIV. OF UTAH
UNIV. OF WASHINGTON
WASHINGTON STATE UNIVERSITY

These supporting institutions contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its contents or policies.


See inside back cover or msp.org/pjm for submission instructions.

The subscription price for 2014 is US \$410/year for the electronic version, and \$535/year for print and electronic. Subscriptions, requests for back issues and changes of subscribers address should be sent to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163, U.S.A. The Pacific Journal of Mathematics is indexed by Mathematical Reviews, Zentralblatt MATH, PASCAL CNRS Index, Referativnyi Zhurnal, Current Mathematical Publications and Web of Knowledge (Science Citation Index).

The Pacific Journal of Mathematics (ISSN 0030-8730) at the University of California, c/o Department of Mathematics, 798 Evans Hall #3840, Berkeley, CA 94720-3840, is published twelve times a year. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163.

PJM peer review and production are managed by EditFlow® from Mathematical Sciences Publishers.

PUBLISHED BY

 **mathematical sciences publishers**
nonprofit scientific publishing

<http://msp.org/>

© 2014 Mathematical Sciences Publishers

PACIFIC JOURNAL OF MATHEMATICS

Volume 269 No. 2 June 2014

Totaro's question for simply connected groups of low rank	257
JODI BLACK and RAMAN PARIMALA	
Uniform hyperbolicity of the curve graphs	269
BRIAN H. BOWDITCH	
Constant Gaussian curvature surfaces in the 3-sphere via loop groups	281
DAVID BRANDER, JUN-ICHI INOBUCHI and SHIMPEI KOBAYASHI	
On embeddings into compactly generated groups	305
PIERRE-EMMANUEL CAPRACE and YVES CORNULIER	
Variational representations for N -cyclically monotone vector fields	323
ALFRED GALICHON and NASSIF GHOUSOUB	
Restricted successive minima	341
MARTIN HENK and CARSTEN THIEL	
Radial solutions of non-Archimedean pseudodifferential equations	355
ANATOLY N. KOCHUBEI	
A Jantzen sum formula for restricted Verma modules over affine Kac–Moody algebras at the critical level	371
JOHANNES KÜBEL	
Notes on the extension of the mean curvature flow	385
YAN LENG, ENTAO ZHAO and HAORAN ZHAO	
Hypersurfaces with prescribed angle function	393
HENRIQUE F. DE LIMA, ERALDO A. LIMA JR. and ULISSES L. PARENTE	
Existence of nonparametric solutions for a capillary problem in warped products	407
JORGE H. LIRA and GABRIELA A. WANDERLEY	
A counterexample to the simple loop conjecture for $\mathrm{PSL}(2, \mathbb{R})$	425
KATHRYN MANN	
Twisted Alexander polynomials of 2-bridge knots for parabolic representations	433
TAKAYUKI MORIFUJI and ANH T. TRAN	
Schwarzian differential equations associated to Shimura curves of genus zero	453
FANG-TING TU	
Polynomial invariants of Weyl groups for Kac–Moody groups	491
ZHAO XU-AN and JIN CHUNHUA	



0030-8730(201406)269:2;1-Z