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In their study of characters of minimal affinizations of representations of orthogonal and symplectic Lie algebras, Chari and Greenstein conjectured that certain Jacobi–Trudi determinants satisfy an alternating sum formula. In this note, we prove their conjecture and slightly more. The proof relies on some symmetries of the ring of symmetric functions discovered by Koike and Terada. Using results of Hernandez, Mukhin and Young, and Naoi, this implies that the characters of minimal affinizations in types B, C, and D are given by a Jacobi–Trudi determinant.

Introduction

In [Chari and Greenstein 2011] (henceforth abbreviated [CG]), the authors study a class of modules over the current algebra $\mathfrak{g} \otimes \mathbb{C}[t]$, where $\mathfrak{g}$ is either a special orthogonal or symplectic Lie algebra (over the complex numbers). These modules are related to the minimal affinizations, a class of irreducible representations for the quantum loop algebra $\mathrm{U}_q(\mathfrak{g} \otimes \mathbb{C}[t, t^{-1}])$. We refer the reader to [CG, §3] for background and references. A character formula, which is similar to a Jacobi–Trudi determinant, for these modules is conjectured in [CG, Conjecture 1.13]. This is inspired by [Nakai and Nakanishi 2006], which conjectures that the characters of minimal affinizations are given by such determinants (see also [Nakai and Nakanishi 2007a; 2007b] for related work).

The aim of this note is to prove [CG, Conjecture 1.13] (see Theorem 1.1). We will give a uniform proof for all types. The conjecture reduces to a combinatorial statement about characters of $\mathfrak{g}$, so we will not need to discuss current or loop algebras any further. In fact, we will prove an extension of the combinatorial statement which removes a restriction on the highest weights considered. Furthermore, using results of Hernandez, Mukhin and Young, and Naoi, this gives a character formula for minimal affinizations of representations of $\mathfrak{g}$ in types B, C, and D (see Remark 1.3).

The method of proof involves passing to a suitable limit (with respect to the rank of the Lie algebra) to take advantage of additional symmetries. This suggests that

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there should be a connection to the categories Rep(O) and Rep(Sp) studied in [Sam and Snowden 2013, §4] and a suitable categorification of the involutions i_{O} and i_{Sp} used in Section 3 (which were introduced in [Koike and Terada 1987]), but we have been unable to find one so far.

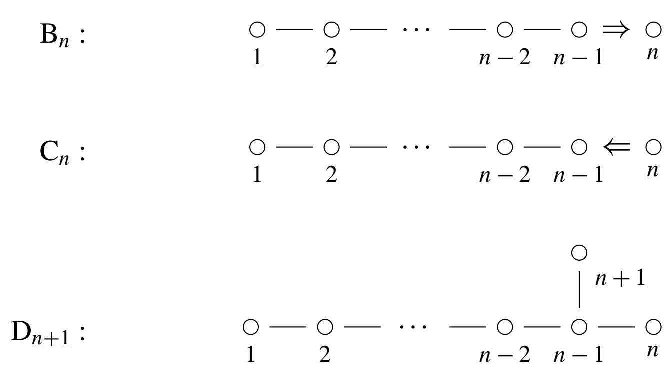
1. Notation

We need some basic terminology of partitions [Macdonald 1995, §I.1]. A partition λ is a sequence of integers $(\lambda_1, \dots, \lambda_r)$ with $\lambda_1 \geq \cdots \geq \lambda_r \geq 0$. We set $|\lambda| = \sum_i \lambda_i$ and $\ell(\lambda) = \max\{i \mid \lambda_i \neq 0\}$. We write $\mu \subseteq \lambda$ if $\mu_i \leq \lambda_i$ for all i and also say that λ contains μ. The notation a^b means the sequence $(a, a, \dots, a)$ where a appears b times. We use $\lambda^\dagger$ to denote the transpose partition of λ, i.e., $\lambda^\dagger_i = \#\{j \mid \lambda_j \geq i\}$ (in terms of Young diagrams, we are flipping across the diagonal). Let $\boldsymbol{S}_\lambda$ denote the corresponding Schur functor [Fulton and Harris 1991, §6.1]; for the purposes of this note, $\boldsymbol{S}_\lambda$ is a functor from the category of complex vector spaces to itself. Special cases are symmetric powers $\boldsymbol{S}_k = \mathrm{Sym}^k$ and exterior powers $\boldsymbol{S}_{(1^k)} = \bigwedge^k$. We use s_λ to denote the Schur function indexed by λ [Macdonald 1995, §I.3] (it is the character of $\boldsymbol{S}_\lambda$). The product of two Schur functions is a linear combination of Schur functions:

$$s_\mu s_\nu = \sum_\lambda c^\lambda_{\mu,\nu} s_\lambda.$$

The $c^\lambda_{\mu,\nu}$ are the Littlewood–Richardson coefficients [Macdonald 1995, §I.9]. If $c^\lambda_{\mu,\nu} \neq 0$, then $|\lambda| = |\mu| + |\nu|$ and also $\mu \subseteq \lambda$ and $\nu \subseteq \lambda$.

Let G be a complex classical group of type B_n, C_n, or D_{n+1}, i.e., G is either $\mathrm{O}_{2n+1}(\mathbb{C})$, $\mathrm{Sp}_{2n}(\mathbb{C})$, or $\mathrm{O}_{2n+2}(\mathbb{C})$, respectively. Let $\mathfrak{g}$ be the Lie algebra of G. Let $\mathrm{rank}(\mathfrak{g})$ be the rank of $\mathfrak{g}$; i.e., it is n in the cases of type B and C, and it is $n+1$ in the case of type D. We use these groups rather than their Lie algebras to avoid having to make technical remarks later. For the representations considered in [CG], this choice will not be important. We number the nodes of the Dynkin diagram according to Bourbaki notation:



Let ω_i be the fundamental weights, and let λ be a dominant integral weight which is a linear combination of $\omega_1, \ldots, \omega_{n-1}$ (so in particular, we avoid the spin representations in the orthogonal case). We will use a basis $e_1, \ldots, e_{\mathrm{rank}(\mathfrak{g})}$ for the weight lattice of G (see [Fulton and Harris 1991, §16.1, §18.1] for details; there the basis is denoted $L_1, \ldots, L_{\mathrm{rank}(\mathfrak{g})}$). Given $\lambda = a_1\omega_1 + \cdots + a_{n-1}\omega_{n-1}$, we associate to it the partition

$$(a_1 + \cdots + a_{n-1}, a_2 + \cdots + a_{n-1}, \ldots, a_{n-1}).$$

So in particular, the notation $\lambda_i = a_i + \cdots + a_{n-1}$ is defined. Then we have $\lambda = \lambda_1 e_1 + \cdots + \lambda_{n-1} e_{n-1}$. Let V_λ be the corresponding highest weight representation of G. We will denote $V = V_1$, the vector representation. We sometimes use the notation V_λ^{O} or V_λ^{Sp} to emphasize that we are dealing with the orthogonal or symplectic case, respectively.

In general, all finite-dimensional irreducible representations V_λ of G can be indexed by partitions λ (see [Fulton and Harris 1991, §17.3, §19.5] or [Sam and Snowden 2013, §4.1]). We may assume that $\ell(\lambda) \le \mathrm{rank}(\mathfrak{g})$ as long as we are ambivalent about the presence of the sign representation in the orthogonal group case. (The reason we do not use the special orthogonal group is because some irreducible representations of the even orthogonal group are not irreducible when restricted to the special orthogonal group, and so the latter group does not behave as well from the perspective of stability.)

Now we rephrase the definitions in [CG, §1.13] in this notation. First, we have $i_\lambda = \ell(\lambda)$. In the orthogonal case, $\Psi_\lambda = \{e_i + e_j \mid 1 \le i < j \le \ell(\lambda)\}$, and in the symplectic case, $\Psi_\lambda = \{e_i + e_j \mid 1 \le i \le j \le \ell(\lambda)\}$. Define the set

$$\Gamma(\lambda, \Psi_\lambda) = \left\{ (\mu, s) \,\middle|\, \lambda = \mu + \sum_{\beta \in \Psi_\lambda} n_\beta \beta,\ n_\beta \in \mathbb{Z}_{\ge 0},\ \sum_{\beta \in \Psi_\lambda} n_\beta = s \right\}.$$

By the definitions of Ψ_λ, we see that $(\mu, s) \in \Gamma(\lambda, \Psi_\lambda)$ implies that $s = (|\lambda| - |\mu|)/2$.

Define $\boldsymbol{h}_k = \mathrm{char}(V_k^{\mathrm{O}})$ in the orthogonal case and $\boldsymbol{h}_k = \sum_{0 \le r \le k/2} \mathrm{char}(V_{k-2r}^{\mathrm{Sp}})$ in the symplectic case. In both cases, define the Jacobi–Trudi determinant

$$\boldsymbol{H}_\lambda = \det(\boldsymbol{h}_{\lambda_i - i + j}).$$

For $(\nu, s) \in \Gamma(\lambda, \Psi_\lambda)$, define

$$C_{\nu,s}^\lambda = \dim \hom_G(V_\nu, \textstyle\bigwedge^s(\mathfrak{g}) \otimes V_\lambda)$$

(see [CG, §2.7], but there it is c instead of C; we use c for Littlewood–Richardson coefficients).

All of the above definitions make sense for any partition λ with $\ell(\lambda) \le \mathrm{rank}(\mathfrak{g})$. To make this clear, we spell out the conversion between partitions and weights now. Let $r = \mathrm{rank}(\mathfrak{g})$ and let $\lambda = (\lambda_1, \ldots, \lambda_r)$ be a partition.

- If $G = \mathrm{Sp}_{2r}(\mathbb{C})$, then V_λ is irreducible with highest weight

$$\sum_{i=1}^{r-1} (\lambda_i - \lambda_{i+1})\omega_i + \lambda_r \omega_r.$$

- If $G = \mathrm{O}_{2r+1}(\mathbb{C})$, then V_λ is irreducible with highest weight

$$\sum_{i=1}^{r-1} (\lambda_i - \lambda_{i+1})\omega_i + 2\lambda_r \omega_r.$$

- If $G = \mathrm{O}_{2r}(\mathbb{C})$, then there are two cases. In both cases, V_λ is an irreducible representation of $\mathrm{O}_{2r}(\mathbb{C})$, but we distinguish between what happens when we pass to the Lie algebra $\mathfrak{so}_{2r}(\mathbb{C})$.
  - If $\lambda_r = 0$, then V_λ is an irreducible representation of $\mathfrak{so}_{2r}(\mathbb{C})$ with highest weight
  $$\sum_{i=1}^{r-2} (\lambda_i - \lambda_{i+1})\omega_i + \lambda_{r-1}(\omega_{r-1} + \omega_r).$$
  - If $\lambda_r > 0$, then as a representation of $\mathfrak{so}_{2r}(\mathbb{C})$, V_λ is the direct sum of irreducible representations with highest weights
  $$\sum_{i=1}^{r-2} (\lambda_i - \lambda_{i+1})\omega_i + (\lambda_{r-1} - \lambda_r)\omega_{r-1} + (\lambda_{r-1} + \lambda_r)\omega_r$$
  and
  $$\sum_{i=1}^{r-2} (\lambda_i - \lambda_{i+1})\omega_i + (\lambda_{r-1} + \lambda_r)\omega_{r-1} + (\lambda_{r-1} - \lambda_r)\omega_r.$$

In the orthogonal case, let d^λ_ν be the multiplicity of V^{Sp}_ν in $\mathbf{S}_\lambda(V^{\mathrm{Sp}})$: here V^{Sp} is the vector representation for $\mathrm{Sp}(2n)$ with $n \geq \ell(\lambda)$ and $\mathbf{S}_\lambda(V^{\mathrm{Sp}})$ is considered as a representation of $\mathrm{Sp}(2n)$. By [Koike and Terada 1987, Proposition 1.5.3], this multiplicity is independent of n as long as $n \geq \ell(\lambda)$, and we have

$$d^\lambda_\nu = \sum_\eta c^\lambda_{\nu,(2\eta)^\dagger}.$$

Similarly, in the symplectic case, let d^λ_ν be the multiplicity of V^{O}_ν in $\mathbf{S}_\lambda(V^{\mathrm{O}})$ (note that we are using branching rules for the *other* group in both cases). Then we have

$$d^\lambda_\nu = \sum_\eta c^\lambda_{\nu,2\eta}.$$

When $\ell(\lambda) \leq n-1$, the following main result proves [CG, Conjecture 1.13].

Theorem 1.1. *Let λ be a partition with $\ell(\lambda) \leq \operatorname{rank}(\mathfrak{g})$. Then*

$$\sum_{(\nu,s)\in\Gamma(\lambda,\Psi_\lambda)} (-1)^s C^{\lambda}_{\nu,s} \boldsymbol{H}_\nu = \operatorname{char}(V_\lambda). \tag{1.2}$$

Also $\boldsymbol{H}_\lambda = \sum_\nu d^\lambda_\nu \operatorname{char}(V_\nu)$.

Remark 1.3. Under the restriction $\ell(\lambda) \leq n-1$, Chari and Greenstein constructed the module $P(\lambda, 0)^{\Gamma(\lambda,\Psi_\lambda)}$ in [CG], and Theorem 1.1 together with [CG, Theorem 2] shows that its character is $\boldsymbol{H}_\lambda$. In types B and C, Naoi [2013, Remark 4.7] shows that these modules are the "graded limits" of the minimal affinizations of the corresponding simple modules V_λ of $\mathfrak{g}$. A similar result is obtained for a special class of highest weights in type D in [Naoi 2014]. In particular, the characters (considered as representations of $\mathfrak{g}$) of both modules are the same. So the character of the minimal affinization is also $\boldsymbol{H}_\lambda$. In type B, this follows from [Hernandez 2007] (see [Naoi 2013, Remark 4.7]) or from [Mukhin and Young 2012, Corollary 7.6]. □

2. Some identities

Let Q_{-1} be the set of partitions with the following inductive definition. The empty partition belongs to Q_{-1}. A nonempty partition μ belongs to Q_{-1} if and only if the number of rows in μ is one more than the number of columns, i.e., $\ell(\mu) = \mu_1 + 1$, and the partition obtained by deleting the first row and column of μ, i.e., $(\mu_2 - 1, \ldots, \mu_{\ell(\mu)} - 1)$, belongs to Q_{-1}. The first few partitions in Q_{-1} are 0, (1, 1), (2, 1, 1), (2, 2, 2). Define $Q_1 = \{\lambda \mid \lambda^\dagger \in Q_{-1}\}$. We record this definition as the following formula:

$$Q_1^\dagger = Q_{-1}. \tag{2.1}$$

The significance of these sets are the following decompositions (see [Macdonald 1995, I.A.7, Examples 4, 5]):

$$\textstyle\bigwedge^i(\operatorname{Sym}^2(E)) = \bigoplus_{\substack{\mu\in Q_1\\ |\mu|=2i}} \boldsymbol{S}_\mu(E), \tag{2.2}$$

$$\textstyle\bigwedge^i\big(\bigwedge^2(E)\big) = \bigoplus_{\substack{\mu\in Q_{-1}\\ |\mu|=2i}} \boldsymbol{S}_\mu(E). \tag{2.3}$$

We need two of Littlewood's identities [Koike and Terada 1987, Proposition 1.5.3]:

$$\operatorname{char}(V^{\mathrm{O}}_\lambda) = \sum_{\mu\in Q_1} (-1)^{|\mu|/2} \sum_\nu c^\lambda_{\mu,\nu} s_\nu, \tag{2.4}$$

$$\operatorname{char}(V^{\mathrm{Sp}}_\lambda) = \sum_{\mu\in Q_{-1}} (-1)^{|\mu|/2} \sum_\nu c^\lambda_{\mu,\nu} s_\nu. \tag{2.5}$$

Lemma 2.6. *Fix* $(\nu, s) \in \Gamma(\lambda, \Psi_\lambda)$, *where* $\ell(\lambda) \le \mathrm{rank}(\mathfrak{g})$ *and* $s = (|\lambda| - |\nu|)/2$. *Then* $C^\lambda_{\nu,s} = \sum_{\mu \in Q_{-1}} c^\lambda_{\mu,\nu}$ *in the orthogonal case* (*for the symplectic case, use* Q_1 *instead of* Q_{-1}).

Conversely, if this sum is nonzero, then $(\nu, s) \in \Gamma(\lambda, \Psi_\lambda)$ *for* $s = (|\lambda| - |\nu|)/2$.

Proof. In the orthogonal case, we have $\mathfrak{g} = V_{1,1} = \bigwedge^2(V)$. So we need to calculate the multiplicity of V_ν in $\bigwedge^s\big(\bigwedge^2(V)\big) \otimes V_\lambda$, where $s = (|\lambda| - |\nu|)/2$. By (2.3), we get

$$\bigwedge^s\big(\bigwedge^2(V)\big) = \bigoplus_{\substack{\mu \in Q_{-1}\\ |\mu| = 2s}} \boldsymbol{S}_\mu(V).$$

(In the symplectic case we instead have $\mathfrak{g} = V_2 = \mathrm{Sym}^2(V)$, so all of the following statements will hold if we replace Q_{-1} with Q_1.) We claim that the multiplicity of V_ν in $\boldsymbol{S}_\mu(V) \otimes V_\lambda$ is the Littlewood–Richardson coefficient $c^\lambda_{\mu,\nu}$.

If $\ell(\mu) \le \mathrm{rank}(\mathfrak{g})$, then as a representation of the orthogonal group (also in the symplectic case), $\boldsymbol{S}_\mu(V)$ is the sum of V_μ and other V_α, where $|\alpha| < |\mu|$ up to twisting V_α with a sign character (this follows from the explicit formula in [Koike and Terada 1987, Proposition 2.5.1]). Also, if V_ν appears in $V_\alpha \otimes V_\lambda$, then we must have $|\nu| \ge |\lambda| - |\alpha|$ by a basic argument with weights. This implies that the multiplicity of V_ν in $\boldsymbol{S}_\mu(V) \otimes V_\lambda$ is the same as the multiplicity of V_ν in $V_\mu \otimes V_\lambda$ under our hypothesis that $|\nu| + |\mu| = |\lambda|$. Furthermore, the multiplicity in this case is the Littlewood–Richardson coefficient $c^\lambda_{\mu,\nu}$ [ibid., Proposition 2.5.2].

If $\ell(\mu) > \mathrm{rank}(\mathfrak{g})$, then the multiplicity of V_ν in $\boldsymbol{S}_\mu(V) \otimes V_\lambda$ is 0 since all V_α in $\boldsymbol{S}_\mu(V)$ satisfy $|\alpha| < |\mu|$. Also, $c^\lambda_{\mu,\nu} = 0$ since $\mu \not\subseteq \lambda$. This proves the claim and the second sentence of the lemma.

Now we handle the last sentence of the lemma. So suppose that $c^\lambda_{\mu,\nu} \ne 0$ for some $\mu \in Q_{-1}$. Set $s = (|\lambda| - |\nu|)/2 = |\mu|/2$. The weights of $\boldsymbol{S}_\mu(V) \subset \bigwedge^s(\mathfrak{g})$ are linear combinations of s roots of $\mathfrak{g}$. In particular, λ is the sum of ν and s roots $\alpha_1, \dots, \alpha_s$ of $\mathfrak{g}$. The possible roots of $\mathfrak{g}$ are $e_i \pm e_j$ and $\pm e_i$. Since $|\nu + e_i - e_j| = |\nu|$ and $|\nu \pm e_i| = |\nu| \pm 1$, the s roots $\alpha_1, \dots, \alpha_s$ must all be of the form $e_i + e_j$, so $(\nu, s) \in \Gamma(\lambda, \Psi_\lambda)$. □

3. Proof of main theorem

Lemma 3.1. *Pick* $\mathrm{X} \in \{\mathrm{B}, \mathrm{C}, \mathrm{D}\}$. *Fix a partition* λ *with* $\ell(\lambda) \le n$. *Then* (1.2) *is true for the representation* V_λ *for* X_n *if and only if it is true for the representation* V_λ *for* X_m *for any* $m \ge n$.

Proof. By [Koike and Terada 1987, Corollary 2.5.3], the tensor product decomposition $V_\lambda \otimes V_\mu$ is independent of m if $m \ge \ell(\lambda) + \ell(\mu)$, and in this case, the tensor product decomposes as a sum of V_α with $\ell(\alpha) \le \ell(\lambda) + \ell(\mu)$. The definition of $\boldsymbol{H}_\lambda$ involves multiplying at most $\ell(\lambda) \le m$ characters, all indexed by one-row partitions, so its definition is independent of m. Certainly the set $\Gamma(\lambda, \Psi_\lambda)$ does not depend on m if $m \geq \ell(\lambda)$. So it remains to show that the coefficients $C^\lambda_{\nu,s}$ are independent of m, but this follows from Lemma 2.6. □

In particular, we may assume that $n = \infty$. In this limit, we can use some additional symmetries of the character ring Λ of $\mathfrak{g}$. Then Λ is the ring of symmetric functions, but is equipped with a new basis which was studied in [ibid.]. Write $s_{[\lambda]} = \mathrm{char}(V_\lambda)$. We use $s^{\mathrm{Sp}}_{[\lambda]}$ or $s^{\mathrm{O}}_{[\lambda]}$ if we need to emphasize the group. Then the $s_{[\lambda]}$, as λ ranges over all partitions, forms a basis for this character ring. The idea is to use (2.4) or (2.5) to exhibit the change of basis between $s_{[\lambda]}$ and the usual Schur functions $s_\mu = \mathrm{char}(\boldsymbol{S}_\mu(V))$. There is an involution (which is an algebra automorphism), denoted i_{O} in the orthogonal case and i_{Sp} in the symplectic case, that sends $s_{[\lambda]}$ to $s_{[\lambda^\dagger]}$ [ibid., Theorem 2.3.4]. Also, we recall that the linear map $\omega\colon s_\lambda \mapsto s_{\lambda^\dagger}$ is an algebra automorphism [Macdonald 1995, §I.3]. We need the following identity [Koike and Terada 1987, Theorem 2.3.2]:

$$\omega(s^{\mathrm{Sp}}_{[\lambda]}) = s^{\mathrm{O}}_{[\lambda^\dagger]}. \tag{3.2}$$

Lemma 3.3. *The involution i_{O} or i_{Sp} sends $\boldsymbol{H}_\nu$ to the Schur function $s_{\nu^\dagger}$.*

Proof. In the orthogonal case, $i_{\mathrm{O}}(\boldsymbol{h}_k) = s_{[1^k]} = \mathrm{char}\big(\bigwedge^k V\big) = s_{1^k}$, and in the symplectic case,

$$i_{\mathrm{Sp}}(\boldsymbol{h}_k) = \sum_{0\leq r\leq k/2} s_{[1^{k-2r}]} = \mathrm{char}\big(\textstyle\bigwedge^k V\big) = s_{1^k}$$

by basic properties of the decomposition of exterior powers under the action of the symplectic group. Since i_{O} and i_{Sp} are algebra homomorphisms, we see that $\boldsymbol{H}_\nu = \det(\boldsymbol{h}_{\nu_i-i+j})$ gets sent to $\det(s_{1^{\nu_i-i+j}})$, which is the Schur function $s_{\nu^\dagger}$ by the Jacobi–Trudi formula [Macdonald 1995, §I.3, equation (3.5)]. □

Now we focus on the orthogonal case (the symplectic case is almost identical). By (2.4),

$$s_{[\lambda]} = \sum_{\mu\in Q_1} (-1)^{|\mu|/2} \sum_\nu c^\lambda_{\mu,\nu} s_\nu.$$

Since $c^\lambda_{\mu,\nu} = c^{\lambda^\dagger}_{\mu^\dagger,\nu^\dagger}$ (use that $s_\mu s_\nu = \sum_\lambda c^\lambda_{\mu,\nu} s_\lambda$ [Macdonald 1995, §I.9] and the involution ω defined above), and $Q_1^\dagger = Q_{-1}$ (2.1), we can rewrite this as

$$s_{[\lambda^\dagger]} = \sum_{\mu\in Q_{-1}} (-1)^{|\mu|/2} \sum_\nu c^\lambda_{\mu,\nu} s_{\nu^\dagger}.$$

In particular, the coefficient of $s_{\nu^\dagger}$ is $\sum_{\mu\in Q_{-1}} (-1)^{(|\lambda|-|\nu|)/2} c^\lambda_{\mu,\nu}$. By Lemma 2.6, we get

$$s_{[\lambda^\dagger]} = \sum_{(\nu,s)\in\Gamma(\lambda,\Psi_\lambda)} (-1)^s C^\lambda_{\nu,s} s_{\nu^\dagger}.$$

Finally, apply the involution i_O to this equation and use Lemma 3.3 to get (1.2). The last part of the theorem follows directly from Lemma 3.3 and (3.2).

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