

*Pacific
Journal of
Mathematics*

**SYSTEMS OF PARAMETERS AND HOLONOMICITY
OF A-HYPERGEOMETRIC SYSTEMS**

CHRISTINE BERKESCH ZAMAERE, STEPHEN GRIFFETH AND EZRA MILLER

SYSTEMS OF PARAMETERS AND HOLONOMICITY OF A -HYPERGEOMETRIC SYSTEMS

CHRISTINE BERKESCH ZAMAERE, STEPHEN GRIFFETH AND EZRA MILLER

We give an elementary proof of holonomicity for A -hypergeometric systems, with no requirements on the behavior of their singularities, a result originally due to Adolphson (1994) after the regular singular case by Gelfand and Gelfand (1986). Our method yields a direct de novo proof that A -hypergeometric systems form holonomic families over their parameter spaces, as shown by Matusevich, Miller, and Walther (2005).

Dedication. Every now and then Andrei Zelevinsky had occasion to write a short and in many ways elementary paper with deep consequences. Particularly close to our hearts are his paper on graded nilpotent classes [Zelevinsky 1985] and his paper with Gelfand and Graev on hypergeometric systems [Gelfand et al. 1987]; both of these had enormous impact on our mathematical careers. It is in that spirit that we dedicate to Andrei this elementary perspective on topics that he influenced substantially for many years.

Introduction

An A -hypergeometric system is the D -module counterpart of a toric ideal. Solutions to A -hypergeometric systems are functions, with a fixed infinitesimal homogeneity, on an affine toric variety. The solution space of an A -hypergeometric system behaves well in part because the system is holonomic, which in particular implies that the vector space of germs of analytic solutions at any nonsingular point has finite dimension.

This note provides an elementary proof of holonomicity for arbitrary A -hypergeometric systems, relying only on the statement that a module over the Weyl algebra in n variables is holonomic precisely when its characteristic variety has dimension at most n (see [Gabber 1981] or [Borel et al. 1987, Theorem 1.12]), along with standard facts about transversality of subvarieties and about Krull

EM had support from NSF grant DMS-1001437. SG acknowledges the financial support of Fondecyt Proyecto Regular 1110072.

MSC2010: primary 33C70; secondary 13N10, 14M25, 16S32, 33C99.

Keywords: hypergeometric, differential operators, toric, holonomic.

dimension. In particular, our proof requires no assumption about the singularities of the A -hypergeometric system; equivalently, the associated toric ideal need not be standard graded. Holonomicity was proved in the regular singular case by Gelfand and Gelfand [1986], and later by Adolphson [1994, §3] regardless of the behavior of the singularities of the system. Adolphson’s proof relies on careful algebraic analysis of the coordinate rings of a collection of varieties whose union is the characteristic variety of the system. Another proof of the holonomicity of an A -hypergeometric system, by Schulze and Walther [2008], yields a more general result: for a weight vector L from a large family of possibilities, the L -characteristic variety for the L -filtration is a union of conormal varieties and hence has dimension n . Holonomicity follows when $L = (0, \dots, 0, 1, \dots, 1)$ induces the order filtration on the Weyl algebra. The L -filtration method uses an explicit combinatorial interpretation of initial ideals of toric ideals, which requires a series of technical lemmas.

Holonomicity of A -hypergeometric systems forms part of the statement and proof, by Matusевич, Miller, and Walther [2005], that A -hypergeometric systems determine holonomic families over their parameter spaces. The new proof of that statement here serves as a model suitable for generalization to hypergeometric systems for reductive groups, in the sense of Kapranov [1998].

The main step (Theorem 1.2) in our proof is an easy geometric argument showing that the Euler operators corresponding to the rows of an integer matrix A form part of a system of parameters on the product $\mathbb{k}^n \times X_A$, where \mathbb{k} is any algebraically closed field and X_A is the toric variety over \mathbb{k} determined by A . This observation leads quickly in Section 2 to the conclusion that the characteristic variety of the associated A -hypergeometric system has dimension at most n , and hence that the system is holonomic. Since the algebraic part of the proof holds when the entries of the parameter β are considered as independent variables that commute with all other variables, the desired stronger consequence is immediate: the A -hypergeometric system forms a holonomic family over its parameter space (Theorem 2.1).

1. Systems of parameters via transversality

Fix a field \mathbb{k} . Let $x = x_1, \dots, x_n$ and $\xi = \xi_1, \dots, \xi_n$ be sets of coordinates on \mathbb{k}^n and let $x\xi$ denote the column vector with entries $x_1\xi_1, \dots, x_n\xi_n$. Given a rectangular matrix L with n columns, write $Lx\xi$ for the vector of bilinear forms given by multiplying L times $x\xi$.

Lemma 1.1. *Let $\mathbb{k}^{2n} = \mathbb{k}_x^n \times \mathbb{k}_\xi^n$ have coordinates (x, ξ) and let $X \subseteq \mathbb{k}_\xi^n$ be a subvariety. If L is an $\ell \times n$ matrix with entries in \mathbb{k} , then the variety $\text{Var}(Lx\xi)$ of $Lx\xi$ in \mathbb{k}^{2n} is transverse to $\mathbb{k}^n \times X$ at any smooth point of $\mathbb{k}^n \times X$ whose ξ -coordinates are all nonzero.*

Proof. It suffices to prove the statement after passing to the algebraic closure of \mathbb{k} , so assume \mathbb{k} is algebraically closed. Let (p, q) be a smooth point of $\mathbb{k}^n \times X$ that lies in $\text{Var}(Lx\xi)$ and has all coordinates of q nonzero. The tangent space to $\mathbb{k}^n \times X$ at (p, q) contains $\mathbb{k}^n \times \{0\}$. The tangent space $T_{(p,q)}$ to $\text{Var}(Lx\xi)$ is the kernel of the $\ell \times 2n$ matrix $[L(q) \ L(p)]$, where $L(p)$ (respectively, $L(q)$) is the $\ell \times n$ matrix that results after multiplying each column of L by the corresponding coordinate of p (respectively, q). Since the q coordinates are all nonzero, $T_{(p,q)}$ projects surjectively onto the last n coordinates; indeed, if $\eta \in \mathbb{k}_\xi^n$ is given, then taking $y_i = -p_i \eta_i / q_i$ yields $y \in \mathbb{k}_x^n$ with $L(q)y + L(p)\eta = 0$. Thus the tangent spaces at (p, q) sum to the ambient space, so the intersection is transverse. \square

The next result applies the lemma to an affine toric variety X . A fixed $d \times n$ integer matrix $A = [a_1 \ a_2 \ \dots \ a_{n-1} \ a_n]$ defines an action of the algebraic torus $T = (\mathbb{k}^*)^d$ on \mathbb{k}_ξ^n by

$$t \cdot \xi = (t^{a_1} \xi_1, \dots, t^{a_n} \xi_n).$$

The orbit $\text{Orb}(A)$ of the point $\mathbf{1} = (1, \dots, 1) \in \mathbb{k}^n$ is the image of an algebraic map $T \rightarrow \mathbb{k}^n$ that sends $t \mapsto t \cdot \mathbf{1}$. The closure of $\text{Orb}(A)$ in \mathbb{k}^n is the affine toric variety $X_A = \text{Var}(I_A)$ cut out by the *toric ideal*

$$I_A = \langle \xi^u - \xi^v \mid Au = Av \rangle \subseteq \mathbb{k}[\xi]$$

of A in the polynomial ring $\mathbb{k}[\xi] = \mathbb{k}[\xi_1, \dots, \xi_n]$. The T -action induces an A -grading on $\mathbb{k}[\xi]$ via $\deg(\xi_i) = a_i$, and the semigroup ring $S_A = \mathbb{k}[\xi]/I_A$ is A -graded [Miller and Sturmfels 2005, Chapters 7–8].

For any face τ of the real cone $\mathbb{R}_{\geq 0}A$ generated by the columns of A , write $\tau \preceq A$ and let $\mathbf{1}^\tau \in \{0, 1\}^n \subset \mathbb{k}^n$ be the vector with nonzero entry $\mathbf{1}_i^\tau = 1$ precisely when A has a nonzero column $a_i \in \tau$. The variety X_A decomposes as a finite disjoint union $X_A = \bigsqcup_{\tau \preceq A} \text{Orb}(\tau)$ of orbits, where $\text{Orb}(\tau) = T \cdot \mathbf{1}^\tau$. Each orbit has dimension $\dim \text{Orb}(\tau) = \text{rank}(A_\tau)$, where A_τ is the submatrix of A consisting of those columns lying in τ , and $\dim X_A = \text{rank}(A)$.

Theorem 1.2. *The ring $\mathbb{k}[x, \xi]/(I_A + \langle Ax\xi \rangle)$ has Krull dimension n . In particular, if A has rank d then the forms $Ax\xi$ are part of a system of parameters for $\mathbb{k}[x] \otimes_{\mathbb{k}} S_A$.*

Proof. Let $\mathbb{k}^\tau \subseteq \mathbb{k}^n$ be the subspace consisting of vectors with 0 in coordinate i if $a_i \notin \tau$, and let $|\tau|$ be its dimension. Since $\mathbb{k}[x, \xi]/I_A = \mathbb{k}[x] \otimes_{\mathbb{k}} S_A$ has dimension $n + \text{rank}(A)$ and the number of \mathbb{k} -linearly independent generators of $\langle Ax\xi \rangle$ is $\text{rank}(A)$, the Krull dimension in question is at least n . Hence it suffices to prove that $(\mathbb{k}^n \times \text{Orb}(\tau)) \cap \text{Var}(Ax\xi) \subseteq \mathbb{k}^n \times \mathbb{k}^\tau$ has dimension at most n . Let x_τ and ξ_τ denote the subsets corresponding to τ in the variable sets x and ξ , respectively. The projection of the intersection onto the subspace $\mathbb{k}^\tau \times \mathbb{k}^\tau$ has image contained in

$$(\mathbb{k}^\tau \times \text{Orb}(\tau)) \cap \text{Var}(A_\tau x_\tau \xi_\tau) \subseteq \mathbb{k}^\tau \times \mathbb{k}^\tau.$$

It therefore suffices to show that the dimension of this latter intersection is at most $|\tau|$. By [Lemma 1.1](#), the intersection is transverse in $\mathbb{k}^\tau \times \mathbb{k}^\tau$. But the dimension of $\text{Orb}(\tau)$ is the codimension of $\text{Var}(A_\tau x_\tau \xi_\tau)$ in $\mathbb{k}^\tau \times \mathbb{k}^\tau$, which completes the proof. \square

2. Hypergeometric holonomicity

In this section, the matrix A is a $d \times n$ integer matrix of full rank d . Let

$$D = \mathbb{C}\langle x, \partial \mid [\partial_i, x_j] = \delta_{ij} \text{ and } [x_i, x_j] = 0 = [\partial_i, \partial_j] \rangle$$

denote the Weyl algebra over the complex numbers \mathbb{C} , where $x = x_1, \dots, x_n$ and ∂_i corresponds to $\partial/\partial x_i$. This is the ring of \mathbb{C} -linear differential operators on $\mathbb{C}[x]$.

For $\beta \in \mathbb{C}^d$, the A -hypergeometric system with parameter β is the left D -module

$$M_A(\beta) = D/D \cdot (I_A^\partial, \{E_i - \beta_i\}_{i=1}^d),$$

where $I_A^\partial = \langle \partial^u - \partial^v \mid Au = Av \rangle \subseteq \mathbb{C}[\partial]$ is the toric ideal associated to A and

$$E_i - \beta_i = \sum_{j=1}^n a_{ij} x_j \partial_j - \beta_i$$

are Euler operators associated to A .

The order filtration F filters D by order of differential operators. The symbol of an operator P is its image $\text{in}(P) \in \text{gr}^F D$. Writing $\xi_i = \text{in}(\partial_i)$, this means $\text{gr}^F D$ is the commutative polynomial ring $\mathbb{C}[x, \xi]$. The characteristic variety of a left D -module M is the variety in \mathbb{A}^{2n} of the associated graded ideal $\text{gr}^F \text{ann}(M)$ of the annihilator of M . A nonzero D -module is holonomic if its characteristic variety has dimension n ; this is equivalent to requiring that the dimension be at most n (see [\[Gabber 1981\]](#) or [\[Borel et al. 1987, Theorem 1.12\]](#)). The rank of a holonomic D -module M is the (always finite) dimension of $\mathbb{C}(x) \otimes_{\mathbb{C}[x]} M$ as a vector space over $\mathbb{C}(x)$; this number equals the dimension of the vector space of germs of analytic solutions of M at any nonsingular point in \mathbb{C}^n [\[Saito et al. 2000, Theorem 1.4.9\]](#).

Viewing the A -hypergeometric system $M_A(\beta)$ as having a varying parameter $\beta \in \mathbb{C}^d$, the rank of $M_A(\beta)$ is upper semicontinuous as a function of β [\[Matusevich et al. 2005, Theorem 2.6\]](#). This follows by viewing $M_A(\beta)$ as a holonomic family [\[ibid., Definition 2.1\]](#) parametrized by $\beta \in \mathbb{C}^d$. By definition, this means not only that $M_A(\beta)$ is holonomic for each β , but also that it satisfies a coherence condition over \mathbb{C}^d : after replacing β with variables $b = b_1, \dots, b_d$, the module $\mathbb{C}(x) \otimes_{\mathbb{C}[x]} M_A(b)$ is finitely generated over $\mathbb{C}(x)[b]$. (The definition of holonomic family cited above allows sheaves of D -modules over arbitrary complex base schemes, but that generality is not needed here.)

The derivation of the holonomic family property for $M_A(b)$ from the holonomicity of the A -hypergeometric system is more or less the same as [ibid., Theorem 7.5], which was phrased in the generality of Euler–Koszul homology of toric modules. The brief deduction here isolates the steps necessary for A -hypergeometric systems; its brevity stems from the special status of affine semigroup rings among all toric modules [ibid., Definition 4.5]. Note further that this proof does not require technical combinatorial arguments using standard pairs, as in [Saito et al. 2000]; indeed, $\text{in}(I_A)$ need not be a monomial ideal.

Theorem 2.1. *The module $M_A(b)$ forms a holonomic family over \mathbb{C}^d with coordinates b . In more detail, as a $D[b]$ -module the parametric A -hypergeometric system $M_A(b)$ satisfies:*

- (1) *the fiber $M_A(\beta) = M_A(b) \otimes_{\mathbb{C}[b]} \mathbb{C}[b]/\langle b - \beta \rangle$ is holonomic for all β ; and*
- (2) *the module $\mathbb{C}(x) \otimes_{\mathbb{C}[x]} M_A(b)$ is finitely generated over $\mathbb{C}(x)[b]$.*

Proof. Since $R = \mathbb{C}[x, \xi]/\langle \text{in}(I_A), Ax\xi \rangle$ surjects onto $\text{gr}^F M_A(\beta)$, it is enough to show that the ring R has dimension n . If $M_A(\beta)$ is standard \mathbb{Z} -graded (equivalently, the rowspan of A over the rational numbers contains the row $[1 \ 1 \ \cdots \ 1 \ 1]$ of length n), then $\text{in}(I_A) = I_A \subseteq \mathbb{C}[\xi]$, and the result follows from Theorem 1.2.

When $M_A(\beta)$ is not standard \mathbb{Z} -graded, let \hat{A} be the $(d+1) \times (n+1)$ matrix obtained by adding a row of 1's across the top of A and then adding as the leftmost column $(1, 0, \dots, 0)$. If ξ_0 denotes a new variable corresponding to the leftmost column of \hat{A} , and $\hat{\xi} = \{\xi_0\} \cup \xi$, then $\mathbb{C}[\xi]/\text{in}(I_A) \cong \mathbb{C}[\hat{\xi}]/\langle I_{\hat{A}}, \xi_0 \rangle$. In particular,

$$\frac{\mathbb{C}[\hat{x}, \hat{\xi}]}{\langle \text{in}(I_A), Ax\xi \rangle} \cong \frac{\mathbb{C}[\hat{x}, \hat{\xi}]}{\langle I_{\hat{A}}, \xi_0, \hat{A}\hat{x}\hat{\xi} \rangle},$$

where $\hat{x} = \{x_0\} \cup x$. Since $\langle I_{\hat{A}}, \xi_0 \rangle$ is \hat{A} -graded and \hat{A} has a row $[1 \ 1 \ \cdots \ 1 \ 1]$, we have reduced to the case where $M_A(\beta)$ is \mathbb{Z} -graded, completing part (1).

With R as in part (1), the ring $R[b]$ surjects onto $\text{gr}^F M_A(b)$, so it suffices for part (2) to show that $R[b]$ becomes finitely generated over $\mathbb{C}(x)[b]$ upon inverting all nonzero polynomials in x . Since the ideal $\langle \text{in}(I_A), Ax\xi \rangle$ has no generators involving b variables, it suffices to show that $R(x)$ itself has finite dimension over $\mathbb{C}(x)$. The desired result follows from the statement proved for part (1): any scheme of dimension n has finite degree over \mathbb{C}_x^n . \square

References

- [Adolphson 1994] A. Adolphson, “Hypergeometric functions and rings generated by monomials”, *Duke Math. J.* **73**:2 (1994), 269–290. MR 96c:33020 Zbl 0804.33013
- [Borel et al. 1987] A. Borel, P.-P. Grivel, B. Kaup, A. Haefliger, B. Malgrange, and F. Ehlers, *Algebraic D-modules*, Perspectives in Mathematics **2**, Academic Press, Boston, 1987. MR 89g:32014 Zbl 0642.32001

- [Gabber 1981] O. Gabber, “The integrability of the characteristic variety”, *Amer. J. Math.* **103**:3 (1981), 445–468. [MR 82j:58104](#) [Zbl 0492.16002](#)
- [Gelfand and Gelfand 1986] I. M. Gelfand and S. I. Gelfand, “Generalized hypergeometric equations”, *Dokl. Akad. Nauk SSSR* **288**:2 (1986), 279–283. In Russian; translated in *Sov. Math., Dokl.* **33** (1986), 643–646. [MR 87h:22013](#) [Zbl 0634.58030](#)
- [Gelfand et al. 1987] I. M. Gelfand, M. I. Graev, and A. V. Zelevinsky, “Holonomic systems of equations and series of hypergeometric type”, *Dokl. Akad. Nauk SSSR* **295**:1 (1987), 14–19. In Russian; translated in *Soviet Math. Dokl.* **36**:1 (1988), 5–10. [MR 88j:58118](#) [Zbl 0661.22005](#)
- [Kapranov 1998] M. Kapranov, “Hypergeometric functions on reductive groups”, pp. 236–281 in *Integrable systems and algebraic geometry* (Kobe/Kyoto, 1997), edited by M.-H. Saito et al., World Sci. Publ., River Edge, NJ, 1998. [MR 2000e:33018](#) [Zbl 0987.33008](#)
- [Matusевич et al. 2005] L. F. Matusевич, E. Miller, and U. Walther, “Homological methods for hypergeometric families”, *J. Amer. Math. Soc.* **18**:4 (2005), 919–941. [MR 2007d:13027](#) [Zbl 1095.13033](#)
- [Miller and Sturmfels 2005] E. Miller and B. Sturmfels, *Combinatorial commutative algebra*, Graduate Texts in Mathematics **227**, Springer, New York, 2005. [MR 2006d:13001](#) [Zbl 1090.13001](#)
- [Saito et al. 2000] M. Saito, B. Sturmfels, and N. Takayama, *Gröbner deformations of hypergeometric differential equations*, Algorithms and Computation in Mathematics **6**, Springer, Berlin, 2000. [MR 2001i:13036](#) [Zbl 0946.13021](#)
- [Schulze and Walther 2008] M. Schulze and U. Walther, “Irregularity of hypergeometric systems via slopes along coordinate subspaces”, *Duke Math. J.* **142**:3 (2008), 465–509. [MR 2009b:13067](#) [Zbl 1144.13012](#)
- [Zelevinsky 1985] A. V. Zelevinsky, “Two remarks on graded nilpotent classes”, *Uspekhi Mat. Nauk* **40**:1(241) (1985), 199–200. In Russian; translated in *Russ. Math. Surv.* **40**:1 (1985), 249–250. [MR 86e:14027](#) [Zbl 0577.20032](#)

Received January 10, 2015. Revised February 19, 2015.

CHRISTINE BERKESCH ZAMAERE
 SCHOOL OF MATHEMATICS
 UNIVERSITY OF MINNESOTA
 MINNEAPOLIS, MN 55455
 UNITED STATES
cberkes@math.umn.edu

STEPHEN GRIFFETH
 INSTITUTO DE MATEMÁTICA Y FÍSICA
 UNIVERSIDAD DE TALCA
 TALCA
 CHILE
sgriffeth@inst-mat.otalca.cl

EZRA MILLER
 MATHEMATICS DEPARTMENT
 DUKE UNIVERSITY
 DURHAM, NC 27708
 UNITED STATES
ezra@math.duke.edu

PACIFIC JOURNAL OF MATHEMATICS

msp.org/pjm

Founded in 1951 by E. F. Beckenbach (1906–1982) and F. Wolf (1904–1989)

EDITORS

Don Blasius (Managing Editor)
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
blasius@math.ucla.edu

Paul Balmer
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
balmer@math.ucla.edu

Robert Finn
Department of Mathematics
Stanford University
Stanford, CA 94305-2125
finn@math.stanford.edu

Sorin Popa
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
popa@math.ucla.edu

Vyjayanthi Chari
Department of Mathematics
University of California
Riverside, CA 92521-0135
chari@math.ucr.edu

Kefeng Liu
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
liu@math.ucla.edu

Jie Qing
Department of Mathematics
University of California
Santa Cruz, CA 95064
qing@cats.ucsc.edu

Daryl Cooper
Department of Mathematics
University of California
Santa Barbara, CA 93106-3080
cooper@math.ucsb.edu

Jiang-Hua Lu
Department of Mathematics
The University of Hong Kong
Pokfulam Rd., Hong Kong
jhlu@maths.hku.hk

Paul Yang
Department of Mathematics
Princeton University
Princeton NJ 08544-1000
yang@math.princeton.edu

PRODUCTION

Silvio Levy, Scientific Editor, production@msp.org

SUPPORTING INSTITUTIONS

ACADEMIA SINICA, TAIPEI
CALIFORNIA INST. OF TECHNOLOGY
INST. DE MATEMÁTICA PURA E APLICADA
KEIO UNIVERSITY
MATH. SCIENCES RESEARCH INSTITUTE
NEW MEXICO STATE UNIV.
OREGON STATE UNIV.

STANFORD UNIVERSITY
UNIV. OF BRITISH COLUMBIA
UNIV. OF CALIFORNIA, BERKELEY
UNIV. OF CALIFORNIA, DAVIS
UNIV. OF CALIFORNIA, LOS ANGELES
UNIV. OF CALIFORNIA, RIVERSIDE
UNIV. OF CALIFORNIA, SAN DIEGO
UNIV. OF CALIF., SANTA BARBARA

UNIV. OF CALIF., SANTA CRUZ
UNIV. OF MONTANA
UNIV. OF OREGON
UNIV. OF SOUTHERN CALIFORNIA
UNIV. OF UTAH
UNIV. OF WASHINGTON
WASHINGTON STATE UNIVERSITY

These supporting institutions contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its contents or policies.


See inside back cover or msp.org/pjm for submission instructions.

The subscription price for 2015 is US \$420/year for the electronic version, and \$570/year for print and electronic. Subscriptions, requests for back issues and changes of subscribers address should be sent to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163, U.S.A. The Pacific Journal of Mathematics is indexed by [Mathematical Reviews](#), [Zentralblatt MATH](#), [PASCAL CNRS Index](#), [Referativnyi Zhurnal](#), [Current Mathematical Publications](#) and [Web of Knowledge \(Science Citation Index\)](#).

The Pacific Journal of Mathematics (ISSN 0030-8730) at the University of California, c/o Department of Mathematics, 798 Evans Hall #3840, Berkeley, CA 94720-3840, is published twelve times a year. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163.

PJM peer review and production are managed by EditFLOW[®] from Mathematical Sciences Publishers.

PUBLISHED BY

 **mathematical sciences publishers**
nonprofit scientific publishing

<http://msp.org/>

© 2015 Mathematical Sciences Publishers

PACIFIC JOURNAL OF MATHEMATICS

Volume 276 No. 2 August 2015

Free evolution on algebras with two states, II MICHAEL ANSHELEVICH	257
Systems of parameters and holonomicity of A -hypergeometric systems CHRISTINE BERKESCH ZAMAERE, STEPHEN GRIFFETH and EZRA MILLER	281
Complex interpolation and twisted Hilbert spaces FÉLIX CABELLO SÁNCHEZ, JESÚS M. F. CASTILLO and NIGEL J. KALTON	287
The ramification group filtrations of certain function field extensions JEFFREY A. CASTAÑEDA and QINGQUAN WU	309
A mean field type flow, II: Existence and convergence JEAN-BAPTISTE CASTÉRAS	321
Isometric embedding of negatively curved complete surfaces in Lorentz–Minkowski space BING-LONG CHEN and LE YIN	347
The complex Monge–Ampère equation on some compact Hermitian manifolds JIANCHUN CHU	369
Topological and physical link theory are distinct ALEXANDER COWARD and JOEL HASS	387
The measures of asymmetry for coproducts of convex bodies QI GUO, JINFENG GUO and XUNLI SU	401
Regularity and analyticity of solutions in a direction for elliptic equations YONGYANG JIN, DONGSHENG LI and XU-JIA WANG	419
On the density theorem for the subdifferential of convex functions on Hadamard spaces MINA MOVAHEDI, DARYOUSH BEHMARDI and SEYEDEHSOMAYEH HOSSEINI	437
L^p regularity of weighted Szegő projections on the unit disc SAMANGI MUNASINGHE and YUNUS E. ZEYTUNCU	449
Topology of complete Finsler manifolds admitting convex functions SORIN V. SABAU and KATSUHIRO SHIOHAMA	459
Variations of the telescope conjecture and Bousfield lattices for localized categories of spectra F. LUKE WOLCOTT	483