

Pacific Journal of Mathematics

**CORRECTION TO THE ARTICLE
CLOSED ORBITS OF A CHARGE IN
A WEAKLY EXACT MAGNETIC FIELD**

WILL J. MERRY

CORRECTION TO THE ARTICLE CLOSED ORBITS OF A CHARGE IN A WEAKLY EXACT MAGNETIC FIELD

WILL J. MERRY

Volume **247**:1 (2010), 189–212

Theorem 5.1 of the titular article is incorrect, as pointed out by Gabriele Benedetti. We describe the error and supply an alternative proof for the article’s main result (Theorem 5.8).

1. Introduction

In this erratum I use the notation and numbering from [Merry 2010]. The problem, pointed out to me by G. Benedetti, resides in its Theorem 5.1; embarrassingly, the function $f : \mathbb{R}^+ \rightarrow \mathbb{R}$ defined by $f(x) := e^{-x}$ already provides a counterexample. One can take \mathcal{F}_n to be the set of singletons $\{x\}$ for $x \in (0, n)$. Theorem 5.1 then erroneously concludes that f has a critical point x_∞ with $f(x_\infty) = 0$, which is, of course, incorrect.

Luckily, the error in Theorem 5.1 does not affect the main result (Theorem 5.8). In fact, whilst attempting to salvage the proof of Theorem 5.8, I realised that the entire argument could be dramatically simplified by the following observation: *Theorem 3.2 still holds in the case $c(g, \sigma) = \infty$.* The proof of this statement is explained below. Once this is established, Contreras’ original argument [2006, Proposition 7.1] can be used directly to obtain [Merry 2010, Theorem 5.8].

L. Asselle and G. Benedetti [2015, Lemma 3.5] independently noticed that Theorem 5.8 could be proved by making use of this observation. In their paper, however, they take these ideas considerably further and extend the main result of [Merry 2010] to cover cases in which the magnetic form is *not* weakly exact.

2. The correction

All references in this section are to [Merry 2010]. Let us explain why Theorem 3.2 continues to hold even in the case $c(g, \sigma) = \infty$. We need only verify that the

MSC2010: 37J45, 70H12.

Keywords: magnetic flow, twisted geodesic flow, periodic orbits, Mañé critical value.

additional hypothesis in Proposition 3.7 — which deals specifically with the case $c(g, \sigma) = \infty$ — is superfluous. More precisely, we show that the hypotheses of Theorem 3.2 automatically imply that the hypotheses of Proposition 3.7 are satisfied, which therefore implies that Theorem 3.2 continues to hold in the case $c(g, \sigma) = \infty$.

Thus, we are given a sequence $(x_n, T_n) \in \mathbb{D}(A, B, k, 0)$, and we must show that there always exists a compact subset $K \subset \tilde{M}$ such that $x_n \in \Lambda_0^K$ for all $n \in \mathbb{N}$. For this it is enough to show that the energy e_n of (x_n, T_n) (defined on the bottom of page 197) is uniformly bounded. This then implies that the length l_n of x_n is bounded (compare Equation (3-1)), which immediately implies that such a compact set $K \subset \tilde{M}$ exists. To see that e_n is bounded, we use Equation (2-6), which tells us

$$\frac{1}{n} \geq \left| \frac{\partial}{\partial T} S_k(x_n, T_n) \right| = \left| \frac{1}{T_n} \int_0^{T_n} (k - E(y_n, \dot{y}_n)) dt \right| = \left| k - \frac{e_n}{T_n} \right|.$$

Since $|T_n| \leq B$ by assumption, e_n is necessarily bounded, as required.

Acknowledgement

I thank Gabriele Benedetti for patiently and repeatedly explaining to me why my Theorem 5.1 was false. I am particularly grateful for the considerable tact he showed while outlining to me how the quintessential function one uses to teach students the necessity of the Palais–Smale condition — namely $x \mapsto e^{-x}$ — provided a counterexample.

References

- [Asselle and Benedetti 2015] L. Asselle and G. Benedetti, “The Lusternik–Fet theorem for autonomous Tonelli Hamiltonian systems on twisted cotangent bundles”, 2015. arXiv 1412.0531v3
- [Contreras 2006] G. Contreras, “The Palais–Smale condition on contact type energy levels for convex Lagrangian systems”, *Calc. Var. Partial Differential Eq.* **27**:3 (2006), 321–395. MR 2007i:37116 Zbl 1105.37037
- [Merry 2010] W. J. Merry, “Closed orbits of a charge in a weakly exact magnetic field”, *Pacific J. Math.* **247**:1 (2010), 189–212. MR 2012b:37168 Zbl 1246.37082

Received January 8, 2015.

WILL J. MERRY
 DEPARTEMENT MATHEMATIK
 ETH ZÜRICH
 RÄMISTRASSE 101
 CH-8092 ZÜRICH
 SWITZERLAND
 merry@math.ethz.ch

PACIFIC JOURNAL OF MATHEMATICS

msp.org/pjm

Founded in 1951 by E. F. Beckenbach (1906–1982) and F. Wolf (1904–1989)

EDITORS

Don Blasius (Managing Editor)
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
blasius@math.ucla.edu

Paul Balmer
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
balmer@math.ucla.edu

Robert Finn
Department of Mathematics
Stanford University
Stanford, CA 94305-2125
finn@math.stanford.edu

Sorin Popa
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
popa@math.ucla.edu

Vyjayanthi Chari
Department of Mathematics
University of California
Riverside, CA 92521-0135
chari@math.ucr.edu

Kefeng Liu
Department of Mathematics
University of California
Los Angeles, CA 90095-1555
liu@math.ucla.edu

Jie Qing
Department of Mathematics
University of California
Santa Cruz, CA 95064
qing@cats.ucsc.edu

Daryl Cooper
Department of Mathematics
University of California
Santa Barbara, CA 93106-3080
cooper@math.ucsb.edu

Jiang-Hua Lu
Department of Mathematics
The University of Hong Kong
Pokfulam Rd., Hong Kong
jhlu@maths.hku.hk

Paul Yang
Department of Mathematics
Princeton University
Princeton NJ 08544-1000
yang@math.princeton.edu

PRODUCTION

Silvio Levy, Scientific Editor, production@msp.org

SUPPORTING INSTITUTIONS

ACADEMIA SINICA, TAIPEI
CALIFORNIA INST. OF TECHNOLOGY
INST. DE MATEMÁTICA PURA E APLICADA
KEIO UNIVERSITY
MATH. SCIENCES RESEARCH INSTITUTE
NEW MEXICO STATE UNIV.
OREGON STATE UNIV.

STANFORD UNIVERSITY
UNIV. OF BRITISH COLUMBIA
UNIV. OF CALIFORNIA, BERKELEY
UNIV. OF CALIFORNIA, DAVIS
UNIV. OF CALIFORNIA, LOS ANGELES
UNIV. OF CALIFORNIA, RIVERSIDE
UNIV. OF CALIFORNIA, SAN DIEGO
UNIV. OF CALIF., SANTA BARBARA

UNIV. OF CALIF., SANTA CRUZ
UNIV. OF MONTANA
UNIV. OF OREGON
UNIV. OF SOUTHERN CALIFORNIA
UNIV. OF UTAH
UNIV. OF WASHINGTON
WASHINGTON STATE UNIVERSITY

These supporting institutions contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its contents or policies.

See inside back cover or msp.org/pjm for submission instructions.

The subscription price for 2016 is US \$440/year for the electronic version, and \$600/year for print and electronic. Subscriptions, requests for back issues and changes of subscribers address should be sent to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163, U.S.A. The Pacific Journal of Mathematics is indexed by Mathematical Reviews, Zentralblatt MATH, PASCAL CNRS Index, Referativnyi Zhurnal, Current Mathematical Publications and Web of Knowledge (Science Citation Index).

The Pacific Journal of Mathematics (ISSN 0030-8730) at the University of California, c/o Department of Mathematics, 798 Evans Hall #3840, Berkeley, CA 94720-3840, is published twelve times a year. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163.

PJM peer review and production are managed by EditFlow® from Mathematical Sciences Publishers.

PUBLISHED BY



mathematical sciences publishers
nonprofit scientific publishing

<http://msp.org/>

© 2016 Mathematical Sciences Publishers

PACIFIC JOURNAL OF MATHEMATICS

Volume 280 No. 1 January 2016

Stable capillary hypersurfaces in a wedge	1
JAIGYOUNG CHOE and MIYUKI KOISO	
The Chern–Simons invariants for the double of a compression body	17
DAVID L. DUNCAN	
Compactness and the Palais–Smale property for critical Kirchhoff equations in closed manifolds	41
EMMANUEL HEBEY	
On the equivalence of the definitions of volume of representations	51
SUNGWON KIM	
Strongly positive representations of even GSpin groups	69
YEANSU KIM	
An Orlik–Raymond type classification of simply connected 6-dimensional torus manifolds with vanishing odd-degree cohomology	89
SHINTARÔ KUROKI	
Solutions with large number of peaks for the supercritical Hénon equation	115
ZHONGYUAN LIU and SHUANGJIE PENG	
Effective divisors on the projective line having small diagonals and small heights and their application to adelic dynamics	141
YÛSUKE OKUYAMA	
Computing higher Frobenius–Schur indicators in fusion categories constructed from inclusions of finite groups	177
PETER SCHAUENBURG	
Chordal generators and the hydrodynamic normalization for the unit ball	203
SEBASTIAN SCHLEISSINGER	
On a question of A. Balog	227
ILYA D. SHKREDOV	
Uniqueness result on nonnegative solutions of a large class of differential inequalities on Riemannian manifolds	241
YUHUA SUN	
Correction to “Closed orbits of a charge in a weakly exact magnetic field”	255
WILL J. MERRY	



0030-8730(2016)280:1;1-6