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WILL J. MERRY

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Theorem 5.1 of the titular article is incorrect, as pointed out by Gabriele Benedetti. We describe the error and supply an alternative proof for the article's main result (Theorem 5.8).

1. Introduction

In this erratum I use the notation and numbering from [Merry 2010]. The problem, pointed out to me by G. Benedetti, resides in its Theorem 5.1; embarrassingly, the function $f : \mathbb{R}^+ \to \mathbb{R}$ defined by $f(x) := e^{-x}$ already provides a counterexample. One can take $\mathcal{F}_n$ to be the set of singletons $\{x\}$ for $x \in (0, n)$. Theorem 5.1 then erroneously concludes that f has a critical point x_∞ with $f(x_\infty) = 0$, which is, of course, incorrect.

Luckily, the error in Theorem 5.1 does not affect the main result (Theorem 5.8). In fact, whilst attempting to salvage the proof of Theorem 5.8, I realised that the entire argument could be dramatically simplified by the following observation: *Theorem 3.2 still holds in the case* $c(g, \sigma) = \infty$. The proof of this statement is explained below. Once this is established, Contreras' original argument [2006, Proposition 7.1] can be used directly to obtain [Merry 2010, Theorem 5.8].

L. Asselle and G. Benedetti [2015, Lemma 3.5] independently noticed that Theorem 5.8 could be proved by making use of this observation. In their paper, however, they take these ideas considerably further and extend the main result of [Merry 2010] to cover cases in which the magnetic form is *not* weakly exact.

2. The correction

All references in this section are to [Merry 2010]. Let us explain why Theorem 3.2 continues to hold even in the case $c(g, \sigma) = \infty$. We need only verify that the

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additional hypothesis in Proposition 3.7 — which deals specifically with the case $c(g, \sigma) = \infty$ — is superfluous. More precisely, we show that the hypotheses of Theorem 3.2 automatically imply that the hypotheses of Proposition 3.7 are satisfied, which therefore implies that Theorem 3.2 continues to hold in the case $c(g, \sigma) = \infty$.

Thus, we are given a sequence $(x_n, T_n) \subset \mathbb{D}(A, B, k, 0)$, and we must show that there always exists a compact subset $K \subset \widetilde{M}$ such that $x_n \in \Lambda_0^K$ for all $n \in \mathbb{N}$. For this it is enough to show that the energy e_n of (x_n, T_n) (defined on the bottom of page 197) is uniformly bounded. This then implies that the length l_n of x_n is bounded (compare Equation (3-1)), which immediately implies that such a compact set $K \subset \widetilde{M}$ exists. To see that e_n is bounded, we use Equation (2-6), which tells us

$$\frac{1}{n} \geq \left| \frac{\partial}{\partial T} S_k(x_n, T_n) \right| = \left| \frac{1}{T_n} \int_0^{T_n} (k - E(y_n, \dot{y}_n))\, dt \right| = \left| k - \frac{e_n}{T_n} \right|.$$

Since $|T_n| \leq B$ by assumption, e_n is necessarily bounded, as required.

Acknowledgement

I thank Gabriele Benedetti for patiently and repeatedly explaining to me why my Theorem 5.1 was false. I am particularly grateful for the considerable tact he showed while outlining to me how the quintessential function one uses to teach students the necessity of the Palais–Smale condition — namely $x \mapsto e^{-x}$ — provided a counterexample.

References

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WILL J. MERRY
DEPARTEMENT MATHEMATIK
ETH ZÜRICH
RÄMISTRASSE 101
CH-8092 ZÜRICH
SWITZERLAND
merry@math.ethz.ch

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