Pacific Journal of Mathematics

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Volume 285 No. 2 December 2016

PACIFIC JOURNAL OF MATHEMATICS
Vol. 285, No. 2, 2016
dx.doi.org/10.2140/pjm.2016.285.283

THE SU(2) CASSON–LIN INVARIANT OF THE HOPF LINK

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We compute the SU(2) Casson–Lin invariant for the Hopf link and determine the sign in the formula of Harper and Saveliev relating this invariant to the linking number.

The Casson–Lin invariant $h(K)$ was defined for knots K by X.-S. Lin [1992] as a signed count of conjugacy classes of irreducible SU(2) representations of the knot group $G_K = \pi_1(S^3 \setminus K)$ with traceless meridional image, and Corollary 2.10 of the same paper shows that $h(K)$ is equal to $\frac{1}{2}\,\mathrm{sign}(K)$, one half the knot signature. E. Harper and N. Saveliev [2010] introduced the Casson–Lin invariant $h_2(L)$ of 2-component links, which they defined as a signed count of certain projective SU(2) representations of the link group $G_L = \pi_1(S^3 \setminus L)$. They showed that $h_2(L)$ equals the linking number of $L = \ell_1 \cup \ell_2$, up to an overall sign: $h_2(L) = \pm\,\mathrm{lk}(\ell_1, \ell_2)$. Harper and Saveliev [2012] also show that $h_2(L)$ can be regarded as an Euler characteristic associated to a certain SU(2) instanton Floer homology theory, defined by Kronheimer and Mrowka [2011].

The purpose of this note is to determine the sign in the formula of Harper and Saveliev, establishing the following.

Theorem 1. *If $L = \ell_1 \cup \ell_2$ is an oriented 2-component link in S^3, then its Casson–Lin invariant satisfies $h_2(L) = -\,\mathrm{lk}(\ell_1, \ell_2)$.*

We remark that the braid approach in [Harper and Saveliev 2010] is close in spirit to Lin's original definition, and it shows that $h_2(L)$ is an invariant of *oriented* links, because the Alexander and Markov theorems hold for oriented links; see Theorems 2.3 and 2.8 of [Kassel and Turaev 2008]. The sign of the invariant $h_2(L)$ depends not only on the choice of orientation on the braid, but also on the more subtle choice of identification of geometric braids with elements in the abstract braid group B_n, viewed as a subgroup of $\mathrm{Aut}(F_n)$. Here we follow Conventions 1.13 of [Kassel and Turaev 2008] in making this choice.

Note that extensions of the Casson–Lin invariants to SU(N) and to oriented links L in S^3 with at least two components are presented in [Boden and Harper

Boden was supported by a grant from the Natural Sciences and Engineering Research Council of Canada.

MSC2010: primary 57M25; secondary 20C15.

Keywords: braids, links, representation spaces, Casson–Lin invariant.

2016], where, as before, they are defined by counting certain projective $\mathrm{SU}(N)$ representations of the link group G_L.

The rest of this paper is devoted to proving Theorem 1.

Proof. The proof of Proposition 5.7 in [Harper and Saveliev 2010] shows that the sign of $\mathrm{lk}(\ell_1, \ell_2)$ in our theorem is independent of L. (See also the proof of their Theorem 2 and their general discussion in Section 5.) Thus Theorem 1 will follow from a single computation.

To that end, we will determine the Casson–Lin invariant for the right-handed Hopf link. Since there is just one irreducible projective $\mathrm{SU}(2)$ representation of the link group, up to conjugation, it suffices to determine the sign associated to this one point.

We identify

$$\mathrm{SU}(2) = \left\{x + yi + zj + wk \;\middle|\; |x|^2 + |y|^2 + |z|^2 + |w|^2 = 1\right\}$$

with the group of unit quaternions and consider the conjugacy class

$$C_i = \left\{yi + zj + wk \;\middle|\; |y|^2 + |z|^2 + |w|^2 = 1\right\} \subset \mathrm{SU}(2)$$

of purely imaginary unit quaternions. Notice that C_i is diffeomorphic to S^2 and coincides with the set of $\mathrm{SU}(2)$ matrices of trace zero.

Let L be an oriented link in S^3, represented as the closure of an n-strand braid $\sigma \in B_n$. We follow Conventions 1.13 on page 17 of [Kassel and Turaev 2008] for writing geometric braids σ as words in the standard generators $\sigma_1, \ldots, \sigma_{n-1}$. In particular, braids are oriented from top to bottom and σ_i denotes a right-handed crossing in which the $(i+1)$-st strand crosses over the i-th strand. The braid group B_n gives a faithful right action on the free group F_n on n generators, and here we follow the conventions in [Boden and Harper 2016] for associating an automorphism of F_n to a given braid $\sigma \in B_n$, which we write as $x_i \mapsto x_i^\sigma$ for $i = 1, \ldots, n$. To be precise, to each braid group generator σ_i we associate the map $\sigma_i : F_n \to F_n$ given by

$$x_i \mapsto x_{i+1}, \quad x_{i+1} \mapsto (x_{i+1})^{-1} x_i x_{i+1}, \quad x_j \mapsto x_j \ (j \neq i,\ i+1),$$

and this is a right action, i.e., if $\sigma, \sigma' \in B_n$ are two braids, then $(x_i)^{\sigma\sigma'} = (x_i^\sigma)^{\sigma'}$ for all $1 \leq i \leq n$. Note that each braid $\sigma \in B_n$ fixes the product $x_1 \cdots x_n$.

A standard application of the Seifert–van Kampen theorem shows that the link complement $S^3 \setminus L$ has fundamental group

$$\pi_1(S^3 \setminus L) = \langle x_1, \ldots, x_n \mid x_i^\sigma = x_i,\ i = 1, \ldots, n \rangle.$$

We can therefore identify representations in $\mathrm{Hom}(\pi_1(S^3 \setminus L), \mathrm{SU}(2))$ with fixed points in $\mathrm{Hom}(F_n, \mathrm{SU}(2))$ under the induced action of the braid σ. We further identify $\mathrm{Hom}(F_n, \mathrm{SU}(2))$ with $\mathrm{SU}(2)^n$ by associating to a homomorphism ϱ the

n-tuple $(X_1, \ldots, X_n) = (\varrho(x_1), \ldots, \varrho(x_n))$. Note that $\sigma : \mathrm{SU}(2)^n \to \mathrm{SU}(2)^n$ is equivariant with respect to conjugation, so that fixed points come in whole orbits.

Every projective SU(2) representation can be identified with a fixed point in $\mathrm{Hom}(F_n, \mathrm{SU}(2))$ under the action of $\varepsilon\sigma$ for some n-tuple $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$ with $\varepsilon_i = \pm 1$ such that $\varepsilon_1 \cdots \varepsilon_n = 1$. Notice that the action of $\varepsilon\sigma$ on $(X_1, \ldots, X_n) \in \mathrm{SU}(2)^n$ preserves the product $X_1 \cdots X_n$ and is equivariant with respect to conjugation. The Casson–Lin invariant $h_2(L)$ is then defined as a signed count of orbits of fixed points of $\varepsilon\sigma$ for a suitably chosen n-tuple $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$. The choice is made so that the resulting projective representations ϱ all have $w_2(\mathrm{Ad}\,\varrho) \neq 0$, meaning that the representations $\mathrm{Ad}\,\varrho$ do not lift to SU(2) representations. It has the consequence that for all fixed points ϱ of $\varepsilon\sigma$, each $\varrho(x_i)$ is a traceless SU(2) element.

We therefore restrict our attention to the subset of traceless representations, which are elements $\varrho \in \mathrm{Hom}(F_n, \mathrm{SU}(2))$ with $\varrho(x_j) \in C_i$ for $j = 1, \ldots, n$. Define $f : C_i^n \times C_i^n \to \mathrm{SU}(2)$ by setting

$$f(X_1, \ldots, X_n, Y_1, \ldots, Y_n) = (X_1 \cdots X_n)(Y_1 \cdots Y_n)^{-1}.$$

We obtain an orientation on $f^{-1}(1)$ by applying the base-fiber rule, using the product orientation on $C_i^n \times C_i^n$ and the standard orientation on the codomain of f. The quotient $f^{-1}(1)/\mathrm{conj}$ is then oriented by another application of the base-fiber rule, using the standard orientation on SU(2). This step uses the fact that, if $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$ is chosen so that the associated SO(3) representation $\mathrm{Ad}\,\varrho$ has nontrivial second Stiefel–Whitney class $w_2 \neq 0$, then every fixed point of $\varepsilon\sigma$ in $\mathrm{Hom}(F_n, \mathrm{SU}(2))$ is necessarily irreducible.

We view conjugacy classes of fixed points of $\varepsilon\sigma$ as points in the intersection $\widehat{\Delta} \cap \widehat{\Gamma}_{\varepsilon\sigma}$, where $\widehat{\Delta} = \Delta/\mathrm{conj}$ is the quotient of the diagonal $\Delta \subset C_i^n \times C_i^n$, and where $\widehat{\Gamma}_{\varepsilon\sigma} = \Gamma_{\varepsilon\sigma}/\mathrm{conj}$ is the quotient of the graph $\Gamma_{\varepsilon\sigma}$ of $\varepsilon\sigma : C_i^n \to C_i^n$.

If the link L is the closure of a 2-strand braid, as it is for the Hopf link, then $\varepsilon = (-1, -1)$ is the only choice whose associated SO(3) bundle has $w_2 \neq 0$. Furthermore, in this case the intersection $\widehat{\Delta} \cap \widehat{\Gamma}_{\varepsilon\sigma}$ takes place in the pillowcase $f^{-1}(1)/\mathrm{conj}$, which is defined as the quotient

$$P = \{(a, b, c, d) \in C_i^4 \mid ab = cd\}/\mathrm{conj}. \tag{1}$$

It is well known that P is homeomorphic to S^2. To see this, first conjugate so that $a = i$, then conjugate by elements of the form $e^{i\theta}$ to arrange that b lies in the (i, j)-circle. A straightforward calculation using the equation $ab = cd$ shows that d must also lie on the (i, j)-circle. Clearly c is determined by a, b, d. We thus obtain an embedded 2-torus of elements of C_i^4 satisfying $ab = cd$, parametrized by

$$g(\theta_1, \theta_2) = (i, e^{k\theta_1} i, e^{k(\theta_2 - \theta_1)} i, e^{k\theta_2} i)$$

for $\theta_1, \theta_2 \in [0, 2\pi)$, which maps onto P. It is easy to verify that this is a two-to-one submersion, except when $\theta_1, \theta_2 \in \{0, \pi\}$. This realizes P as a quotient of the torus by the hyperelliptic involution. In particular, this involution is orientation-preserving, and away from the four singular points of P, we can lift all orientation questions up to the torus.

Let L be the right-handed Hopf link, which we view as the closure of the braid $\sigma = \sigma_1^2 \in B_2$, and suppose $\varepsilon = (-1, -1)$. The intersection $\widehat{\Delta} \cap \widehat{\Gamma}_{\varepsilon\sigma}$ consists of only one point, the conjugacy class of $g\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$, that is, the point $[(i, j, i, j)] \in P$. (This is easily verified using the action of σ_1^2 on $F_2 = \langle x, y\rangle$; see Figure 1.) Thus, in order to pin down the sign of the Casson–Lin invariant $h_2(L)$, we must determine the orientations of $\widehat{\Delta}$, $\widehat{\Gamma}_{\varepsilon\sigma}$, and P at this point.

Notice that

$$\frac{\partial}{\partial\theta_1} g(\theta_1, \theta_2) = (0, e^{k\theta_1} j, -e^{k(\theta_2-\theta_1)} j, 0),$$
$$\frac{\partial}{\partial\theta_2} g(\theta_1, \theta_2) = (0, 0, e^{k(\theta_2-\theta_1)} j, e^{k\theta_2} j).$$

Evaluating at $\theta_1 = \theta_2 = \frac{\pi}{2}$ gives two tangent vectors $u_1 := (0, -i, -j, 0)$ and $u_2 := (0, 0, j, -i)$ to C_i^4 which span a complementary subspace in $\ker df$ to the orbit tangent space. Therefore, an ordering of these vectors determines an orientation on $P = f^{-1}(1)/\mathrm{conj}$.

The orbit tangent space is spanned by the three tangent vectors

$$v_1 := \frac{\partial}{\partial t}\Big|_{t=0} e^{it}(i, j, i, j)e^{-it} = (0, 2k, 0, 2k),$$
$$v_2 := \frac{\partial}{\partial t}\Big|_{t=0} e^{jt}(i, j, i, j)e^{-jt} = (-2k, 0, -2k, 0),$$
$$v_3 := \frac{\partial}{\partial t}\Big|_{t=0} e^{kt}(i, j, i, j)e^{-kt} = (2j, -2i, 2j, -2i).$$

Then $\{u_1, u_2, v_1, v_2, v_3\}$ is a basis for $\ker(df|_{(i,j,i,j)}) = T_{(i,j,i,j)} f^{-1}(1)$. We choose vectors $w_1 = (k, 0, 0, 0)$, $w_2 = (0, k, 0, 0)$, $w_3 = (j, 0, 0, 0)$ to extend this to a basis for $T_{(i,j,i,j)} C_i^4$.

The orientation conventions in the definition of $h_2(L)$ (see Section 5d of [Harper and Saveliev 2010]) involve pulling back the orientation from $\mathfrak{su}(2) = T_1 \mathrm{SU}(2)$ by df to obtain a coorientation for $\ker(df|_{(i,j,i,j)})$. With that in mind, we compute the action of df on $\{w_1, w_2, w_3\}$, namely, $df(w_1) = -j$, $df(w_2) = i$, $df(w_3) = k$.

Notice that the ordered triple $\{df(w_1), df(w_2), df(w_3)\} = \{-j, i, k\}$ gives the same orientation as the standard basis for $\mathfrak{su}(2)$. Thus, the base-fiber rule gives the coorientation $\{w_1, w_2, w_3\}$ on $\ker df$, so we choose the orientation $\mathcal{O}_{\ker df}$ on $\ker df$ such that $\mathcal{O}_{\{w_1,w_2,w_3\}} \oplus \mathcal{O}_{\ker df}$ agrees with the product orientation on $C_i^2 \times C_i^2$.

The orientation on the pillowcase P is then obtained by applying the base-fiber rule a second time to the quotient (1), using $\mathcal{O}_{\ker df}$ to orient $f^{-1}(1)$ and giving the

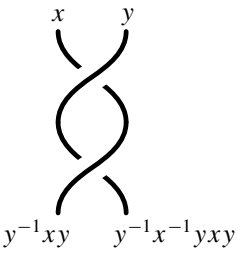

Figure 1. The action of $\sigma = \sigma_1^2$ on $F_2 = \langle x, y\rangle$.

orbit tangent space the orientation induced from that on SU(2) as well. We claim that the basis $\{u_1, u_2\}$ for the tangent space to the pillowcase has the opposite orientation. To see this, we note that $\{v_1, v_2, v_3\}$ is the fiber orientation for $\mathrm{SO}(3) \to f^{-1}(1) \to P$ and compare $S = \{w_1, w_2, w_3, u_1, u_2, v_1, v_2, v_3\}$ to the product orientation on $C_i^2 \times C_i^2$. Using the basis $\{(j, 0), (k, 0), (0, k), (0, i)\}$ for $T_{(i,j)}(C_i^2)$, we see that

$$\beta = \{(j, 0, 0, 0), (k, 0, 0, 0), (0, k, 0, 0), (0, i, 0, 0), (0, 0, j, 0), (0, 0, k, 0), (0, 0, 0, k), (0, 0, 0, i)\}$$

is an oriented basis for $T_{(i,j,i,j)}C_i^4 = T_{(i,j)}C_i^2 \times T_{(i,j)}C_i^2$ with the product orientation.[1]

Let M be the matrix expressing the vectors in S in terms of the basis β. Since

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 \\ 1 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & -2 \end{bmatrix},$$

one easily computes that $\det M = -8$, confirming our claim that $\{u_2, u_1\}$ is a positively oriented basis for the pillowcase tangent space.

Recall that L is the right-handed Hopf link, which we represent as the closure of the braid $\sigma = \sigma_1^2 \in B_2$. For $\varepsilon = (-1, -1)$, as in Figure 1, one can verify that

$$\varepsilon\sigma(X, Y) = (-Y^{-1}XY, -Y^{-1}X^{-1}YXY).$$

Consider the curve $\alpha(\theta) = (i, e^{k\theta}i)$, passing through the point $(i, j) \in C_i^2$ when $\theta = \frac{\pi}{2}$, which is transverse to the orbit $[(i, j)]$. Then $(\alpha(\theta), \alpha(\theta))$ and $(\alpha(\theta), \varepsilon\sigma \circ \alpha(\theta))$ are curves in Δ and $\Gamma_{\varepsilon\sigma}$, respectively, and both are necessarily transverse to the orbit in

[1] As explained in Section 5d of [Harper and Saveliev 2010], the invariant $h_2(L)$ is independent of the choice of orientation on C_i. In fact, C_i^2 can be oriented arbitrarily provided one uses the *product* orientation on $C_i^2 \times C_i^2$.

C_i^4/conj. Thus, we can compare the orientations induced by the parametrizations $[(\alpha(\theta), \alpha(\theta))]$ and $[(\alpha(\theta), \varepsilon\sigma\circ\alpha(\theta))]$ of $\widehat{\Delta}$ and $\widehat{\Gamma}_{\varepsilon\sigma}$ to the pillowcase orientation determined above, namely $\{u_2, u_1\}$. The velocity vectors for the paths $(\alpha(\theta), \alpha(\theta)) = (i, e^{k\theta}i, i, e^{k\theta}i)$ and $(\alpha(\theta), \varepsilon\sigma\circ\alpha(\theta)) = (i, e^{k\theta}i, -e^{2k\theta}i, -e^{3k\theta}i)$ at $\theta = \frac{\pi}{2}$ are given by $(0, -i, 0, -i) = u_1 + u_2$ and $(0, -i, 2j, -3i) = u_1 + 3u_2$, respectively.

The Casson–Lin invariant is defined as the intersection number $h_2(L) = \langle\widehat{\Delta}, \widehat{\Gamma}_{\varepsilon\sigma}\rangle$, and in our case the sign of the unique intersection point in $\widehat{\Delta}\cap\widehat{\Gamma}_{\varepsilon\sigma}$ is determined by comparing the orientation of $\{u_1 + u_2, u_1 + 3u_2\}$ with $\{u_2, u_1\}$. Since the change of basis matrix $\left[\begin{smallmatrix}1 & 3\\ 1 & 1\end{smallmatrix}\right]$ has negative determinant, it follows that $h_2(L) = -1$. □

Acknowledgements

The authors thank Eric Harper and Nikolai Saveliev for many helpful discussions.

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Received January 22, 2016. Revised April 20, 2016.

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PACIFIC JOURNAL OF MATHEMATICS

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