Pacific Journal of Mathematics

NEW CHARACTERIZATIONS OF LINEAR WEINGARTEN SPACELIKE HYPERSURFACES IN THE DE SITTER SPACE

LUIS J. ALÍAS, HENRIQUE F. DE LIMA AND FÁBIO R. DOS SANTOS

Volume 292 No. 1 January 2018

PACIFIC JOURNAL OF MATHEMATICS
Vol. 292, No. 1, 2018
dx.doi.org/10.2140/pjm.2018.292.1

NEW CHARACTERIZATIONS OF LINEAR WEINGARTEN SPACELIKE HYPERSURFACES IN THE DE SITTER SPACE

LUIS J. ALÍAS, HENRIQUE F. DE LIMA AND FÁBIO R. DOS SANTOS

We deal with complete linear Weingarten spacelike hypersurfaces immersed in the de Sitter space, that is, spacelike hypersurfaces of de Sitter space whose mean and scalar curvatures are linearly related. In this setting, we apply a suitable extension of the generalized maximum principle of Omori–Yau to show that either such a spacelike hypersurface must be totally umbilical or there holds a sharp estimate for the norm of its total umbilicity tensor, with equality characterizing hyperbolic cylinders of de Sitter space. We also study the parabolicity of these spacelike hypersurfaces with respect to a Cheng–Yau modified operator.

1. Introduction

The last few decades have seen a steadily growing interest in the study of the geometry of spacelike hypersurfaces immersed into a Lorentzian space. Recall that a hypersurface M^n immersed into a Lorentzian space is said to be *spacelike* if the metric induced on M^n from that of the ambient space is positive definite. Apart from physical motivations, from the mathematical point of view this interest is mostly due to the fact that such hypersurfaces exhibit nice Bernstein-type properties, and one can truly say that the first remarkable results in this branch were the rigidity theorems of E. Calabi [1970] and S. Y. Cheng and S. T. Yau [1976], who showed (the former for $n \leq 4$, and the latter for general n) that the only maximal complete, noncompact, spacelike hypersurfaces of the Lorentz–Minkowski space $\mathbb{L}^{n+1}$ are the spacelike hyperplanes. However, in the case that the mean curvature is a positive constant, A. E. Treibergs [1982] astonishingly showed that there are many entire solutions of the corresponding constant mean curvature equation in $\mathbb{L}^{n+1}$, which he was able to classify by their projective boundary values at infinity.

When the ambient is the de Sitter space $\mathbb{S}_1^{n+1}$, A. J. Goddard [1977] conjectured that every complete spacelike hypersurface with constant mean curvature H in $\mathbb{S}_1^{n+1}$ should be totally umbilical. Although the conjecture turned out to be false in its

MSC2010: primary 53C42; secondary 53A10, 53C20, 53C50.
Keywords: de Sitter space, linear Weingarten hypersurfaces, spacelike hypersurfaces, totally umbilical hypersurfaces, hyperbolic cylinders, parabolicity.

original statement, it motivated a great deal of work of several authors trying to find a positive answer to the conjecture under appropriate additional hypotheses. For instance, J. Ramanathan [1987] proved Goddard's conjecture for $\mathbb{S}_1^3$ and $0 \leq H \leq 1$. Moreover, if $H > 1$ he showed that the conjecture is false as can be seen from an example due to M. Dajczer and K. Nomizu [1981]. K. Akutagawa [1987] proved that Goddard's conjecture is true when $n = 2$ and $H^2 \leq 1$ or when $n \geq 3$ and $H^2 < 4(n-1)/n^2$. He also constructed complete spacelike rotation surfaces in $\mathbb{S}_1^3$ with constant H satisfying $H > 1$ which are not totally umbilical.

S. Montiel [1988] proved that Goddard's conjecture is true provided that M^n is compact. Furthermore, he exhibited examples of complete spacelike hypersurfaces in $\mathbb{S}_1^{n+1}$ with constant H satisfying $H^2 \geq 4(n-1)/n^2$ and being nontotally umbilical, the so-called *hyperbolic cylinders*, which are isometric to the Riemannian product $\mathbb{H}^1(r) \times \mathbb{S}^{n-1}(\sqrt{1+r^2})$ of a hyperbolic line of radius $r > 0$ and an $(n-1)$-dimensional Euclidean sphere of radius $\sqrt{1+r^2}$. S. Montiel [1996] characterized such hyperbolic cylinders as the only complete noncompact spacelike hypersurfaces in $\mathbb{S}_1^{n+1}$ with constant mean curvature $H = 2\sqrt{n-1}/n$ and having at least two ends. A. Brasil Jr., A. G. Colares and O. Palmas [Brasil et al. 2003] obtained a sort of extension of Montiel's result, showing that the hyperbolic cylinders are the only complete spacelike hypersurfaces in $\mathbb{S}_1^{n+1}$ with constant mean curvature, nonnegative Ricci curvature and having at least two ends. They also characterized all complete spacelike hypersurfaces of constant mean curvature with two distinct principal curvatures as rotation hypersurfaces or generalized hyperbolic cylinders $\mathbb{H}^k(r) \times \mathbb{S}^{n-k}(\sqrt{1+r^2})$. Proceeding with the ideas related to Goddard's conjecture, it is interesting to obtain characterizations of the so-called *linear Weingarten* spacelike hypersurfaces (that is, spacelike hypersurfaces whose mean and scalar curvatures are linearly related) immersed in a certain Lorentzian space. Many authors have approached problems in this branch. For instance, when the ambient space is $\mathbb{S}_1^{n+1}$, we refer to the readers the works [Chao 2013; Cheng 1990; de Lima and Velásquez 2013; Hou and Yang 2010; Li 1997].

Here, our purpose is to obtain new characterizations concerning complete linear Weingarten spacelike hypersurfaces immersed in $\mathbb{S}_1^{n+1}$. Under appropriated constrains on the scalar curvature function, we apply a suitable extension of the generalized maximum principle of Omori–Yau (see Proposition 7) in order to give a sharp estimate of the total umbilicity tensor of the hypersurface, which allows us to characterize hyperbolic cylinders

$$\mathbb{H}^1(r) \times \mathbb{S}^{n-1}(\sqrt{1+r^2})$$

of $\mathbb{S}_1^{n+1}$ when $n \geq 3$ (see Theorem 8 and Corollary 9) and totally umbilic 2-spheres in $\mathbb{S}_1^3$ when $n = 2$ (see Theorem 10 and Corollary 11). We also study the parabolicity of these spacelike hypersurfaces with respect to a Cheng–Yau modified operator (see Theorem 12 and Proposition 13).

2. Preliminaries

Let $\mathbb{R}_1^{n+2}$ be an $(n+2)$-dimensional real vector space endowed with an inner product of index 1 given by

$$\langle x, y\rangle = \sum_{j=1}^{n+1} x_j y_j - x_{n+2} y_{n+2},$$

where $x = (x_1, x_2, \ldots, x_{n+2})$ is the natural coordinate of $\mathbb{R}_1^{n+2}$.

$\mathbb{R}_1^{n+2} = \mathbb{L}^{n+2}$ is called the $(n+2)$-dimensional *Lorentz–Minkowski space*. We define the $(n+1)$-dimensional *de Sitter space* $\mathbb{S}_1^{n+1}$ as the following hyperquadric of $\mathbb{L}^{n+2}$:

$$\mathbb{S}_1^{n+1} = \{(x_1, x_2, \ldots x_{n+2}) \in \mathbb{R}_1^{n+2} : \langle x, x\rangle = 1\}.$$

The induced metric $\langle\,\cdot\,,\cdot\,\rangle$ makes $\mathbb{S}_1^{n+1}$ a Lorentzian manifold with constant sectional curvature 1.

An n-dimensional hypersurface M^n in $\mathbb{S}_1^{n+1}$ is said to be *spacelike* if the metric on M^n induced from the metric of $\mathbb{S}_1^{n+1}$ is positive definite.

From now on, we will consider complete spacelike hypersurfaces M^n of $\mathbb{S}_1^{n+1}$. We choose a local field of semi-Riemannian orthonormal frame $\{e_A\}_{1\le A\le n+1}$ in $\mathbb{S}_1^{n+1}$, with dual coframe $\{\omega_A\}_{1\le A\le n+1}$, such that, at each point of M^n, $e_1, \ldots, e_n$ are tangent to M^n and e_{n+1} is normal to M^n. We will use the following convention for the indices

$$1 \le A, B, C, \ldots \le n+1, \qquad 1 \le i, j, k, \ldots \le n.$$

In this setting, denoting by $\{\omega_{AB}\}$ the connection forms of $\mathbb{S}_1^{n+1}$, the structure equations of $\mathbb{S}_1^{n+1}$ are given by

$$d\omega_A = \sum_i \omega_{Ai}\wedge\omega_i - \omega_{An+1}\wedge\omega_{n+1}, \qquad \omega_{AB}+\omega_{BA}=0,$$

$$d\omega_{AB} = \sum_C \varepsilon_C\omega_{AC}\wedge\omega_{CB} - \frac{1}{2}\sum_{C,D}\varepsilon_C\varepsilon_D K_{ABCD}\omega_C\wedge\omega_D,$$

$$K_{ABCD} = \varepsilon_A\varepsilon_B(\delta_{AC}\delta_{BD} - \delta_{AD}\delta_{BC}),$$

where $\varepsilon_i = 1$ and $\varepsilon_{n+1} = -1$.

Next, we restrict all the tensors to M^n. First of all, $\omega_{n+1} = 0$ on M^n, so $\sum_i \omega_{n+1i}\wedge\omega_i = d\omega_{n+1} = 0$ and by Cartan's lemma we can write

$$\omega_{n+1i} = \sum_j h_{ij}\omega_j, \qquad h_{ij} = h_{ji}. \tag{2-1}$$

This gives the second fundamental form of M^n, $A = \sum_{ij} h_{ij}\omega_i \otimes \omega_j e_{n+1}$. Furthermore, the mean curvature H of M^n is defined by $H = 1/n \sum_i h_{ii}$.

The structure equations of M^n are given by

$$d\omega_i = \sum_j \omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0,$$
$$d\omega_{ij} = \sum_k \omega_{ik} \wedge \omega_{kj} - \frac{1}{2}\sum_{k,l} R_{ijkl}\omega_k \wedge \omega_l.$$

Using the structure equations we obtain the Gauss equation

$$R_{ijkl} = \delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk} - h_{ik}h_{jl} + h_{il}h_{jk}, \tag{2-2}$$

where R_{ijkl} are the components of the curvature tensor of M^n.

The Ricci curvature and the normalized scalar curvature of M^n are given, respectively, by

$$R_{ij} = (n-1)\delta_{ij} - nHh_{ij} + \sum_k h_{ik}h_{kj} \tag{2-3}$$

and

$$R = \frac{1}{n(n-1)}\sum_i R_{ii}. \tag{2-4}$$

From (2-3) and (2-4) we obtain

$$S = n^2H^2 + n(n-1)(R-1) = nH^2 + n(n-1)(H^2 - H_2), \tag{2-5}$$

where $S = \sum_{i,j} h_{ij}^2$ is the square of the length of the second fundamental form A of M^n, and $H_2 = 2S_2/(n(n-1))$ denotes the mean value of the second elementary symmetric function S_2 on the eigenvalues of A. In particular, it follows from (2-5) that M^n is totally umbilical if and only if $S = nH^2$.

The components h_{ijk} of the covariant derivative ∇A satisfy

$$\sum_k h_{ijk}\omega_k = dh_{ij} + \sum_k h_{ik}\omega_{kj} + \sum_k h_{jk}\omega_{ki}. \tag{2-6}$$

Observe that,

$$|\nabla A|^2 = \sum_{i,j,k} h_{ijk}^2.$$

Then, by exterior differentiation of (2-1), we obtain the *Codazzi equation*

$$h_{ijk} = h_{jik} = h_{ikj}. \tag{2-7}$$

Similarly, the components h_{ijkl} of the second covariant derivative $\nabla^2 B$ are given by

$$\sum_l h_{ijkl}\omega_l = dh_{ijk} + \sum_l h_{ljk}\omega_{li} + \sum_l h_{ilk}\omega_{lj} + \sum_l h_{ijl}\omega_{lk}.$$

By exterior differentiation of (2-6), we can get the following *Ricci formula*

$$h_{ijkl} - h_{ijlk} = \sum_m h_{im}R_{mjkl} + \sum_m h_{jm}R_{mikl}. \tag{2-8}$$

The Laplacian Δh_{ij} of h_{ij} is defined by $\Delta h_{ij} = \sum_k h_{ijkk}$. From (2-7) and (2-8), we have

$$\Delta h_{ij} = \sum_k h_{kkij} + \sum_{k,l} h_{kl}R_{lijk} + \sum_{k,l} h_{li}R_{lkjk}. \tag{2-9}$$

Since $\Delta S = 2\big(\sum_{i,j} h_{ij}\Delta h_{ij} + \sum_{i,j,k} h_{ijk}^2\big)$, from (2-9) we get

$$\tfrac{1}{2}\Delta S = |\nabla A|^2 + \sum_{i,i,k} h_{ij}h_{kkij} + \sum_{i,j,k,l} h_{ij}h_{lk}R_{lijk} + \sum_{i,j,k,l} h_{ij}h_{il}R_{lkjk}. \tag{2-10}$$

Consequently, taking a (local) orthonormal frame $\{e_1, \dots, e_n\}$ on M^n such that $h_{ij} = \lambda_i\delta_{ij}$, from (2-10) we obtain the following Simons-type formula:

$$\tfrac{1}{2}\Delta S = |\nabla A|^2 + \sum_i \lambda_i (nH)_{,ii} + \frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i - \lambda_j)^2. \tag{2-11}$$

Now, let $\phi = \sum_{i,j}\phi_{ij}\omega_i\omega_j$ be a symmetric tensor on M^n defined by

$$\phi_{ij} = nH\delta_{ij} - h_{ij}.$$

Following [Cheng and Yau 1977], we introduce a operator $\square$ associated to ϕ acting on any smooth function f by

$$\square f = \sum_{i,j}\phi_{ij}f_{ij} = \sum_{i,j}(nH\delta_{ij} - h_{ij})f_{ij}. \tag{2-12}$$

Setting $f = nH$ in (2-12) and taking into account equations (2-5) and (2-11), with a straightforward computation we obtain

$$\square(nH) = |\nabla A|^2 - n^2|\nabla H|^2 - \tfrac{1}{2}n(n-1)\Delta R + \frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i - \lambda_j)^2. \tag{2-13}$$

3. Linear Weingarten hypersurfaces in $\mathbb{S}_1^{n+1}$

In what follows, we will consider M^n as being a *linear Weingarten* spacelike hypersurface immersed in $\mathbb{S}_1^{n+1}$, that is, a spacelike hypersurface of $\mathbb{S}_1^{n+1}$ whose mean curvature H and normalized scalar curvature R satisfy

$$R = aH + b,$$

for some $a, b \in \mathbb{R}$. We first state some auxiliary results.

Lemma 1 [de Lima and Velásquez 2013]. *Let M^n be a linear Weingarten spacelike hypersurface in $\mathbb{S}_1^{n+1}$, such that $R = aH + b$ for some $a, b \in \mathbb{R}$. Suppose that*

$$(n-1)a^2 + 4n(1-b) \geq 0. \tag{3-1}$$

Then

$$|\nabla A|^2 \geq n^2 |\nabla H|^2. \tag{3-2}$$

Moreover, if the inequality (3-1) *is strict and the equality holds in* (3-2) *on M^n, then H is constant on M^n.*

Now, we will consider the following Cheng–Yau's modified operator:

$$L = \square + \frac{n-1}{2} a \Delta. \tag{3-3}$$

In other words, for any $u \in \mathcal{C}^2(M)$,

$$L(u) = \operatorname{tr}(P \circ \nabla^2 u), \tag{3-4}$$

with

$$P = \left(nH + \frac{n-1}{2} a \right) I - A, \tag{3-5}$$

where I is the identity in the algebra of smooth vector fields on M^n and $\nabla^2 u$ stands for the self-adjoint linear operator metrically equivalent to the Hessian of u.

Remark 2. From Equation (2-5), since $R = aH + b$, we have that

$$n^2 H^2 = S - n(n-1)(aH + b - 1). \tag{3-6}$$

When $b < 1$, it follows from (3-6) that $H(p) \neq 0$ for every $p \in M^n$. In this case, we can choose the orientation of M^n such that $H > 0$. On the other hand, when $b = 1$, we will assume that H does not change sign on M^n and we will choose the orientation of M^n such that $H \geq 0$.

The next lemma gives a sufficient criterion for the ellipticity of the operator L.

Lemma 3. *Let M^n be a linear Weingarten spacelike hypersurface in $\mathbb{S}_1^{n+1}$ such that $R = aH + b$. Let μ_- and μ_+ be, respectively, the minimum and the maximum of the eigenvalues of the operator P at every point $p \in M^n$. If $b < 1$, then the operator L is elliptic, with*

$$\mu_- > 0 \quad \textit{and} \quad \mu_+ < 2nH + (n-1)a.$$

In the case where $b = 1$, assume further that the mean curvature function H does not change sign and $R \geq 1$. Then the operator L is semielliptic, with

$$\mu_- \geq 0 \quad \textit{and} \quad \mu_+ \leq 2nH + (n-1)a,$$

unless M^n is totally geodesic.

Proof. Let us consider $b < 1$ and choose the orientation on M^n for which $H > 0$ (see Remark 2). From (3-6), we have that

$$n^2H^2 = S + n(n-1)(1 - aH - b) \geq \lambda_i^2 - n(n-1)aH,$$

for each principal curvature λ_i of M^n, $i = 1, \ldots, n$.

On the other hand, with a straightforward computation we verify that

$$\begin{aligned} \lambda_i^2 &\leq n^2H^2 + n(n-1)aH \\ &= \left(nH + \frac{n-1}{2}a\right)^2 - \frac{(n-1)^2}{4}a^2 \\ &\leq \left(nH + \frac{n-1}{2}a\right)^2. \end{aligned} \tag{3-7}$$

From (3-6) we also have that

$$nH(nH + (n-1)a) = S + n(n-1)(1-b) > 0. \tag{3-8}$$

We claim that $nH + \frac{1}{2}(n-1)a > 0$. When $a \geq 0$, our assertion is immediate since

$$nH + \left(\frac{n-1}{2}\right)a \geq nH > 0.$$

When $a < 0$, from (3-8) we get $nH + (n-1)a > 0$. So, $nH + \frac{1}{2}(n-1)a > nH + (n-1)a > 0$.

So, from (3-7) we obtain

$$-nH - \left(\frac{n-1}{2}\right)acn - 12a < \lambda_i < nH + \left(\frac{n-1}{2}\right)acn - 12a, \quad i = 1, \ldots, n.$$

Therefore, for every i, we get

$$0 < nH + \left(\frac{n-1}{2}\right)a - \lambda_i < 2nH + (n-1)a.$$

However, $\mu_i = nH + \frac{1}{2}(n-1)a - \lambda_i$ are precisely the eigenvalues of P. In particular, we conclude that $\mu_- > 0$ and $\mu_+ < 2nH + (n-1)a$.

If $b = 1$, then choose the orientation of M^n for which $H \geq 0$. From (3-6), we have that

$$n^2H^2 = S - n(n-1)aH \geq \lambda_i^2 - n(n-1)aH,$$

for each principal curvature λ_i of M^n, $i = 1, \dots, n$ and

$$\lambda_i^2 \le \left(nH + \frac{n-1}{2}a\right)^2.$$

From (3-6) we also have that

$$nH(nH + (n-1)a) = S \ge 0.$$

Since $R = aH + 1 \ge 1$, we have $aH \ge 0$. If $a \ge 0$ then $nH + \frac{1}{2}(n-1)a \ge 0$ and, similarly as in the case $b < 1$, we conclude that $\mu_- \ge 0$ and $\mu_+ \le 2nH + (n-1)a$.

On the other hand, if $a < 0$ we have $H \equiv 0$ and then $R \equiv 1$ and $S \equiv 0$, which means that M^n must be totally geodesic. □

Remark 4. Also related to the ellipticity of operator L, when M^n is totally geodesic, we observe that the operator L reduces to $L = \frac{1}{2}(n-1)a\Delta$, which is elliptic if and only if $a > 0$. For this reason, in order to keep the validity of Lemma 3 when $b = 1$, even in the totally geodesic case, we will choose a to be a positive constant.

We close this section recalling a classic algebraic lemma due to M. Okumura [1974], which was completed with the equality case by H. Alencar and M. P. do Carmo [1994].

Lemma 5. *Let $\kappa_1, \dots, \kappa_n$ be real numbers such that $\sum_i \kappa_i = 0$ and $\sum_i \kappa_i^2 = \beta^2$, with $\beta \ge 0$. Then,*

$$-\frac{(n-2)}{\sqrt{n(n-1)}}\beta^3 \le \sum_i \kappa_i^3 \le \frac{(n-2)}{\sqrt{n(n-1)}}\beta^3,$$

and equality holds if and only if at least $(n-1)$ of the numbers κ_i are equal.

4. Characterizations of linear Weingarten spacelike hypersurfaces

From now on, we will also consider the following symmetric tensor

$$\Phi = \sum_{i,j} \Phi_{ij}\omega_i \otimes \omega_j$$

associated to the second fundamental form of a hypersurface M^n in $\mathbb{S}_1^{n+1}$, whose components are given by $\Phi_{ij} = h_{ij} - H\delta_{ij}$. Let $|\Phi|^2 = \sum_{i,j} \Phi_{ij}^2$ be the square of the length of Φ. It is not difficult to check that Φ is traceless and, from (2-4), we get

$$|\Phi|^2 = S - nH^2 = n(n-1)H^2 + n(n-1)(R-1). \tag{4-1}$$

In particular, $|\Phi|^2 = 0$ at the umbilic points of M^n. For that reason Φ is usually called the total umbilicity tensor of M^n.

In order to establish our characterization results, we will need the following lower bound for the operator L acting on the square length of the traceless operator of a linear Weingarten hypersurface.

Proposition 6. *Let M^n be a linear Weingarten spacelike hypersurface immersed in $\mathbb{S}_1^{n+1}$, $n \geq 2$, such that $R = aH + b$ with $b \leq 1$. In the case where $b = 1$, assume that the mean curvature function H does not change sign and $R \geq 1$. Then,*

$$L(|\Phi|^2) \geq 2(n-1)|\Phi|^2 \varphi_{a,b}(|\Phi|)\sqrt{\frac{|\Phi|^2}{n(n-1)} + \frac{a^2}{4} - b + 1},$$

where

$$\varphi_{a,b}(x) = \frac{n-2}{n-1}x^2 + \left(na - \frac{n(n-2)}{\sqrt{n(n-1)}}x\right)\sqrt{\frac{x^2}{n(n-1)} + \frac{a^2}{4} - b + 1} + \frac{n(n-2)}{\sqrt{n(n-1)}}\frac{a}{2}x - n\left(\frac{a^2}{2} - b\right). \tag{4-2}$$

Proof. Let us choose a local orthonormal frame $\{e_1, \ldots, e_n\}$ on M^n such that $h_{ij} = \lambda_i \delta_{ij}$. Taking into account equations (2-10) and (2-13), we get from (3-3) that

$$L(nH) = \sum_{i,j,k} h_{ijk}^2 - n^2|\nabla H|^2 + \frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i - \lambda_j)^2 \tag{4-3}$$

On the one hand, by a straightforward computation we can check

$$\frac{1}{2}\sum_{i,j} R_{ijij}(\lambda_i - \lambda_j)^2 = \frac{1}{2}\sum_{i,j}(1 - \lambda_i\lambda_j)(\lambda_i - \lambda_j)^2 = S^2 - nH\sum_i \lambda_i^3 + n|\Phi|^2. \tag{4-4}$$

But, on the other hand, since we are assuming that $b \leq 1$, we have that the relation (3-1) holds, and hence we can apply Lemma 1 to guarantee that

$$\sum_{i,j,k} h_{ijk}^2 - n^2|\nabla H|^2 \geq 0. \tag{4-5}$$

Thus, from (4-3), (4-4) and (4-5) we have

$$L(nH) \geq S^2 - nH\sum_i \lambda_i^3 + n|\Phi|^2. \tag{4-6}$$

Taking a (local) orthonormal frame $\{e_1, \ldots, e_n\}$ at $p \in M^n$ such that

$$h_{ij} = \lambda_i \delta_{ij} \quad \text{and} \quad \phi_{ij} = \kappa_i \delta_{ij},$$

it is not difficult to verify the algebraic relations

$$\sum_i \kappa_i = 0, \quad \sum_i \kappa_i^2 = |\Phi|^2 \quad \text{and} \quad \sum_i \kappa_i^3 = \sum_i \lambda_i^3 - 3H|\Phi|^2 - nH^3. \tag{4-7}$$

When $n \geq 3$, using Lemma 5 and equations (4-1) and (4-7) we have

$$\begin{aligned} S^2 - nH\sum_{i=1}^n \lambda_i^3 &= (|\Phi|^2 + nH^2)^2 - nH\sum_i \kappa_i^3 - 3nH^2|\Phi|^2 - n^2H^4 \\ &= |\Phi|^4 - nH^2|\Phi|^2 - nH\sum_i \kappa_i^3 \\ &\geq |\Phi|^2\left(|\Phi|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}}H|\Phi| - nH^2\right). \end{aligned} \tag{4-8}$$

In the case that $n = 2$, since $\kappa_1 + \kappa_2 = 0$ we also have $\kappa_1^3 + \kappa_2^3 = 0$, and from (4-1) and (4-7) we obtain

$$\begin{aligned} S^2 - 2H\sum_{i=1}^2 \lambda_i^3 &= (\lambda_1^2 + \lambda_2^2)^2 - (\lambda_1 + \lambda_2)(\lambda_1^3 + \lambda_2^3) \\ &= |\Phi|^2(|\Phi|^2 - 2H^2). \end{aligned} \tag{4-9}$$

Hence, inserting either (4-8), when $n \geq 3$, or (4-9), when $n = 2$, into (4-6) we get

$$L(nH) \geq |\Phi|^2\left(|\Phi|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}}H|\Phi| - n(H^2 - 1)\right). \tag{4-10}$$

On the other hand, from (4-1) and $R = aH + b$, we have

$$\frac{1}{n-1}|\Phi|^2 = nH^2 + naH + n(b-1). \tag{4-11}$$

If M^n is totally geodesic then the operator L reduces to $L = \frac{1}{2}(n-1)a\Delta$ where $a > 0$ is any positive constant (see Remark 4). In this case $|\Phi|^2 \equiv 0$ and the inequality in Proposition 6 holds trivially. On the other hand, if M^n is not totally geodesic then Lemma 3 guarantees that the operator P is positive definite if $b < 1$, and P is positive semidefinite if $b = 1$. In any case, from (4-11) we have

$$\begin{aligned} \frac{1}{n-1}L(|\Phi|^2) &= 2HL(nH) + 2n\langle P(\nabla H), \nabla H\rangle + aL(nH) \\ &\geq 2\left(H + \frac{a}{2}\right)L(nH), \end{aligned} \tag{4-12}$$

once (3-4) guarantees that $L(u^2) = 2uL(u) + 2\langle P(\nabla u), \nabla u\rangle$ for every $u \in \mathcal{C}^2(M)$.

Therefore, taking into account that $H + a/2 \geq 0$, from (4-10) and (4-12) we get

$$\frac{1}{2(n-1)}L(|\Phi|^2) \geq \left(H + \frac{a}{2}\right)|\Phi|^2\left(|\Phi|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}}H|\Phi| - n(H^2 - 1)\right). \tag{4-13}$$

Besides, from (4-11) we have

$$H^2 = \frac{1}{n(n-1)}|\Phi|^2 - aH - b + 1,$$

and consequently, we can write

$$H + \frac{a}{2} = \sqrt{\frac{|\Phi|^2}{n(n-1)} + \frac{a^2}{4} - b + 1}. \tag{4-14}$$

From (4-14) and (4-11), after a straightforward computation, we get

$$|\Phi|^2 - \frac{n(n-2)}{\sqrt{n(n-1)}} H|\Phi| - n(H^2 - 1) = \varphi_{a,b}(|\Phi|), \tag{4-15}$$

where $\varphi_{a,b}(x)$ is defined as in (4-2). Therefore, replacing (4-14) and (4-15) in (4-13), we obtain the desired inequality. □

Let us consider on a Riemannian manifold M^n a semielliptic operator of the form $\mathcal{L} = \operatorname{tr}(\mathcal{P} \circ \operatorname{Hess})$, where $\mathcal{P} : TM \to TM$ is a positive semidefined symmetric tensor. We say that *the Omori–Yau maximum principle* holds on M^n for the operator $\mathcal{L}$ if, for any function $u \in \mathcal{C}^2(M)$ with $u^* = \sup_M u < \infty$, there exists a sequence $\{p_k\}_{k\in\mathbb{N}} \subset M^n$ with the properties

$$u(p_k) > u^* - \frac{1}{k}, \quad |\nabla u(p_k)| < \frac{1}{k} \quad \text{and} \quad \mathcal{L}u(p_k) < \frac{1}{k}$$

for every $k \in \mathbb{N}$.

As an application of Theorem 6.13 of [Alías et al. 2016] (see also Lemma 4.2 of [Alías et al. 2012]), we establish the following Omori–Yau maximum principle which will be our analytical key tool for the proofs of our main results.

Proposition 7. *Let M^n be complete noncompact linear Weingarten spacelike hypersurface immersed in $\mathbb{S}_1^{n+1}$ such that $R = aH + b$ with $b \le 1$. In the case where $b = 1$, assume that the mean curvature function H does not change sign and $R \ge 1$. If $\sup_M |\Phi|^2 < +\infty$, then the Omori–Yau maximum principle holds on M^n for the operator L.*

Proof. Taking into account that $R = aH + b$, from (4-1) we get

$$|\Phi|^2 = n(n-1)(H^2 + aH) + n(n-1)(b-1). \tag{4-16}$$

Since we are assuming $\sup_M |\Phi|^2 < +\infty$, from (4-16) it follows that $\sup_M H < +\infty$. Thus, from (3-5) we have

$$\operatorname{tr}(P) = n(n-1)H + \frac{n(n-1)}{2}a$$

and hence,

$$\sup_M \operatorname{tr}(P) < +\infty. \tag{4-17}$$

On the other hand, recall from the proof of Lemma 3 that $nH+\frac{1}{2}(n-1)a>0$ and

$$-nH-\frac{n-1}{2}a<\lambda_i<nH+\frac{n-1}{2}a, \quad i=1,\dots,n.$$

Therefore, from (2-2) we see that the sectional curvatures of M^n satisfy

$$R_{ijij}=1-\lambda_i\lambda_j\geq 1-\left(nH+\frac{n-1}{2}a\right)^2>-\infty. \tag{4-18}$$

Furthermore, Lemma 3 guarantees us that the operator L is semielliptic. Therefore, taking into account (3-4), (4-17) and (4-18), we can apply Theorem 6.13 of [Alías et al. 2016] in the particular case where the sectional curvatures of M^n are bounded from below by a constant to conclude the desired result. □

Now, we are in position to state and prove our main characterization result concerning linear Weingarten hypersurfaces immersed in $\mathbb{S}_1^{n+1}$.

Theorem 8. *Let M^n be a complete linear Weingarten spacelike hypersurface isometrically immersed in the de Sitter space $\mathbb{S}_1^{n+1}$, $n\geq 3$, such that $R=aH+b$ with $0<b\leq R<(n-2)/n$. Then*

(i) *either* $\sup_M|\Phi|^2=0$ *and M^n is a totally umbilical hypersurface,*

(ii) *or*

$$\sup_M|\Phi|^2\geq\alpha(n,a,b)>0,$$

where $\alpha(n,a,b)$ is a positive constant depending only on n, a and b.

In (ii), *a necessary and sufficient condition for equality to hold and the supremum to be attained at some point of M^n is that M^n be isometric to a hyperbolic cylinder $\mathbb{H}^1(r)\times\mathbb{S}^{n-1}(\sqrt{1+r^2})$ of radius $r>0$.*

Proof. If $\sup_M|\Phi|^2=0$, then M^n is totally umbilical and, hence, item (i) holds. If $\sup_M|\Phi|^2=+\infty$, then (ii) is trivially satisfied. So, let us suppose that $0<\sup_M|\Phi|^2<+\infty$ and let us take $u=|\Phi|^2$. Then, from Proposition 6 we get

$$L(u)\geq f(u), \tag{4-19}$$

where

$$f(u)=2(n-1)u\varphi_{a,b}(\sqrt{u})\sqrt{\frac{u}{n(n-1)}+1-b+\frac{a^2}{4}}$$

and $\varphi_{a,b}(x)$ is given by (4-2).

If M^n is compact, there exists a point $p_0\in M^n$ such that $u(p_0)=u^*=\sup_M u$. Consequently, $\nabla u(p_0)=0$ and $Lu(p_0)\leq 0$. Therefore, from (4-19) we get $f(u^*)\leq 0$. Now, assume that M^n is complete and noncompact. Since $u^*<+\infty$, Proposition 7 guarantees that there exists a sequence of points $\{p_k\}_{k\in\mathbb{N}} \subset M^n$ satisfying

$$u(p_k) > u^* - \frac{1}{k} \quad \text{and} \quad Lu(p_k) < \frac{1}{k}, \tag{4-20}$$

for every $k \in \mathbb{N}$. Therefore from (4-19) and (4-20), we get

$$\frac{1}{k} > Lu(p_k) \geq f(u(p_k)). \tag{4-21}$$

Taking the limit $k \to +\infty$ in (4-21), by continuity, we have

$$f(u^*) = 2(n-1)u^*\varphi_{a,b}(\sqrt{u^*})\sqrt{\frac{u^*}{n(n-1)} + 1 - b + \frac{a^2}{4}} \leq 0.$$

Since $u^* > 0$ and $b < 1$, we obtain that

$$\varphi_{a,b}(\sqrt{u^*}) \leq 0. \tag{4-22}$$

Recall from Remark 2 that $H > 0$ on M^n. Thus, since we are assuming that $n \geq 3$ and $0 < b \leq R < (n-2)/n$, it is not difficult to verify that $\varphi_{a,b}$ has an unique positive root $x_0 = \sqrt{\alpha(n, a, b)} > 0$. Moreover, we have that $\varphi_{a,b}(x) > 0$, for $0 \leq x < x_0$, and $\varphi_{a,b}(x) < 0$, for $x > x_0$.

Therefore, (4-22) implies

$$u^* \geq x_0^2 = \alpha(n, a, b),$$

that is,

$$\sup_M |\Phi|^2 \geq \alpha(n, a, b).$$

This proves the inequality of item (ii).

Moreover, the equality $\sup_M |\Phi|^2 = \alpha(n, a, b)$ holds if and only if $\sqrt{u^*} = x_0$. Thus $\varphi_{a,b}(\sqrt{u}) \geq 0$ on M^n, which jointly with (4-19) implies that

$$L(u) \geq 0 \quad \text{on } M^n.$$

On the other hand, since $b < 1$ we know from Lemma 3 that the operator L is elliptic. Therefore, if there exists a point $p_0 \in M^n$ such that $|\Phi(p_0)| = \sup_M |\Phi|$, from the maximum principle the function $u = |\Phi|^2$ must be constant and, consequently, $|\Phi| \equiv x_0$. Thus,

$$0 = L(|\Phi|^2) \geq 2(n-1)|\Phi|^2\varphi_{a,b}(|\Phi|)\sqrt{\frac{|\Phi|^2}{n(n-1)} + 1 - b + \frac{a^2}{4}}.$$

Hence, all the inequalities in the proof of Proposition 6 must be equalities. In particular, since L is elliptic if and only if P is positive definite, returning to (4-12) we obtain that $\nabla H = 0$ and H is constant. Moreover, equality occurs in (4-5) as well, or equivalently,

$$|\nabla A|^2=\sum_{i,j,k}h_{ijk}^2=n^2|\nabla H|^2=0.$$

So, it follows that λ_i is constant for every $i=1,\dots,n$, that is, M^n is an isoparametric hypersurface. Finally, (4-8) must also be an equality, which guarantees that the equality in Lemma 5 occurs. This implies that the hypersurface has exactly two distinct principal curvatures one of which is simple. Therefore, we can apply a classical congruence theorem due to Abe et al. [1987, Theorem 5.1] to conclude that M^n must be one of the two following standard product embeddings into $\mathbb{S}_1^{n+1}$:

(a) $\mathbb{H}^1(r)\times\mathbb{S}^{n-1}(\sqrt{1+r^2})$,

(b) $\mathbb{H}^{n-1}(r)\times\mathbb{S}^1(\sqrt{1+r^2})$,

in either case with a positive radius $r>0$. In case (a), for a given radius $r>0$ the standard product embedding $\mathbb{H}^1(r)\times\mathbb{S}^{n-1}(\sqrt{1+r^2})\hookrightarrow\mathbb{S}_1^{n+1}$ has constant principal curvatures given by

$$\lambda_1=\frac{\sqrt{1+r^2}}{r},\qquad \lambda_2=\cdots=\lambda_n=\frac{r}{\sqrt{1+r^2}}.$$

Therefore,

$$nH=\frac{1+nr^2}{r\sqrt{1+r^2}},\qquad S=\frac{1+2r^2+nr^4}{r^2(1+r^2)},\qquad\text{and}\qquad |\Phi|^2=\frac{n-1}{nr^2(1+r^2)},$$

and its constant scalar curvature is given

$$R=\frac{n-2}{n(1+r^2)},$$

which satisfies our hypothesis, since

$$0<R<\frac{n-2}{n}<1$$

for every $r>0$. On the other hand, in case (b) and for a given radius $r>0$ the standard product embedding $\mathbb{H}^{n-1}(r)\times\mathbb{S}^1(\sqrt{1+r^2})\hookrightarrow\mathbb{S}_1^{n+1}$ has constant principal curvatures given by

$$\lambda_1=\cdots=\lambda_{n-1}=\frac{\sqrt{1+r^2}}{r},\qquad \lambda_n=\frac{r}{\sqrt{1+r^2}}.$$

Therefore,

$$nH=\frac{(n-1)+nr^2}{r\sqrt{1+r^2}}\qquad S=\frac{n-1+2(n-1)r^2+nr^4}{r^2(1+r^2)},\qquad\text{and}\qquad |\Phi|^2=\frac{n-1}{nr^2(1+r^2)},$$

and its constant scalar curvature is given by

$$R = -\frac{n-2}{nr^2} < 0,$$

which does not satisfy our hypothesis. □

When the spacelike hypersurface has constant scalar curvature (which corresponds to the case $a = 0$), we also have the following consequence of Theorem 8.

Corollary 9. *Let M^n be a complete spacelike hypersurface isometrically immersed in de Sitter space $\mathbb{S}_1^{n+1}$, $n \geq 3$, with constant scalar curvature R satisfying $0 < R < (n-2)/n$. Then*

(i) *either* $\sup_M |\Phi|^2 = 0$ *and M^n is a totally umbilical hypersurface,*

(ii) *or*

$$\sup_M |\Phi|^2 \geq \beta(n, R) > 0,$$

where

$$\beta(n, R) = \alpha(n, 0, R) = \frac{n(n-1)R^2}{(n-2)(n-2-nR)}.$$

In (ii), *a necessary and sufficient condition for equality to hold and the supremum to be attained at some point of M^n is that M^n be isometric to a hyperbolic cylinder $\mathbb{H}^1(r) \times \mathbb{S}^{n-1}(\sqrt{1+r^2})$ of radius $r > 0$.*

For the proof of Corollary 9 simply observe that when $a = 0$ (and hence $R = b$) the positive root x_0 of $\varphi_{0,R}(x) = 0$ is given explicitly by

$$x_0^2 = \frac{n(n-1)R^2}{(n-2)(n-2-nR)}.$$

On the other hand, when $n = 2$ it is easy to see that, supposing $0 < b < 1$ and $R \geq b$, the function $\varphi_{a,b}(x)$ is increasing for $x \geq 0$, with $\varphi_{a,b}(x) \geq \varphi_{a,b}(0) > 0$. Therefore in this case, and taking into account that $R = K$ is the Gaussian curvature of M^2, Theorem 8 can be written as follows.

Theorem 10. *Let M^2 be a complete linear Weingarten spacelike surface isometrically immersed in the de Sitter space $\mathbb{S}_1^3$ such that $K = aH + b$ with $0 < b < 1$ and $K \geq b$. If $\sup_M |\Phi|^2 < +\infty$ then M^2 is a totally umbilical surface.*

In other words, taking into account that the only totally umbilical surfaces in $\mathbb{S}_1^3$ having $K > 0$ are the totally umbilical 2-spheres $\mathbb{S}^2(r) \subset \mathbb{S}_1^3$, with radius $r > 1$, Theorem 10 says:

> *The only complete linear Weingarten spacelike surfaces in de Sitter space $\mathbb{S}_1^3$ satisfying $K = aH + b$ with $0 < b < 1$ and $K \geq b$ for which $|\Phi|^2$ is bounded are the totally umbilical 2-spheres.*

The proof of Theorem 10 follows from that of Theorem 8 after observing that when $n = 2$ it cannot happen that $0 < \sup_M |\Phi|^2 < +\infty$ because that would imply $0 < \varphi_{a,b}(\sqrt{u^*}) \leq 0$. Thus if $\sup_M |\Phi|^2 < +\infty$ we must have $|\Phi|^2 \equiv 0$ and M^2 is a totally umbilical surface.

Finally, when $a = 0$ and $n = 2$, from Theorem 10 we also obtain the following:

Corollary 11. *The only complete spacelike surfaces in the de Sitter space $\mathbb{S}_1^3$ with constant Gaussian curvature $0 < K < 1$ for which $|\Phi|^2$ is bounded (or, equivalently, H is bounded) are the totally umbilical 2-spheres $\mathbb{S}^2(r) \subset \mathbb{S}_1^3$, with radius $r > 1$.*

5. L-parabolicity of linear Weingarten hypersurfaces

Recall that a Riemannian manifold M^n is said to be parabolic if the constant functions are the only subharmonic functions on M^n which are bounded from above; that is, for a function $u \in \mathcal{C}^2(M)$

$$\Delta u \geq 0 \quad \text{and} \quad u \leq u^* < +\infty \quad \text{imply } u = \text{constant}.$$

So, considering the Cheng–Yau modified operator L given in (3-3), we say that M^n is L-parabolic if the only solutions of the inequality $L(u) \geq 0$ which are bounded from above are the constant functions. In this setting, and motivated by Theorem 3 in [Alías et al. 2012], we have the following result.

Theorem 12. *Let M^n be a complete linear Weingarten spacelike hypersurface immersed in de Sitter space $\mathbb{S}_1^{n+1}$, $n \geq 3$, such that $R = aH + b$ with $0 < b \leq R < (n-2)/n$. Suppose that M^n is not totally umbilical. If M^n is L-parabolic, then*

$$\sup_M |\Phi|^2 \geq \alpha(n, a, b) > 0, \tag{5-1}$$

with equality if and only if M^n is isometric to a hyperbolic cylinder $\mathbb{H}^1(r) \times \mathbb{S}^{n-1}(\sqrt{1+r^2})$ of radius $r > 0$.

Proof. If $\sup_M |\Phi|^2 = +\infty$ then there is nothing to prove. Since M^n is not totally umbilical, we consider the case that $0 < \sup_M |\Phi|^2 < +\infty$. In this case, reasoning as in the first part of the proof of Theorem 8, we guarantee that $\sup_M |\Phi|^2 \geq \alpha(n, a, b)$. Moreover, if equality holds in (5-1), then we have $\varphi_{a,b}(|\Phi|) \geq 0$ and, consequently, $L(|\Phi|^2) \geq 0$ on M^n. Hence, from the L-parabolicity of M^n we conclude that the function $u = |\Phi|^2$ must be constant and equal to $\alpha(n, a, b)$. Therefore, at this point, we can reason as in the proof of Theorem 8. □

We close our paper establishing the following L-parabolicity criterion.

Proposition 13. *Let M^n be a complete linear Weingarten spacelike hypersurface immersed in $\mathbb{S}_1^{n+1}$ such that $R = aH + b$ with $b \leq 1$. In the case $b = 1$, assume further that mean curvature function H does not change sign and $b \leq R$. If*

$\sup_M |\Phi|^2 < +\infty$ *and, for some reference point* $o \in M^n$,

$$\int_0^{+\infty} \frac{dr}{\mathrm{vol}(\partial B_r)} = +\infty, \tag{5-2}$$

then M^n *is L-parabolic. Here* B_r *denotes the geodesic ball of radius* r *in* M^n *centered at the origin* o.

Proof. By a straightforward computation we can check from (3-4) that

$$L(u) = \mathrm{div}(P(\nabla u)), \tag{5-3}$$

for any $u \in \mathcal{C}^2(M)$, where P is defined in (3-5).

Now, we consider on M^n the symmetric $(0, 2)$ tensor field h given by $h(X, Y) = \langle PX, Y\rangle$, or, equivalently, $h(\nabla u, \cdot)^\sharp = P(\nabla u)$, where $^\sharp : T^*M \to TM$ denotes the musical isomorphism. Thus, from (5-3) we get

$$L(u) = \mathrm{div}(h(\nabla u, \cdot)^\sharp).$$

On the other hand, as $\sup_M |\Phi|^2 < +\infty$, from (4-16), we have that $\sup_M H < +\infty$. So, we can define a positive continuous function h_+ on $[0, +\infty)$, by

$$h_+(r) = 2n \sup_{\partial B_r} H + (n-1)a. \tag{5-4}$$

Thus, from (5-4) we have

$$h_+(r) = 2n \sup_{\partial B_r} H + (n-1)a \leq 2n \sup_M H + (n-1)a < +\infty. \tag{5-5}$$

Hence, from (5-2) and (5-5) we get

$$\int_0^{+\infty} \frac{dr}{h_+(r)\mathrm{vol}(\partial B_r)} = +\infty.$$

Therefore, we can apply Theorem 2.6 of [Pigola et al. 2005] to conclude the proof. □

Acknowledgements

The authors would like to thank the anonymous referee for valuable suggestions and corrections which greatly improved the final version of this paper.

This work is a result of the activity developed within the framework of the Programme in Support of Excellence Groups of the Región de Murcia, Spain, by Fundación Séneca, Science and Technology Agency of the Región de Murcia. The first author was partially supported by MINECO/FEDER project reference MTM2015-65430-P and Fundación Séneca project reference 19901/GERM/15, Spain, and Ciência sem Fronteiras, Programa PVE, project A012/2013, CAPES, Brazil. The second author was partially supported by CNPq, Brazil, grant 303977/2015-9.

References

[Abe et al. 1987] N. Abe, N. Koike, and S. Yamaguchi, "Congruence theorems for proper semi-Riemannian hypersurfaces in a real space form", *Yokohama Math. J.* **35**:1-2 (1987), 123–136. MR Zbl

[Akutagawa 1987] K. Akutagawa, "On spacelike hypersurfaces with constant mean curvature in the de Sitter space", *Math. Z.* **196**:1 (1987), 13–19. MR Zbl

[Alencar and do Carmo 1994] H. Alencar and M. do Carmo, "Hypersurfaces with constant mean curvature in spheres", *Proc. Amer. Math. Soc.* **120**:4 (1994), 1223–1229. MR Zbl

[Alías et al. 2012] L. J. Alías, D. Impera, and M. Rigoli, "Spacelike hypersurfaces of constant higher order mean curvature in generalized Robertson–Walker spacetimes", *Math. Proc. Cambridge Philos. Soc.* **152**:2 (2012), 365–383. MR Zbl

[Alías et al. 2016] L. J. Alías, P. Mastrolia, and M. Rigoli, *Maximum principles and geometric applications*, Springer, Cham, 2016. MR Zbl

[Brasil et al. 2003] A. Brasil, Jr., A. G. Colares, and O. Palmas, "Complete spacelike hypersurfaces with constant mean curvature in the de Sitter space: a gap theorem", *Illinois J. Math.* **47**:3 (2003), 847–866. MR Zbl

[Calabi 1970] E. Calabi, "Examples of Bernstein problems for some nonlinear equations", pp. 223–230 in *Global analysis* (Berkeley, CA, 1968), edited by S.-s. Chern and S. Smale, Proc. Sympos. Pure Math. **15**, American Mathematical Society, Providence, RI, 1970. MR Zbl

[Chao 2013] X. Chao, "Complete spacelike hypersurfaces in the de Sitter space", *Osaka J. Math.* **50**:3 (2013), 715–723. MR Zbl

[Cheng 1990] Q. M. Cheng, "Complete space-like hypersurfaces of a de Sitter space with $r = kH$", *Mem. Fac. Sci. Kyushu Univ. Ser. A* **44**:2 (1990), 67–77. MR Zbl

[Cheng and Yau 1976] S. Y. Cheng and S. T. Yau, "Maximal space-like hypersurfaces in the Lorentz–Minkowski spaces", *Ann. of Math.* (2) **104**:3 (1976), 407–419. MR Zbl

[Cheng and Yau 1977] S. Y. Cheng and S. T. Yau, "Hypersurfaces with constant scalar curvature", *Math. Ann.* **225**:3 (1977), 195–204. MR Zbl

[Dajczer and Nomizu 1981] M. Dajczer and K. Nomizu, "On flat surfaces in S_1^3 and H_1^3", pp. 71–108 in *Manifolds and Lie groups* (Notre Dame, IN, 1980), edited by S. Murakami et al., Progr. Math. **14**, Birkhäuser, Boston, 1981. MR Zbl

[Goddard 1977] A. J. Goddard, "Some remarks on the existence of spacelike hypersurfaces of constant mean curvature", *Math. Proc. Cambridge Philos. Soc.* **82**:3 (1977), 489–495. MR Zbl

[Hou and Yang 2010] Z. H. Hou and D. Yang, "Linear Weingarten spacelike hypersurfaces in de Sitter space", *Bull. Belg. Math. Soc. Simon Stevin* **17**:5 (2010), 769–780. MR Zbl

[Li 1997] H. Li, "Global rigidity theorems of hypersurfaces", *Ark. Mat.* **35**:2 (1997), 327–351. MR Zbl

[de Lima and Velásquez 2013] H. F. de Lima and M. A. L. Velásquez, "On the geometry of linear Weingarten spacelike hypersurfaces in the de Sitter space", *Bull. Braz. Math. Soc.* (*N.S.*) **44**:1 (2013), 49–65. MR Zbl

[Montiel 1988] S. Montiel, "An integral inequality for compact spacelike hypersurfaces in de Sitter space and applications to the case of constant mean curvature", *Indiana Univ. Math. J.* **37**:4 (1988), 909–917. MR Zbl

[Montiel 1996] S. Montiel, "A characterization of hyperbolic cylinders in the de Sitter space", *Tohoku Math. J.* (2) **48**:1 (1996), 23–31. MR Zbl

[Okumura 1974] M. Okumura, "Hypersurfaces and a pinching problem on the second fundamental tensor", *Amer. J. Math.* **96** (1974), 207–213. MR Zbl

[Pigola et al. 2005] S. Pigola, M. Rigoli, and A. G. Setti, "A Liouville-type result for quasi-linear elliptic equations on complete Riemannian manifolds", *J. Funct. Anal.* **219**:2 (2005), 400–432. MR Zbl

[Ramanathan 1987] J. Ramanathan, "Complete spacelike hypersurfaces of constant mean curvature in de Sitter space", *Indiana Univ. Math. J.* **36**:2 (1987), 349–359. MR Zbl

[Treibergs 1982] A. E. Treibergs, "Entire spacelike hypersurfaces of constant mean curvature in Minkowski space", *Invent. Math.* **66**:1 (1982), 39–56. MR Zbl

Received December 6, 2016.

LUIS J. ALÍAS
DEPARTAMENTO DE MATEMÁTICAS
UNIVERSIDAD DE MURCIA
CAMPUS DE ESPINARDO
MURCIA
SPAIN
ljalias@um.es

HENRIQUE F. DE LIMA
DEPARTAMENTO DE MATEMÁTICA
UNIVERSIDADE FEDERAL DE CAMPINA GRANDE
CAMPINA GRANDE, PB
BRAZIL
henrique@mat.ufcg.edu.br

FÁBIO R. DOS SANTOS
DEPARTAMENTO DE MATEMÁTICA
UNIVERSIDADE FEDERAL DE CAMPINA GRANDE
CAMPINA GRANDE, PB
BRAZIL
fabio@mat.ufcg.edu.br

PACIFIC JOURNAL OF MATHEMATICS

Founded in 1951 by E. F. Beckenbach (1906–1982) and F. Wolf (1904–1989)

msp.org/pjm

See inside back cover or msp.org/pjm for submission instructions.

The subscription price for 2018 is US $475/year for the electronic version, and $640/year for print and electronic.
Subscriptions, requests for back issues and changes of subscriber address should be sent to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163, U.S.A. The Pacific Journal of Mathematics is indexed by Mathematical Reviews, Zentralblatt MATH, PASCAL CNRS Index, Referativnyi Zhurnal, Current Mathematical Publications and Web of Knowledge (Science Citation Index).

The Pacific Journal of Mathematics (ISSN 0030-8730) at the University of California, c/o Department of Mathematics, 798 Evans Hall #3840, Berkeley, CA 94720-3840, is published twelve times a year. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163.

PJM peer review and production are managed by EditFLOW® from Mathematical Sciences Publishers.

PUBLISHED BY
mathematical sciences publishers
nonprofit scientific publishing

http://msp.org/

PACIFIC JOURNAL OF MATHEMATICS

Volume 292 No. 1 January 2018

0030-8730(201801)292:1;1-Z