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MERIDIONAL RANK AND BRIDGE NUMBER FOR A CLASS OF LINKS

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We prove that links with meridional rank 3 whose 2-fold branched covers are graph manifolds are 3-bridge links. This gives a partial answer to a question by S. Cappell and J. Shaneson on the relation between the bridge numbers and meridional ranks of links. To prove this result, we also show that the meridional rank of any satellite knot is at least 4.

1. Introduction

An *n-bridge sphere* of a link L in the 3-sphere S^3 is a 2-sphere which meets L in $2n$ points and cuts (S^3, L) into n-string trivial tangles. Here, an *n-string trivial tangle* is a pair (B^3, t) of the 3-ball B^3 and n arcs properly embedded in B^3 parallel to the boundary of B^3. It is known that every link admits an n-bridge sphere for some positive integer n. We call a link L an *n-bridge link* if L admits an n-bridge sphere and does not admit an $(n-1)$-bridge sphere. We call n the *bridge number* of the link L and denote it by $b(L)$.

If a link admits an n-bridge sphere, then it is easy to see that $\pi_1(S^3 \setminus L)$ can be generated by n meridians, where a meridian is an element of the fundamental group that is represented by a curve that is freely homotopic to a meridian of L. This implies that the minimal number of meridians needed to generate the group $\pi_1(S^3 \setminus L)$ is less than or equal to $b(L)$. We denote by $w(L)$ the minimal number of meridians needed to generate $\pi_1(S^3 \setminus L)$ and call it the *meridional rank* of L. Thus for any link L we have $b(L) \geq w(L)$.

S. Cappell and J. Shaneson [Kirby 1978, Problem 1.11], as well as K. Murasugi, have asked whether the converse holds:

Question 1.1. Does the equality $b(L) = w(L)$ hold for any link L in S^3?

This is known to be true for (generalized) Montesinos links by [Boileau and Zieschang 1985], torus links by [Rost and Zieschang 1987] and for another class

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of knots (also referred to as generalized Montesinos knots) by [Lustig and Moriah 1993]. More recently the equality has been established for a large class of iterated torus knots using knot contact homology [Cornwell and Hemminger 2016]; see also [Cornwell 2014]. It is a consequence of Dehn's Lemma that $b(L) = 1$ if and only if $w(L) = 1$. Moreover in [Boileau and Zimmermann 1989] it is proved that $b(L) = 2$ if and only if $w(L) = 2$.

The main purpose of this paper is to prove the following theorem.

Theorem 1.2. *Let L be a link in the 3-sphere S^3, and suppose that the 2-fold branched cover of S^3 branched along L is a graph manifold. If $w(L) = 3$, then $b(L) = 3$, i.e., L is a 3-bridge link.*

Here a graph manifold is a compact orientable prime 3-manifold whose geometric decomposition contains only Seifert fibered pieces.

The above theorem, together with the result in [Boileau and Zimmermann 1989], implies that $b(L) = 3$ if and only if $w(L) = 3$ for links whose 2-fold branched covers are graph manifolds. In particular we obtain the following:

Corollary 1.3. *Let $L \subset S^3$ be a link whose 2-fold branched cover is a graph manifold. If $b(L) = 4$, then $w(L) = 4$.*

We obtain also the following corollary which answers [Boileau and Weidmann 2008, Question 2] positively for graph manifolds.

Corollary 1.4. *For a closed orientable graph manifold M, any inversion of $\pi_1(M)$ is hyperelliptic.*

We remark that Question 1.1, posed by Cappell and Shaneson, is related, by taking the 2-fold branched covering, to the question of whether or not the Heegaard genus of a 3-manifold equals the rank of its fundamental group. For the latter question many counterexamples are known; see [Boileau and Weidmann 2005; Boileau and Zieschang 1983; 1984; Li 2013; Schultens and Weidmann 2007; Weidmann 2003]. Thus there exist manifolds such that the ranks of their fundamental groups are smaller than their Heegaard genera. To the question of Cappell and Shaneson, however, no counterexample is known to date.

We also remark that if we replace $w(L)$ with the rank of the link group $\pi_1(S^3 \setminus L)$ then we can easily find examples where the differences between the two numbers are arbitrarily large. For example, the rank of the group $\pi_1(S^3 \setminus K(p, q))$ of a torus link $K(p, q)$ is 2 while $b(K(p, q)) = \min(p, q)$ by [Schubert 1954].

To prove Theorem 1.2 we distinguish two cases, namely the case when the link L is arborescent in the sense of Bonahon and Siebenmann [2016] and the case when L is not arborescent. We will make use of the following theorem, which is interesting in its own right.

Theorem 1.5. *Let K be a prime knot such that $S^3 \setminus K$ has a nontrivial JSJ-decomposition and let m_1, m_2, m_3 be meridians. Then one of the following holds*:

(1) $\langle m_1, m_2, m_3 \rangle$ *is free.*

(2) $\langle m_1, m_2, m_3 \rangle$ *is conjugate into the subgroup of $\pi_1(S^3 \setminus K)$ corresponding to the peripheral piece of $S^3 \setminus K$.*

Corollary 1.6. *Let K be a prime knot such that $S^3 \setminus K$ has a nontrivial JSJ-decomposition. Then $w(K) \geq 4$.*

Corollary 1.7. *Let $K \subset S^3$ be a knot. If $w(K) \leq 3$, then K is either a hyperbolic knot or a torus knot or a connected sum of two 2-bridge knots.*

Theorem 1.5 suggests this strengthening of Question 1.1 for a hyperbolic knot:

Question 1.8. Let $K \subset S^3$ be a hyperbolic knot. Is a subgroup of $\pi_1(S^3 \setminus K)$ generated by at most $b(K) - 1$ meridians free?

In the case of torus knots the conclusion of Question 1.8 has been established by M. Rost and H. Zieschang [1987]. The case of hyperbolic 3-bridge knots follows from a general result for subgroups generated by two meridians in a knot group; see Proposition 4.2. It should be noted that the conclusion of Question 1.8 does obviously not hold for connected sums of knots, and it is moreover not difficult to come up with examples of prime knots with nontrivial JSJ-decomposition for which the conclusion does not hold either.

There is a natural partial order on the set of links in S^3 given by degree-one maps: We say that a link $L \subset S^3$ *dominates* a link $L' \subset S^3$ and write $L \geq L'$ if there is a proper degree-one map $f : E(L) \to E(L')$ between the exteriors of L and L' whose restriction to the boundary is a homeomorphism which extends to the regular neighborhoods of L and L'. It defines a partial order on the set of links in S^3, and it is an open problem to characterize minimal elements. In particular the behavior of the bridge number with respect to this order is far from being understood:

Question 1.9. Let L and L' be links in S^3. Does $L \geq L'$ imply $b(L) \geq b(L')$?

It follows from the definition that the epimorphism $f_\star : \pi_1(S^3 \setminus L) \to \pi_1(S^3 \setminus L')$ induced by the degree-one map $f : E(L) \to E(L')$ preserves the meridians and so that $w(L) \geq w(L')$ whenever $L \geq L'$. Therefore an affirmative answer to Question 1.1 would imply an affirmative answer to Question 1.9.

The answer to Question 1.9 is certainly positive when $b(L') = 2$ as in this case any knot L with $L \geq L'$ cannot be trivial. Our results moreover imply the following:

Proposition 1.10. *Let $L \geq L'$ be two links in S^3.*

(a) *If $b(L') = 3$, then $b(L) \geq 3$.*

(b) *If $b(L') = 4$ and the 2-fold cover of S^3 branched along L is a graph manifold, then $b(L) \geq 4$.*

In Section 2, we recall the definition and some properties of arborescent links and show that an arborescent link L with $w(L) = 3$ is hyperbolic. Section 3 is devoted to the proof of Theorem 1.2 for arborescent links. Section 4 contains the proof of Theorem 1.5. In Section 5 we complete the proof of Theorem 1.2 for the case of non-arborescent links. Then Section 6 contains the proof of Proposition 1.10.

2. Arborescent links

A (3, 1)-*manifold pair* is a pair (M, L) of a compact oriented 3-manifold M and a proper 1-submanifold L of M. By a *surface F in* (M, L), we mean a surface F in M intersecting L transversely. Two surfaces F and F' in (M, L) are said to be *pairwise isotopic* (*isotopic*, in brief) if there is a homeomorphism $f : (M, L) \to (M, L)$ such that $f(F) = F'$ and f is pairwise isotopic to the identity. We call a (3, 1)-manifold pair a *tangle* if M is homeomorphic to B^3.

A *trivial tangle* is a (3, 1)-manifold pair (B^3, L), where L is the union of two properly embedded arcs in the 3-ball B^3 which together with arcs on the boundary of B^3 bound disjoint disks. A *rational tangle* is a trivial tangle (B^3, L) endowed with a homeomorphism from $\partial(B^3, L)$ to the "standard" pair of the 2-sphere and the union of four points on the sphere. It is well known that rational tangles (up to isotopy fixing the boundaries) correspond to elements of $\mathbb{Q} \cup \{\infty\}$, called the *slopes* of the rational tangles. For example, the rational tangle of slope β/α can be illustrated as in Figure 1, where α, β are defined by the continued fraction

$$(*)\qquad \begin{aligned}\frac{\beta}{\alpha} &= -a_0 + [a_1, -a_2, \dots, \pm a_m] \\ &:= -a_0 + \cfrac{1}{a_1 + \cfrac{1}{-a_2 + \cfrac{1}{\dots + \cfrac{1}{\pm a_m}}}}\end{aligned}$$

together with the condition that α and β are relatively prime and $\alpha \geq 0$. Here, the numbers a_i denote the numbers of right-hand half twists.

A *Montesinos pair* is a (3, 1)-manifold pair which is built from the pair in Figure 2 (left) or Figure 2 (right) by plugging some of the holes with rational tangles of finite slopes. We say that a Montesinos pair is *trivial* if it is homeomorphic to a rational tangle or $(S, P) \times I$, where S is a 2-sphere, P is the union of four distinct points on S and I is a closed interval. A *Montesinos link* is a link obtained by plugging the remaining holes of a Montesinos pair in Figure 2 (left) with rational tangles of finite slopes, as shown in Figure 3. Unless otherwise stated, we assume that the slope β_i/α_i of each rational tangle is not an integer, that is, $\alpha_i > 1$. The above Montesinos link is denoted by $L(-b\,; \beta_1/\alpha_1, \dots, \beta_r/\alpha_r)$. (We note that this is denoted by $m(0 \mid b\,; (\alpha_1, \beta_1), (\alpha_2, \beta_2), \dots, (\alpha_r, \beta_r))$ in [Boileau and Zieschang 1985].)

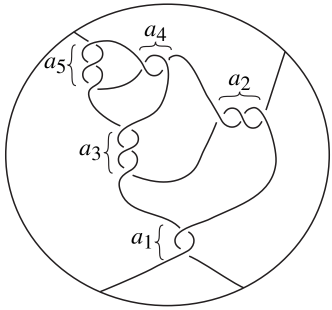


Figure 1. Rational tangle of slope $\beta/\alpha = 31/50$, which has the expression (∗) with $m = 5$, $a_0 = 0$, $a_1 = 2$, $a_2 = 3$, $a_3 = 3$, $a_4 = 2$, $a_5 = 3$.

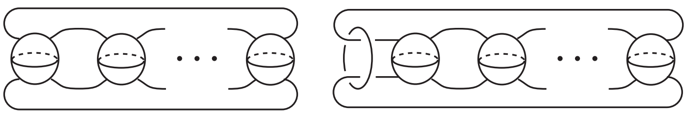


Figure 2. Starting points for a Montesinos pair.

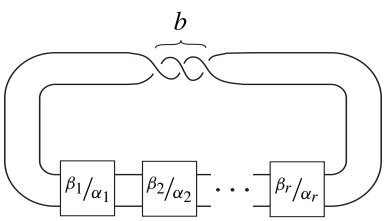


Figure 3. A Montesinos link with $b = 3$.

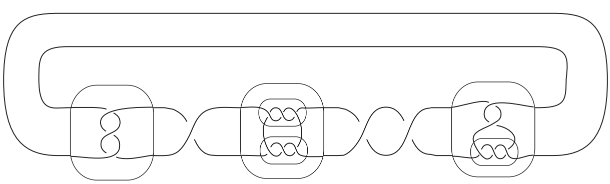


Figure 4. An arborescent link.

An *arborescent link* is a link in S^3 obtained by gluing some Montesinos pairs in their boundaries as in Figure 4; see [Bonahon and Siebenmann 2016].

The main result of this section is the following proposition which is used to prove Theorem 1.2 in Section 3 when the link L is an arborescent link.

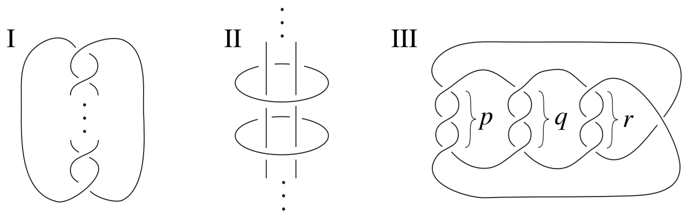
Proposition 2.1. *Let L be an arborescent link which is not a generalized Montesinos link, and suppose that $w(L) = 3$. Then L is hyperbolic.*

Proof. Let L be an arborescent link which is not a generalized Montesinos link, and suppose that $w(L) = 3$. Assume on the contrary that L is not hyperbolic. By [Bonahon and Siebenmann 2016] (see also [Futer and Guéritaud 2009] or [Jang 2011, Proposition 3]), we are in one of three cases, illustrated below:

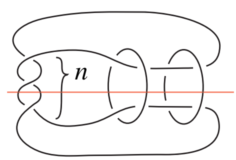
(I) L is a torus knot or link of type $(2, n)$ for some integer n.

(II) L has two parallel components, each of which bounds a twice-punctured disk properly embedded in $S^3 \setminus L$.

(III) L or its reflection is the pretzel link $P(p, q, r, -1) := L\big(-1; \frac{1}{p}, \frac{1}{q}, \frac{1}{r}\big)$, where $p, q, r \geq 2$ and $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} \geq 1$.



By the assumptions that L is not a generalized Montesinos link and that $w(L) = 3$, L must be equivalent to a link in case II, namely, L has two parallel components, each of which bounds a twice-punctured disk properly embedded in $S^3 \setminus L$. Moreover, since $w(L) = 3$, L must have 3 components. Recall that the 2-fold branched cover of S^3 branched along L is a graph manifold. By [Boileau and Weidmann 2008, Proposition 20(2)], the union of any two components of L is a 2-bridge link. Then, by arguments in the proof of [Jang 2011, Proposition 4(1)], we see that L is equivalent to this link:



However, this link is a generalized Montesinos link, which contradicts the assumption. Hence, L is hyperbolic. □

3. Proof of Theorem 1.2 for arborescent links

Let L be an arborescent link and suppose that $w(L) = 3$. If L is a generalized Montesinos link, then we have $b(L) = 3$ by [Boileau and Zieschang 1985]. Thus we assume that L is not a generalized Montesinos link in the remainder of this proof. Then, by Proposition 2.1, L is hyperbolic. Let $P = P_1 \cup \cdots \cup P_k$ be the union of Conway spheres which gives the characteristic decomposition of L. (See [Bonahon and Siebenmann 2016] for a definition of the characteristic decomposition of a link; by [Boileau et al. 2003], this decomposition corresponds to the geometric decomposition of the 3-orbifold with underlying space S^3 and singular locus L with branching index 2.) Let $M := M_2(L)$ be the 2-fold cover of S^3 branched along L, and let T_i be the preimage of P_i in M ($i = 1, \dots, k$). Then each T_i is a separating torus in M and $T = T_1 \cup \dots \cup T_k$ gives the JSJ-decomposition of M, by [Jang 2011, Proposition 4]. Let τ_L be the covering involution of the 2-fold branched cover. By construction, the following hold.

(T1) Each T_i is τ_L-invariant and $\tau|_{T_i}$ is hyperelliptic.

(T2) τ_L preserves each JSJ piece and each exceptional fiber of Seifert pieces.

Recall that we have an exact sequence

$$1 \to \pi_1(M) \to \pi_1(S^3 \setminus L)/N \to \mathbb{Z}/2\mathbb{Z} \to 1,$$

where N is the subgroup of $\pi_1(S^3 \setminus L)$ normally generated by the squares of the meridians. Let m_1, m_2 and m_3 be meridians of $\pi_1(S^3 \setminus L)$ generating the group. For $1 \le i \le 3$ we denote the image of m_i in $\pi_1(S^3 \setminus L)/N$ again by m_i. Since $\pi_1(M)$ can be regarded as an index-2 subgroup of $\pi_1(S^3 \setminus L)/N$ by the above exact sequence, any element of $\pi_1(M)$ can be represented as a product of even numbers of m_1, m_2 and m_3. Set $g_1 := m_1 m_2$ and $g_2 := m_1 m_3$. Then g_1 and g_2 generate $\pi_1(M)$. Let α be the automorphism of $\pi_1(S^3 \setminus L)/N$ induced by the conjugation by m_1. Then τ_L is a realization of α. We see $\alpha(g_i) = m_1 g_i m_1^{-1} = g_i^{-1}$ for each $i = 1, 2$, and hence, $\alpha|_{\pi_1(M)}$ is an automorphism of $\pi_1(M)$ which sends each generator g_i to g_i^{-1}. Namely, α is an *inversion* of $\pi_1(M)$ (see [Boileau and Weidmann 2008]). Since M is a graph manifold which admits an inversion, the Heegaard genus of M is 2 by [Boileau and Weidmann 2008, Theorem 3]. Recall that $T = T_1 \cup \dots \cup T_k$ gives the nontrivial JSJ-decomposition of M, where each T_i is a separating torus in M. By [Jang 2011, Proposition 4], M satisfies one of the following conditions (M1), (M2), (M3) and (M4) which originally come from [Kobayashi 1984].

(M1) M is obtained from a Seifert fibered space M_1 over a disk with two exceptional fibers and the exterior M_2 of a nonhyperbolic 1-bridge knot K in a lens space by gluing their boundaries so that the meridian of K is identified with the regular fiber of M_1.

(M2) M is obtained from a Seifert fibered space M_1 over a disk with two or three exceptional fibers and the exterior M_2 of a nonhyperbolic 2-bridge knot K in S^3 by gluing their boundaries so that the meridian of K is identified with the regular fiber of M_1.

(M3) M is obtained from a Seifert fibered space M_1 over a Möbius band with one or two exceptional fibers and the exterior M_2 of a nonhyperbolic 2-bridge knot K in S^3 by gluing their boundaries so that the meridian of K is identified with the regular fiber of M_1.

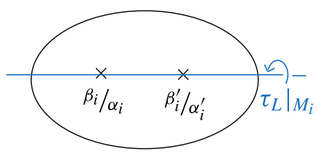
(M4) M is obtained from two Seifert fibered spaces M_1 and M_2 over a disk with two exceptional fibers and the exterior M_3 of a nonhyperbolic 2-bridge link $L = K_1 \cup K_2$ in S^3 by gluing $\partial(M_1 \cup M_2)$ and ∂M_3 so that the meridian of K_i is identified with the regular fiber of M_i $(i = 1, 2)$.

Assume that M satisfies the condition (M1). That is, M is obtained from a Seifert fibered space M_1 over a disk with two exceptional fibers and the exterior M_2 of a nonhyperbolic 1-bridge knot K in a lens space by gluing their boundaries so that the meridian of K is identified with the regular fiber of M_1. By [Kobayashi 1984], M_2 satisfies one of the following.

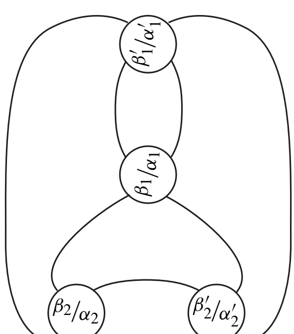
(M1-a) M_2 is a Seifert fibered space over a disk with two exceptional fibers, or

(M1-b) M_2 is a Seifert fibered space over a Möbius band with one exceptional fiber.

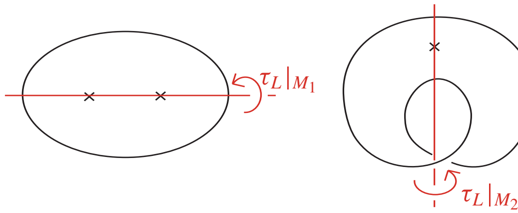
First we assume that M_2 satisfies (M1-a). Recall that the covering involution τ_L satisfies the conditions (T1) and (T2). Since the center of $\pi_1(M)$ is trivial, the strong equivalence class of τ_L is determined by its image in the mapping class group by [Tollefson 1981, Theorem 7.1]. By [Jang 2011, Lemma 4(1)] (or [Jang 2011, Proposition 6(1)]), we may assume that the restriction $\tau_L|_{M_i}$ $(i = 1, 2)$ is a fiber-preserving involution of M_i which induces the involution on the base orbifold:



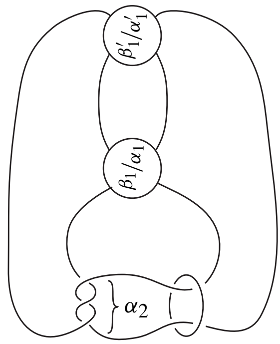
Each quotient orbifold $(M_i, \mathrm{Fix}\tau_L|_{M_i})/\tau_L|_{M_i}$ $(i = 1, 2)$ is a Montesinos pair with two rational tangles. By gluing them so that the image of the meridian of K is identified with the image of the regular fiber of M_1, we see that L must be a 3-bridge link like this (see also [Jang 2011, Section 7, Case 1.1]):



Assume that M_2 satisfies (M1-b). By [Jang 2011, Lemma 4(1) and (2)] together with [Tollefson 1981, Theorem 7.1], we may assume that the restriction $\tau_L|_{M_i}$ ($i = 1, 2$) is a fiber-preserving involution of M_i that induces the involution on the base orbifold as illustrated here:



By considering the quotient orbifold $(M, \mathrm{Fix}\tau_L)/\tau_L$, we see that L is equivalent to a 3-bridge link of this form (see also [Jang 2011, Section 7, Case 1.2]):



The remaining cases can be treated similarly except for the case where M satisfies the condition (M3). Thus, in the rest of this section, we assume that M satisfies the condition (M3). That is, M is obtained from a Seifert fibered space M_1 over a Möbius band with one or two exceptional fibers and the exterior M_2 of a nonhyperbolic 2-bridge knot K in S^3 by gluing their boundaries so that the meridian of K is identified with the regular fiber of M_1. By an argument similar to those for the previous cases, we can see that L is equivalent to the link in Figure 5 on the next page. For that link, we may assume that the rational number β_1/α_1 is not an integer, and that the rational number β_2/α_2 is an integer or not an integer according to whether the number of the exceptional fibers of M_1 is one or two. We can see that the bridge number of the link $K_1 \cup K_2$ in the figure is at least 4, since K_1 is a 3-bridge link by [Boileau and Zieschang 1985] and [Jang 2011]. However, by [Boileau and Zieschang 1985, Lemma 1.7] and [Boileau and Zimmermann 1989, Corollary 3.3], we have $w(K_1 \cup K_2) \geq w(K_1) + w(K_2) = 3 + 1 = 4$, which contradicts the assumption that $w(L) = 3$.

This completes the proof of Theorem 1.2 for arborescent knots.

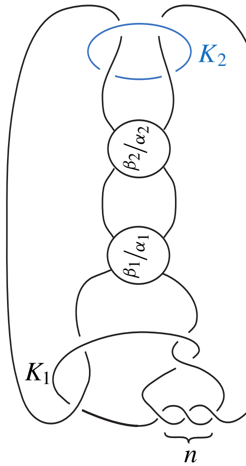


Figure 5. Link equivalent to L when M satisfies condition (M3); see previous page.

4. Subgroups generated by meridians

In this section we study subgroups of knot and link groups that are generated by two or three meridians and we give a proof of Theorem 1.5.

For L a link in S^3 and the link space $E(L)$, choose annuli and tori as follows:

(1) Let $\{A_1, \dots, A_n\}$ be a maximal collection of disjoint nonparallel and properly embedded essential annuli in $E(L)$ whose boundaries are meridians. Thus the closures of the components of $E(L) \setminus \bigcup_{1 \le i \le n} A_i$ are the link spaces $E(L_1), \dots, E(L_k)$ of the prime factors L_i of L.

(2) Let $\{T_1, \dots, T_m\}$ be the union of the characteristic families of tori of the manifolds $E(L_i)$ for $1 \le i \le n$.

Thus the closures of the components of

$$E(L) \setminus \left(\left(\bigcup_{1 \le i \le n} A_i \right) \cup \left(\bigcup_{1 \le i \le m} T_i \right) \right)$$

are the pieces of the JSJ-decompositions of the link spaces $E(L_i)$ with $1 \le i \le n$. We call such a piece peripheral if it meets a boundary component of $E(L)$.

Now, let $G = \pi_1(E(L))$. Let $\mathbb{A}_L$ be the graph of group decomposition of G corresponding to the splitting of $E(L)$ along the A_i and T_i. Thus the vertex groups are the fundamental groups of pieces of the JSJ-decompositions of the $E(L_i)$ and the edge groups are infinite cyclic or isomorphic to $\mathbb{Z}^2$.

Lemma 4.1. *Let L be as above, $G := \pi_1(E(L))$ and $m_1, \dots, m_k \in G$ be meridians (not necessarily corresponding to the same component of L).*

Then either $\langle m_1, \dots, m_k\rangle$ is free or there exist meridians $m'_1, \dots, m'_k \in G$ such that the following hold:

(1) $(m_1, \dots, m_k)$ *is Nielsen-equivalent to* $(m'_1, \dots, m'_k)$ *and* m_i *is conjugate to* m'_i *for* $1 \le i \le k$.

(2) *There exist* $i \ne j \in \{1, \dots, k\}$ *such that* $\langle m'_i, m'_j\rangle$ *is conjugate to a vertex group of* $\mathbb{A}_L$ *that corresponds to a peripheral piece of some* $E(L_i)$. *Moreover* m'_i *and* m'_j *are conjugate to meridians in this vertex group.*

Proof. We consider the action of G on the Bass–Serre tree T corresponding to $\mathbb{A}_L$. Any m_i acts elliptically and the fixed point set of m_i coincides with the fixed point set of m_i^n for any $n \ne 0$. This is true as m_i is a peripheral element and therefore not a proper root of the regular fiber of any Seifert piece.

Moreover for all $i \in \{1, \dots, k\}$ the element m_i (and therefore also m_i^n with $n \ne 0$) fixes no edge corresponding to a canonical torus of the JSJ-decomposition of some $E(L_i)$ as no power of the meridian is freely homotopic to a curve in one of these tori.

It now follows from [Weidmann 2002, Theorem 7] applied to $(\{m_1\}, \dots, \{m_k\}, \varnothing)$ that either $\langle m_1, \dots, m_k\rangle$ is free or that there exist elements $m'_1, \dots, m'_k$ such that the following hold:

(1) $(m_1, \dots, m_k)$ is Nielsen-equivalent to $(m'_1, \dots, m'_k)$.

(2) m_i is conjugate to m'_i for $1 \le i \le k$.

(3) There exist $i \ne j \in \{1, \dots, k\}$ such that nontrivial powers of m'_i and m'_j fix a common vertex of T.

This implies in particular that m'_i is a meridian for $1 \le i \le k$. The above remark further implies that not only powers of m'_i and m'_j but m'_i and m'_j themselves fix a common vertex v of T that is therefore also fixed by $\langle m'_i, m'_j\rangle$. As both m'_i and m'_j only fix vertices of T that correspond to peripheral pieces, it follows that v corresponds to a peripheral piece. As no meridian is conjugate in a peripheral piece to an element corresponding to one of the characteristic tori it follows moreover that m'_i and m'_j are conjugate to meridians in the stabilizer of v. □

Proposition 4.2. *Let K be a knot in S^3 and $G := \pi_1(E(K))$. If $m_1, m_2 \in G$ are meridians that generate a nonfree subgroup of G then K has a prime factor K_1 that is a 2-bridge knot and $\langle m_1, m_2\rangle$ is conjugate to the subgroup of G corresponding to K_1.*

Proof. It follows from Lemma 4.1 that $\langle m_1, m_2\rangle$ lies in the subgroup corresponding to a peripheral piece of $E(K)$. Thus $\langle m_1, m_2\rangle$ is contained in the subgroup corresponding to the peripheral piece M of the JSJ-decomposition of a prime factor K_1

of K. Moreover m_1 and m_2 are in this subgroup conjugate to the meridian. We distinguish two cases:

Suppose first that M is Seifert fibered. Thus M is a torus knot space or a cable space. In the first case it follows from [Rost and Zieschang 1987] that either $\langle m_1, m_2\rangle$ is free or that $\langle m_1, m_2\rangle = \pi_1(M)$ and that M is the exterior of a 2-bridge knot which proves the claim. In the second case M is the mapping torus of a disk with finitely many punctures with respect to an automorphism of finite order. Moreover (like all elements conjugate to a meridian) both m_1 and m_2 lie in the free fundamental group of the fiber which implies that $\langle m_1, m_2\rangle$ is free.

Suppose now that M is hyperbolic. We may assume that $\langle m_1, m_2\rangle$ is not abelian as two conjugates of the meridian that generate an abelian group must lie in the same conjugate of the same peripheral subgroup and therefore generate a cyclic subgroup.

It follows from Proposition 2 of [Boileau and Weidmann 2005] that either $\langle m_1, m_2\rangle = \pi_1(M)$ and that M is the exterior of a 2-bridge knot or that $|\pi_1(M) : \langle m_1, m_2\rangle| = 2$ and the 2-sheeted cover $\tilde{M}$ of M corresponding to $\langle m_1, m_2\rangle$ is the exterior of a 2-bridge link with 2 components.

In the first case the conclusion is immediate. Suppose now that the second case occurs. As m_1 and m_2 is conjugate in $\pi_1(M)$ it follows that both boundary components of $\tilde{M}$ cover the same boundary component of M, in particular M is a knot exterior. Now $\langle m_1, m_2\rangle$ contains a conjugate of the peripheral subgroup of $\pi_1(M)$ and is normal in $\pi_1(M)$. It follows that $\langle m_1, m_2\rangle$ contains all parabolic elements of $\pi_1(M)$. As $\pi_1(M)$ is a knot group, it is generated by parabolic elements. It follows that $\pi_1(M) = \langle m_1, m_2\rangle$ which yields a contradiction. □

The rest of this section is devoted to the proofs of Theorem 1.5 and Corollary 1.7.

Proof of Theorem 1.5. It follows from Lemma 4.1 that we may assume that $\langle m_1, m_2\rangle$ fixes a vertex v of the Bass–Serre tree that corresponds to the peripheral piece M of $S^3 \setminus K$ and m_1 and m_2 are conjugate to meridians in $\pi_1(M)$. By Proposition 4.2 the group $\langle m_1, m_2\rangle$ is free.

Choose a torus T of the characteristic family of tori for $S^3 \setminus K$ such that T cuts $S^3 \setminus K$ into two pieces, a geometric knot space N and its complement $\widehat{M}$. Clearly M is contained in $\widehat{M}$. Let $W = S^3 \setminus \mathrm{int(N)}$ be the solid torus containing M. Since m_1 and m_2 are conjugate to meridians in $\pi_1(M)$, they are null-homologous in W and so is any element of $\langle m_1, m_2\rangle$. The meridian of N and its powers are the only elements of $\pi_1(T) = \partial W$ which are null-homologous in W, therefore the subgroup $\langle m_1, m_2\rangle$ intersects any conjugate of the free abelian subgroup $\pi_1(T) \subset G = \pi_1(S^3 \setminus K)$ at most in a subgroup of the cyclic group generated by the meridian of N. Consider the action of G on the Bass–Serre tree corresponding to the amalgamated product $\pi_1(N) *_{\pi_1(T)} \pi_1(\widehat{M})$. Let v be the vertex fixed by $\langle m_1, m_2\rangle$, note that v corresponds to $\pi_1(\widehat{M})$. As the meridian of N does not agree with the fiber of N if N is Seifert fibered, it follows that no element of $\langle m_1, m_2\rangle$ fixes a vertex at distance more than 1 from v. Moreover m_3 fixes a single vertex that corresponds to $\pi_1(\widehat{M})$. By applying Theorem 7 of [Weidmann 2002] to $(\{m_1, m_2\}, \{m_3\})$ it follows that either m_3 also fixes v or that $\langle m_1, m_2, m_3\rangle \cong \langle m_1, m_2\rangle * \langle m_3\rangle \cong F_3$. This proves the claim. □

Corollary 1.6 is a direct consequence of Theorem 1.5. We prove now Corollary 1.7.

Proof of Corollary 1.7. Let $K \subset S^3$ be a knot such that $w(K) = 3$. If K is prime, then Theorem 1.5 implies that K is a hyperbolic knot or a torus knot. If $K = K_1 \sharp K_2$ is a nontrivial connected sum, then the 2-fold cover $M_2(K)$ of S^3 branched along K is the nontrivial connected sum $M_2(K_1)\sharp M_2(K_2)$ of the 2-fold branched covers of K_1 and K_2. Since $w(K) = 3$, it follows that $\pi_1(M_2(K))$ is generated by two elements. Since

$$\pi_1(M_2(K)) = \pi_1(M_2(K_1)) * \pi_1(M_2(K_2))$$

is a free product of nontrivial groups, by the orbifold theorem (see [Boileau and Porti 2001]), it follows that each group $\pi_1(M_2(K_1))$ and $\pi_1(M_2(K_2))$ is cyclic. Again the orbifold theorem allows us to conclude that K_1 and K_2 are 2-bridge knots. □

5. Proof of Theorem 1.2

Let L be a link in S^3, and suppose that the 2-fold branched cover $M := M_2(L)$ of S^3 branched along L is a graph manifold. Since we have already treated the case when L is an arborescent link in Section 3, we assume here that L is not an arborescent link and that $w(L) = 3$.

We first assume that M is a Seifert fibered space. Then L is either a (generalized) Montesinos link or a Seifert link, i.e., $S^3 \setminus L$ admits a Seifert fibration. If L is a (generalized) Montesinos link or a torus link, then we have $b(L) = 3$ by [Boileau and Zieschang 1985; Rost and Zieschang 1987]. So we assume that L is a Seifert link which is not a torus link. By [Burde and Murasugi 1970], we see that L is the union of a torus knot of type $(2, b)$ and its *core* of index 2, in which case it is easy to see that $b(L) = 3$.

Next we assume that M is not a Seifert fibered space. Let $T = T_1 \cup \cdots \cup T_k$ be tori which give the JSJ-decomposition of M. As in Section 3, we can see that M is a genus-2 manifold and the covering involution τ_L is a realization of an inversion of $\pi_1(M)$. Let $\alpha := (\tau_L)_*$ be the automorphism of $\pi_1(M)$ and let g and h be a pair of generators for $\pi_1(M)$. By [Boileau and Weidmann 2008, Proposition 20], τ_L respects the JSJ-decomposition of M and the Seifert fibered structures on the JSJ pieces. Let Q be the oriented circle bundle over the Möbius band. We follow the argument in [Boileau and Weidmann 2005, Section 3], under the assumption that M is a genus-2 closed manifold. We first deal with the following case.

5.1. ***The JSJ-decomposition has a separating torus and no piece homeomorphic to Q.*** Let T_1 be the separating torus by changing order if necessary, and let M_A and M_B be the two submanifold of M divided by T_1. By the argument in [Boileau and Weidmann 2005], we see that M_A is a Seifert fibered space, g is a root of a fiber of M_A and $g^n \in \pi_1(T_1)$. Moreover, one of the following holds.

(i) M_A is a Seifert fibered space over a disk with two exceptional fibers and M_B is the exterior of a 1-bridge knot in a lens space.

(ii) M_A is a Seifert fibered space over a disk with two exceptional fibers and M_B is the exterior of a nonhyperbolic 2-bridge knot in S^3.

(iii) M_A is a Seifert fibered space over a disk with two exceptional fibers and M_B is decomposed by T_2 into two pieces $M_B^{(1)}$ and $M_B^{(2)}$, where $M_B^{(1)}$ is the exterior of a 2-component nonhyperbolic 2-bridge link in S^3 and where $M_B^{(2)}$ is a Seifert fibered space over a disk with two exceptional fibers.

(iv) M_A is a Seifert fibered space over a Möbius band with one or two exceptional fibers and M_B is the exterior of a nonhyperbolic 2-bridge knot in S^3.

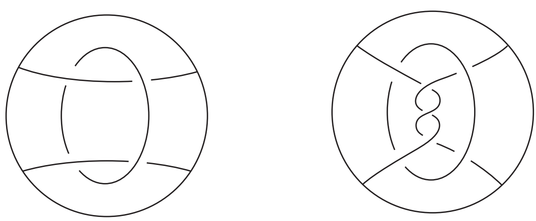
(v) M_A is a Seifert fibered space over a disk with three exceptional fibers and M_B is the exterior of a nonhyperbolic 2-bridge knot in S^3.

Here, the boundaries of M_A and M_B are glued so that the fiber of M_A is identified with the meridian of M_B.

First assume that (i) is satisfied. Since

$$\alpha(g^n) = g^{-n},$$

we see that $\tau_L|_{T_1}$ is hyperelliptic. Note that $\tau_L|_{T_1}$ extends to M_B in a unique way and the quotient of M_B by $\tau_L|_{M_B}$ gives a tangle as in the figure below, right (see

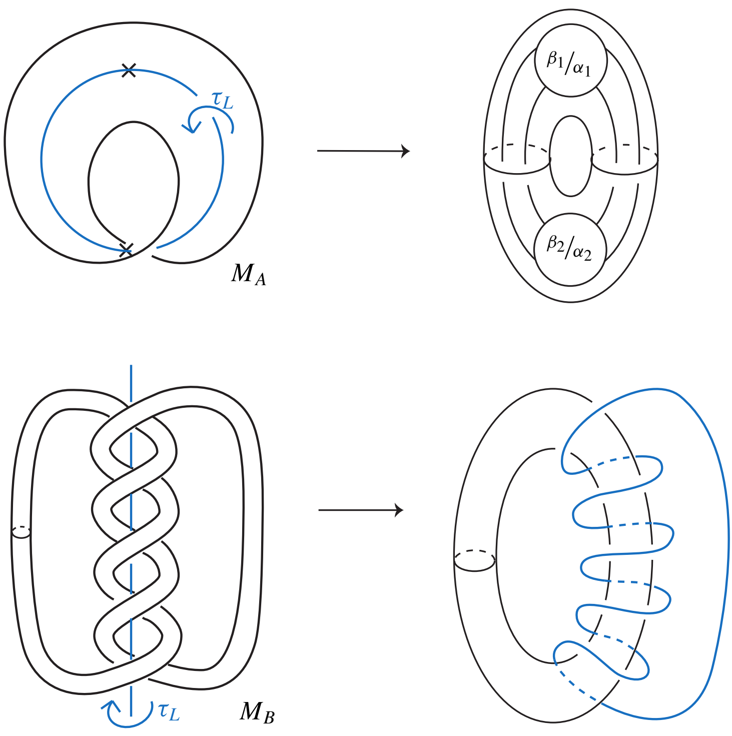


[Jang 2011, Lemma 9]). Since we assume that L is not an arborescent link, we see that τ_L exchanges the two exceptional fibers of M_A. This implies that the two exceptional fibers of M_A have the same index. Then the quotient of M_A by $\tau_L|_{M_A}$ is obtained from the tangle in the figure above, left, by applying Dehn surgery along the loop component in the tangle, where the surgery slope is the reciprocal of the index of the exceptional fibers of M_A. Hence the quotient of M by τ_L is a nontrivial lens space, a contradiction.

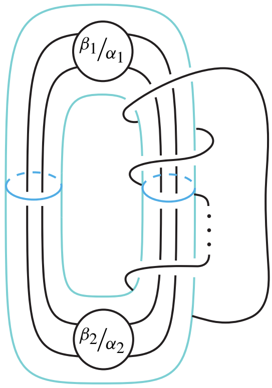
Assume that (ii) is satisfied. Note that M_B is a Seifert fibered space over a disk with two exceptional fibers of indices $1/2$ and $-n/(2n+1)$. Thus the involution on M_B which is hyperelliptic on the boundary is unique (see [Jang 2011, Lemma 4(1)] for example). By an argument similar to that for the previous case, we can lead to a contradiction.

Assume that (iii) is satisfied. Then we see that either $\tau_L(T_i) = T_i$ and $\tau_L|_{T_i}$ is hyperelliptic ($i = 1, 2$) or $\tau_L(T_1) = T_2$. In the former case, we can use arguments similar to those in the previous cases to lead to a contradiction. In the latter case, M_A and $M_B^{(2)}$ are homeomorphic and τ_L interchanges the two pieces. Denote by N the quotient of $M_B^{(1)}$ by $\tau_L|_{M_B^{(1)}}$, which is a solid torus, and denote by F the image of the fixed point set. Then the exterior of F in N is homeomorphic to the exterior of a torus link of type $(2, 2m)$. The quotient of M by τ_L, which is supposed to be S^3, is obtained by gluing M_A and a solid torus, which implies that M_A is homeomorphic to the exterior of a torus knot (see [Burde and Murasugi 1970]). Thus L is a nontrivial cable knot of a torus knot. By Corollary 1.6, we have $w(L) \geq 4$, a contradiction.

Assume that (iv) is satisfied. By arguments similar to those for the previous cases, we can see that $\tau_L|_{M_A}$ and $\tau_L|_{M_B}$ are equivalent to the involutions illustrated here:



Hence, the quotient of M_A gives a 2-bridge link in a solid torus and the quotient of M_B gives a component of a torus link of type $(2, 2m)$ with the regular neighborhood of the other component removed. Then we obtain this 3-bridge link (see [Jang 2012]):



Assume that (v) is satisfied. We can lead to a contradiction by arguments similar to those for the previous cases.

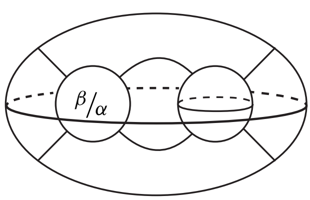
5.2. ***The JSJ-decomposition has a nonseparating torus.*** Since the genus of M is 2, M consists of one or two Seifert pieces.

We first deal with the case when M consists of one Seifert piece. By an argument of [Boileau and Weidmann 2005], we have the following two cases.

(i) The torus T cuts M into the exterior of a 2-component nonhyperbolic 2-bridge link, and g and hgh^{-1} are the meridians.

(ii) The torus T cuts M into a Seifert fibered space over an annulus with two exceptional fibers, whose boundary components are glued so that the fibers are identified.

When (ii) holds, M is a Seifert fibered space, a contradiction. Hence assume that (i) holds. Note that the closure of $M \setminus T$ is a Seifert fibered space, say M', over an annulus with one exceptional fiber. Since we assume that M is not a Seifert fibered space, the fibers on the two boundary components of M' do not match. Since g is a meridian of the 2-bridge link, we can see that $\tau_L|_T$ is hyperelliptic. Then the quotient of M' by $\tau_L|_{M'}$ gives a (3, 1)-manifold pair in the following diagram:



The quotient of M by τ_L is obtained from $S^3 \setminus (B_1 \cup B_2)$, where B_1 and B_2 are open 3-balls, by gluing the two 2-spheres ∂B_1 and ∂B_2, and hence the quotient of M cannot be homeomorphic to S^3, a contradiction.

Next we deal with the case when M consists of two Seifert pieces M_A and M_B. By [Kobayashi 1984], M_A is a Seifert fibered space over an annulus with one or two exceptional fibers and M_B is the exterior of a 2-component nonhyperbolic 2-bridge link. By arguments similar to those for previous cases (compare [Jang 2012]), we can see that L is equivalent to a link having the form shown at the top of the previous page.

5.3. ***There exists a piece homeomorphic to Q.*** By [Kobayashi 1984], we have the following cases.

(i) M consists of two JSJ pieces homeomorphic to Q.

(ii) M consists of two JSJ pieces, one of which is homeomorphic to Q, and the other is either a Seifert fibered space over a disk with two exceptional fibers or a Seifert fibered space over a Möbius band with one exceptional fiber.

(iii) M consists of three JSJ pieces, one of which is homeomorphic to Q, the second piece is the exterior of a 2-component nonhyperbolic 2-bridge link and the third piece is a Seifert fibered space over a disk with two exceptional fibers.

Assume that (i) is satisfied. By [Boileau and Weidmann 2005, Lemma 17], the regular fibers of the two pieces, considered as a Seifert fibered space over a disk with two exceptional fibers, intersect in one point, and g^2 is a fiber of one piece. Then we see that $\tau_L|_T$ is hyperelliptic, and we can lead to a contradiction by using arguments similar to those in the previous cases.

Assume that (ii) is satisfied. By an argument in [Boileau and Weidmann 2005, Proof of Lemma 18], we can see that $\tau_L|_T$ is hyperelliptic, and we can lead to a contradiction by using arguments similar to those in the previous cases.

Assume that (iii) is satisfied. Similarly, we can see that either $\tau_L(T_i) = T_i$ and $\tau_L|_{T_i}$ is hyperelliptic ($i = 1, 2$) or $\tau_L(T_1) = T_2$. In the former case, we can lead to a contradiction by using arguments similar to those in the previous cases. In the latter case, we can see that the quotient of M by τ_L is the union of Q and a solid torus, which cannot be homeomorphic to S^3, a contradiction.

This completes the proof of Theorem 1.2.

Proof of Corollary 1.4. Let M be a closed orientable graph manifold which admits an inversion, i.e., $\pi_1(M)$ is generated by two elements g and h and there exists an automorphism α of $\pi_1(M)$ which sends g and h to g^{-1} and h^{-1}, respectively. If M is a Seifert fibered space, then α is hyperelliptic by [Boileau and Weidmann 2008, Theorem 5]. If M is not a Seifert fibered space, then α is hyperelliptic by Theorem 1.2 and [Boileau and Weidmann 2008, Proposition 20(3)]. $\square$

6. Degree-one maps

Proof of Proposition 1.10. (a) Let $L' \subset S^3$ such that $b(L') = 3$, then $w(L') = 3$ by [Boileau and Zimmermann 1989]. Thus if $L \geq L'$, then $b(L) \geq w(L) \geq w(L') = 3$.

(b) Let $L' \subset S^3$ such that $b(L') = 4$. Assume that $L \geq L'$ and that the 2-fold branched cover M of L is a graph manifold. The degree-one map $f : E(L) \to E(L')$ between the exteriors of L and L' which preserves the meridians lifts to a degree-one map $\tilde{f} : \tilde{E}(L) \to \tilde{E}(L')$ between their 2-fold covers, which extends to a degree-one map $\tilde{f} : M \to M'$ between their 2-fold branched covers $M := M_2(L)$ and $M' = M_2(L')$. Since M is a graph manifold, its simplicial volume $\|M\| = 0$. The existence of the degree-one map $\tilde{f} : M \to M'$ implies that $\|M'\| \leq \|M\|$ and thus $\|M'\| = 0$. By the orbifold theorem [Boileau and Porti 2001] M' admits a geometric decomposition and thus is a connected sum of graph manifolds. Therefore L' is a connected sum of links whose 2-fold branched covers are graph manifolds.

If L' is prime, it follows from Corollary 1.3 that $w(L') = 4$ and therefore $b(L) \geq w(L) \geq w(L') = 4$.

If L' is not prime, then $L' = L'_1 \sharp L'_2$ with $b(L'_1) = 2 = w(L'_1)$ and $b(L'_2) = 3 = w(L'_2)$ by [Boileau and Zimmermann 1989]. The exterior $E(L')$ can be split along a properly embedded essential annulus A into two pieces homeomorphic to $E(L'_1)$ and $E(L'_2)$ so that $\pi_1(E(L') = \pi_1(E(L'_1)) *_{\pi_1(A)} \pi_1(E(L'_2)$, where $\pi_1(A) \cong \mathbb{Z}$ is generated by a meridian of L'_1 and L'_2. By killing the meridians of L'_2 which are not conjugate to the generator of $\pi_1(A)$, one can define an epimorphism $\phi_1 : \pi_1(E(L')) \to \pi_1(E(L'_1))$ such that the restriction of ϕ_1 to $\pi_1(E(L'_1))$ is the identity and $\phi_1(\pi_1(E(L'_2))) = \pi_1(A)$. In the same way one can define an epimorphism $\phi_2 : \pi_1(E(L')) \to \pi_1(E(L'_2))$ such that the restriction of ϕ_2 to $\pi_1(E(L'_2))$ is the identity and $\phi_2(\pi_1(E(L'_1))) = \pi_1(A)$. These epimorphisms imply that $w(L') = w(L'_1) + w(L'_2) - 1 = 4$, and thus $b(L) \geq w(L) \geq w(L') = 4$. □

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