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MONOTONICITY AND RADIAL SYMMETRY RESULTS FOR SCHRÖDINGER SYSTEMS WITH FRACTIONAL DIFFUSION

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We consider a nonlinear Schrödinger system with fractional diffusion

$$\begin{cases} (-\Delta)^{\alpha/2}u(x)+A(x)u(x)=v^p(x) & \text{in } \Omega,\\ (-\Delta)^{\beta/2}v(x)+B(x)v(x)=u^q(x) & \text{in } \Omega,\\ u(x)\geq 0,\, v(x)\geq 0 & \text{in } \Omega,\\ u(x)=v(x)=0 & \text{on } \Omega^C, \end{cases}$$

where Ω is an unbounded parabolic domain. We first establish a narrow region principle. Using this principle and a direct method of moving planes, we obtain the monotonicity of nonnegative solutions and the Liouville-type result for the nonlinear Schrödinger system with fractional diffusion. We also obtain the radially symmetric result of positive solutions for the system in the unit ball when $A(x)$ and $B(x)$ are constants.

1. Introduction

We are interested in the following nonlinear Schrödinger system with fractional diffusion:

$$\begin{cases} (-\Delta)^{\alpha/2}u(x)+A(x)u(x)=v^p(x) & \text{in } \Omega,\\ (-\Delta)^{\beta/2}v(x)+B(x)v(x)=u^q(x) & \text{in } \Omega,\\ u(x)\geq 0,\, v(x)\geq 0 & \text{in } \Omega,\\ u(x)=v(x)=0 & \text{on } \Omega^C, \end{cases} \tag{1-1}$$

where $\alpha, \beta \in (0,2)$, $p, q > 1$, $A(x)$ and $B(x)$ are bounded from below and Ω is an unbounded parabolic domain in $\mathbb{R}^n$ defined by

$$\Omega = \{x = (x', x_n) \in \mathbb{R}^n \mid x_n > |x'|^2,\, x' = (x_1, x_2, \dots, x_{n-1})\}.$$

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Here, $(-\Delta)^{\alpha/2}$ and $(-\Delta)^{\beta/2}$ are nonlocal pseudodifferential operators defined by

$$(-\Delta)^{\alpha/2}u(x) = C_{n,\alpha} P.V. \int_{\mathbb{R}^n} \frac{u(x)-u(y)}{|x-y|^{n+\alpha}}\, dy, \tag{1-2}$$

$$(-\Delta)^{\beta/2}v(x) = C_{n,\beta} P.V. \int_{\mathbb{R}^n} \frac{v(x)-v(y)}{|x-y|^{n+\beta}}\, dy, \tag{1-3}$$

where $P.V.$ stands for the Cauchy principal value, and $C_{n,\alpha}$, $C_{n,\beta}$ are normalization positive constants. Let

$$F = L_\alpha \cap C^{1,1}_{loc}(\Omega), \quad G = L_\beta \cap C^{1,1}_{loc}(\Omega),$$

where

$$L_\alpha = \left\{ u \,\middle|\, u \in L^1_{\text{loc}},\ \int_{\mathbb{R}^n} \frac{|u(x)|}{1+|x|^{n+\alpha}}\, dx < \infty \right\},$$

and

$$L_\beta = \left\{ v \,\middle|\, v \in L^1_{\text{loc}},\ \int_{\mathbb{R}^n} \frac{|v(x)|}{1+|x|^{n+\beta}}\, dx < \infty \right\}.$$

For $u \in F,\ v \in G$, the integral on the left-hand side of the equations in (1-1) is well defined (see [Chen et al. 2017b]).

Linear and nonlinear equations and systems involving the fractional Laplacian have received growing attention in recent years. It can be used to model diverse physical phenomena, such as turbulence and water waves, molecular dynamics, and pseudorelativistic boson stars (see [Bouchaud and Georges 1990], [Caffarelli and Vasseur 2010], [Constantin 2006], [Tarasov and Zaslavsky 2006]). The operator $(-\Delta)^{\alpha/2}$ can also be used in mathematical finance (see [Applebaum 2009], [Bertoin 1996]). But they are still much less understood than nonfractional counterparts.

When $\alpha = \beta = 2,\ A(x) = B(x) = 0$, (1-1) becomes the classical Lane–Emden system:

$$\begin{cases} -\Delta u = v^p, \\ -\Delta v = u^q. \end{cases} \tag{1-4}$$

When $1/(p+1)+1/(q+1) > (n-2)/n$, the system (1-4) has no positive radial solutions in all dimension (see [Mitidieri 1996]). D. G. de Figueiredo and P. L. Felmer [1994] studied a Liouville type theorem for (1-4) by introducing superharmonic functions when $n \geq 3$. The main tool they used is the method of moving planes. For $n = 3$, J. Serrin and H. Zou [1996] proved that the system (1-4) has no positive solutions when $1/(p+1)+1/(q+1) > (n-2)/n$ under assumption that (u, v) has at most polynomial growth at infinity. After Serrin's work, there are some interesting works about Lane–Emden systems and related Schrödinger systems on whole space and half space; see [Montaru and Souplet 2014; Poláčik et al. 2007; Souplet 2009].

For classical semilinear elliptic system, the symmetry and monotonicity of positive solutions have been widely studied (see [Busca and Sirakov 2000; Chen and Li 2010; Liu and Ma 2012; 2013; Ma and Liu 2010]). A powerful tool to obtain these properties of such equations and systems is the method of moving planes, which was introduced by Alexandrov [1962]. Serrin [1971] and Gidas, Ni, Nirenberg [Gidas et al. 1979; 1981] adapted this method in partial differential equations and made great contributions to improving this method.

As we know, the fractional Laplacian is nonlocal; that is, it is not differentiable pointwise, but is globally integrable with respect to a singular kernel. The nonlocality causes the main difficulty in studying corresponding problems. To circumvent this difficulty, Caffarelli and Silvestre [2007] introduced the extension method that reduced this nonlocal problem in $\mathbb{R}^n$ into a local one in $\mathbb{R}^{n+1}_+$ through constructing a Dirichlet to Neumann operator of a degenerate elliptic equation. This extension method has been applied successfully to study equations involving the fractional Laplacian, and a series of fruitful results have been obtained; see [Brändle et al. 2013; Chen and Zhu 2016].

Due to technical restrictions, they have to assume $\alpha \geq 1$. It seems that this condition cannot be weakened if one wants to carry out the method of moving planes on the extended equation. Actually, the case $0 < \alpha < 1$ can be treated by considering the corresponding integral equation. Using the method of moving planes (or spheres) in integral forms [Chen and Li 2009; Chen et al. 2005a; 2005b; 2006; 2015; Fall and Weth 2016; Fang and Zhang 2013; Li and Ma 2008; Ma and Chen 2008; Ma and Zhao 2008, one can obtain the radial symmetry properties of the fractional Laplacian equation. For the fractional Laplacian system, here we mention the work by Zhuo, Chen, Cui and Yuan [Zhuo et al. 2016]. They considered the system

$$(-\Delta)^{\alpha/2} u_i(x) = f_i(u_1(x), u_2(x), \dots, u_m(x)), \quad x \in \mathbb{R}^n, \quad i = 1, 2, \dots, m. \tag{1-5}$$

By establishing the equivalence between (1-5) and its corresponding integral system, the authors obtained the symmetry result and the nonexistence of positive solutions.

Either by extension or by integral equations, one needs to impose extra conditions on the solutions. Can one carry out the method of moving planes directly on fractional equation? The answer was provided in [Jarohs and Weth 2016] by Jarohs and Weth. They introduced antisymmetric maximum principles and applied them to carry out the method of moving planes directly on nonlocal problems to show the symmetry of solutions. However, their maximum principles only apply to bounded regions.

Recently, Chen, Li and Li [Chen et al. 2017b] developed a direct method of moving planes to study the fractional Laplacian, which worked directly on the nonlocal operator. The key ingredients of this method are the antisymmetric properties. They used this property to develop some techniques needed in the direct method of moving planes in the whole space $\mathbb{R}^n$ and the upper half space $\mathbb{R}^n_+$, such as the narrow region principle, decay at infinity. The direct method of moving planes is very useful. This method has been applied to fully nonlinear fractional order-equations and systems in [Chen et al. 2017a]. In [Cheng et al. 2017], the authors considered the symmetry and monotonicity properties for positive solutions of fractional Laplacian equations by the direct method. Using the spirit of direct method of moving planes in [Chen et al. 2017b], Cai and Mei [2017] studied the fractional Lane–Emden system in $\mathbb{R}^n$ and obtain the symmetry properties and Liouville-type result of positive solutions. Liu and Ma [2016] studied symmetry properties of the general fractional Laplacian system:

$$\begin{cases} (-\Delta)^{\alpha/2}u(x)) = f(u,v) & \text{in } \mathbb{R}^n, \\ (-\Delta)^{\alpha/2}v(x) = g(u,v) & \text{in } \mathbb{R}^n, \\ u(x) \geq 0,\, v(x) \geq 0 & \text{in } \mathbb{R}^n \end{cases} \tag{1-6}$$

under a strong decay condition on the solutions at infinity.

The goal of this paper is to generalize the direct method of moving planes to the Schrödinger system. We first establish the narrow region principle for Schrödinger systems with fractional diffusion. We write $x = (x', x_n) \in \mathbb{R}^n$ with $x' = (x_1, x_2, \ldots, x_{n-1})$. Assume $A(x)$ and $B(x)$ are independent of x_n, that is,

$$A(x) = A(x'), \quad B(x) = B(x').$$

Let

$$T_\lambda = \{x \in \mathbb{R}^n \mid x_n = \lambda, \lambda \in \mathbb{R}, \lambda > 0\}$$

be the moving plane and denote

$$H_\lambda = \{x \in \mathbb{R}^n \mid x_n < \lambda\}, \quad \Sigma_\lambda = \{x \in \Omega \mid 0 < x_n < \lambda\}.$$

For each point $x = (x', x_n) \in \Sigma_\lambda$, let $x^\lambda = (x', 2\lambda - x_n)$ be the reflection point about the plane T_λ. Denote

$$U_\lambda(x) = u(x^\lambda) - u(x) = u_\lambda(x) - u(x), \quad V_\lambda(x) = v(x^\lambda) - v(x) = v_\lambda(x) - v(x).$$

It follows that for $x \in \Sigma_\lambda$,

$$\begin{aligned} (-\Delta)^{\alpha/2}U_\lambda(x) &= (-\Delta)^{\alpha/2}u_\lambda(x) - (-\Delta)^{\alpha/2}u(x) \\ &= p\xi^{p-1}(x)V_\lambda(x) - A(x')U_\lambda(x), \end{aligned} \tag{1-7}$$

and

$$\begin{aligned} (-\Delta)^{\beta/2}V_\lambda(x) &= (-\Delta)^{\beta/2}v_\lambda(x) - (-\Delta)^{\beta/2}v(x) \\ &= q\eta^{q-1}(x)U_\lambda(x) - B(x')V_\lambda(x), \end{aligned} \tag{1-8}$$

where $\xi(x)$ is between $v_\lambda(x)$ and $v(x)$ and $\eta(x)$ is between $u_\lambda(x)$ and $u(x)$. It is obvious that $U_\lambda(x)$ and $V_\lambda(x)$ satisfy the antisymmetry property:

$$U_\lambda(x^\lambda) = -U_\lambda(x), \quad V_\lambda(x^\lambda) = -V_\lambda(x), \qquad x \in H_\lambda. \tag{1-9}$$

Lemma 1.1 (narrow region principle). *Let $(u, v) \in F \times G$ be a nonnegative solution of system (1-1) with $0 < \alpha, \beta < 2$. Assume that $1 < p, q < \infty$, $A(x) = A(x')$ and $B(x) = B(x')$ are bounded from below in Ω, where $x = (x', x_n) \in \Omega$, $x' = (x_1, x_2, \dots, x_{n-1})$. Then for all systems (1-7) and (1-8) and for sufficiently small δ,*

(i) *if there exists $x_1^* \in \Sigma_{\lambda,\delta} = \{x \in \Sigma_\lambda \mid \lambda - \delta < x_n < \lambda\}$ satisfying $U_\lambda(x_1^*) = \min_{x\in\bar{\Sigma}_\lambda} U_\lambda(x) < 0$, then*

$$V_\lambda(x_1^*) < U_\lambda(x_1^*) < 0;$$

(ii) *if there exists $x_2^* \in \Sigma_{\lambda,\delta} = \{x \in \Sigma_\lambda \mid \lambda - \delta < x_n < \lambda\}$ satisfying $V_\lambda(x_2^*) = \min_{x\in\bar{\Sigma}_\lambda} V_\lambda(x) < 0$, then*

$$U_\lambda(x_2^*) < V_\lambda(x_2^*) < 0.$$

Based on Lemma 1.1, we can obtain the following result.

Theorem 1.2. *Let $(u, v) \in F \times G$ be a nonnegative solution of system (1-1) with $0 < \alpha, \beta < 2$. If $1 < p, q < \infty$, $A(x) = A(x')$ and $B(x) = B(x')$ are bounded from below in Ω, where $x = (x', x_n) \in \Omega$, $x' = (x_1, x_2, \dots, x_{n-1})$. Then (u, v) is monotonically increasing in x_n.*

When $\alpha = \beta = 2$, this result is contained in the series papers of Berestycki, Caffarelli and Nirenberg [Berestycki and Nirenberg 1992; Berestycki et al. 1993; 1996; 1997] and they have used the classical method of moving planes. Hence our result by using the direct method of moving planes due to [Chen et al. 2017b] and [Jarohs and Weth 2016] can be considered as an extension of theirs to the nonlocal system.

As an immediate application, we obtain the following Liouville type result.

Corollary 1.3. *Let $(u, v) \in F \times G$ be a nonnegative solution of system (1-1) with $0 < \alpha, \beta < 2$. Assume that $1 < p, q < \infty$, $A(x) = A(x')$ and $B(x) = B(x')$ are bounded from below in Ω, where $x = (x', x_n) \in \Omega$, $x' = (x_1, x_2, \dots, x_{n-1})$. If*

$$\lim_{x\to\infty} u(x) = 0, \qquad \lim_{x\to\infty} v(x) = 0, \tag{1-10}$$

Then $u \equiv 0$, $v \equiv 0$.

We consider the system (1-1) when $A(x) = A$ and $B(x) = B$, where A, B are two constants in the unit ball $\mathbb{B}_1(0)$ and obtain the radial symmetry and monotonicity of positive solutions.

Theorem 1.4. *Assume $(u, v) \in L_\alpha \cap C^{1,1}_{loc}(\mathbb{B}_1(0)) \times L_\beta \cap C^{1,1}_{loc}(\mathbb{B}_1(0))$ is a positive solution of the following system*

$$\begin{cases} (-\Delta)^{\alpha/2} u(x) + Au(x) = v^p(x) & \text{in } \mathbb{B}_1(0), \\ (-\Delta)^{\beta/2} v(x) + Bv(x) = u^q(x) & \text{in } \mathbb{B}_1(0), \\ u(x) \ge 0, v(x) \ge 0 & \text{in } \mathbb{B}_1(0), \\ u(x) = v(x) = 0 & \text{on } \mathbb{B}_1^C(0), \end{cases} \tag{1-11}$$

with $0 < \alpha, \beta < 2$ *and* $1 < p, q < \infty$. *Then each positive solution* $(u(x), v(x))$ *must be radially symmetric and monotone decreasing about the origin.*

The paper is organized as follows. Section 2 is devoted to proving Lemma 1.1, the narrow region principle for (1-1). In Section 3, we study the monotonicity of positive solutions of (1-1) in Ω and prove Theorem 1.2. Finally, the proof of Theorem 1.4 will be presented in Section 4. Note that in the following, C will be a positive constant which can be different from line to line.

2. Preliminaries

In this section, we will prove Lemma 1.1, which plays an important role in the proof of Theorem 1.2 and Theorem 1.4.

Proof. (i) Without loss of generality, let

$$x_1^* \in \Sigma_{\lambda,\delta} \quad \text{and} \quad U_\lambda(x_1^*) = \min_{x \in \overline{\Sigma}_\lambda} U_\lambda(x) < 0.$$

It follows that

$$\begin{aligned}(-\Delta)^{\alpha/2} U_\lambda(x_1^*) &= C_{n,\alpha} P.V. \int_{\mathbb{R}^n} \frac{U_\lambda(x_1^*) - U_\lambda(y)}{|x_1^* - y|^{n+\alpha}} dy \\ &= C_{n,\alpha} P.V. \Big(\int_{H_\lambda} \frac{U_\lambda(x_1^*) - U_\lambda(y)}{|x_1^* - y|^{n+\alpha}} dy + \int_{\mathbb{R}^n \setminus H_\lambda} \frac{U_\lambda(x_1^*) - U_\lambda(y)}{|x_1^* - y|^{n+\alpha}} dy \Big) \\ &= C_{n,\alpha} P.V. \Big(\int_{H_\lambda} \frac{U_\lambda(x_1^*) - U_\lambda(y)}{|x_1^* - y|^{n+\alpha}} dy + \int_{H_\lambda} \frac{U_\lambda(x_1^*) - U_\lambda(y^\lambda)}{|x_1^* - y^\lambda|^{n+\alpha}} dy \Big).\end{aligned}$$

Note that $|x_1^* - y| \le |x_1^* - y^\lambda|$ when $x_1^*, y \in \Sigma_\lambda$, and using the antisymmetry of $U_\lambda(x)$, we have

$$\begin{aligned}(-\Delta)^{\alpha/2} U_\lambda(x_1^*) &\le C_{n,\alpha} P.V. \Big(\int_{H_\lambda} \frac{U_\lambda(x_1^*) - U_\lambda(y)}{|x_1^* - y^\lambda|^{n+\alpha}} dy + \int_{H_\lambda} \frac{U_\lambda(x_1^*) + U_\lambda(y)}{|x_1^* - y^\lambda|^{n+\alpha}} dy \Big) \\ &= C_{n,\alpha} P.V. \int_{H_\lambda} \frac{2U_\lambda(x_1^*)}{|x_1^* - y^\lambda|^{n+\alpha}} dy.\end{aligned}$$

Let $D = B_{2\delta}(x_1^*) \cap H_\lambda$. Then we obtain

$$\int_{H_\lambda} \frac{1}{|x_1^* - y^\lambda|^{n+\alpha}} dy \ge \int_D \frac{1}{|x_1^* - y^\lambda|^{n+\alpha}} dy \ge C \int_{B_{2\delta}(x_1^*)} \frac{1}{|x_1^* - y^\lambda|^{n+\alpha}} dy \ge \frac{C}{\delta^\alpha}. \tag{2-1}$$

Thus,

$$(-\Delta)^{\alpha/2} U_\lambda(x_1^*) \le \frac{C U_\lambda(x_1^*)}{\delta^\alpha} < 0.$$

According to (1-7), we get

$$p\xi^{p-1}(x_1^*) V_\lambda(x_1^*) - A(x_1^*) U_\lambda(x_1^*) = (-\Delta)^{\alpha/2} U_\lambda(x_1^*) \le \frac{C}{\delta^\alpha} U_\lambda(x_1^*) < 0.$$

Note that $A(x) = A(x')$ is bounded from below and $\xi(x)$ is between $v_\lambda(x)$ and $v(x)$, $v(x) \in G = L_\beta \cap C^{1,1}_{loc}(\Omega)$, hence, for δ sufficiently small, we have

$$V_\lambda(x_1^*) < \frac{A(x_1^*) + \frac{C}{\delta^\alpha}}{pv^{p-1}(x_1^*)} U_\lambda(x_1^*) < U_\lambda(x_1^*) < 0. \tag{2-2}$$

(ii) If there exists $x_2^* \in \Sigma_{\lambda,\delta} = \{x \in \Sigma_\lambda \mid \lambda - \delta < x_n < \lambda\}$ such that

$$V_\lambda(x_2^*) = \min_{x \in \overline{\Sigma}_\lambda} V_\lambda(x) < 0,$$

similarly to the proof of (i), we can obtain

$$(-\Delta)^{\beta/2} V_\lambda(x_2^*) \le \frac{C V_\lambda(x_2^*)}{\delta^\beta} < 0. \tag{2-3}$$

Note that $B(x)$ is bounded from below and $u(x) \in F = L_\alpha \cap C^{1,1}_{loc}(\Omega)$, hence, for δ sufficiently small, according to (1-8), we have

$$U_\lambda(x_2^*) < \frac{B(x_2^*) + \frac{C}{\delta^\beta}}{qu^{q-1}(x_2^*)} V_\lambda(x_2^*) < V_\lambda(x_2^*) < 0. \tag{2-4}$$

Thus we have completed the proof of Lemma 1.1. □

3. The proof of Theorem 1.2

In this section, we will carry out the direct method of moving planes on the solution $(u(x), v(x))$ along x_n direction to prove Theorem 1.2.

Proof. The proof of Theorem 1.2 is divided into two steps.

Step 1: We show that for $\lambda > 0$ sufficiently close to zero,

$$U_\lambda(x) \ge 0, \quad V_\lambda(x) \ge 0, \quad x \in \Sigma_\lambda. \tag{3-1}$$

If (3-1) does not hold, then there exists a point $x_1^* \in \Sigma_\lambda$ such that $U_\lambda(x_1^*) < 0$. Without loss of generality, we assume

$$U_\lambda(x_1^*) = \min_{x \in \overline{\Sigma}_\lambda} U_\lambda(x) < 0.$$

By Lemma 1.1, for $\lambda > 0$ sufficiently close to zero,

$$V_\lambda(x_1^*) < \frac{A(x_1^*) + \frac{C}{\lambda^\alpha}}{pv^{p-1}(x_1^*)} U_\lambda(x_1^*) < 0. \tag{3-2}$$

Note that $V_\lambda(x) = 0$ for $x \in T_\lambda$ and $V_\lambda(x) \ge 0$ for $x \in \partial\Sigma_\lambda$, hence, there exists $x_2^* \in \Sigma_\lambda$ such that

$$V_\lambda(x_2^*) = \min_{x \in \overline{\Sigma}_\lambda} V_\lambda(x) < 0.$$

Similarly to the proof of Lemma 1.1, for $\lambda > 0$ sufficiently close to zero, we have

$$U_\lambda(x_2^*) < \frac{B(x_2^*) + \frac{C}{\lambda^\beta}}{qu^{q-1}(x_2^*)} V_\lambda(x_2^*). \tag{3-3}$$

Therefore, together with (3-2) and (3-3), we get

$$\begin{aligned} V_\lambda(x_1^*) < \frac{A(x_1^*) + \frac{C}{\lambda^\alpha}}{pv^{p-1}(x_1^*)} U_\lambda(x_1^*) &\le \frac{A(x_1^*) + \frac{C}{\lambda^\alpha}}{pv^{p-1}(x_1^*)} U_\lambda(x_2^*) \\ &< \frac{\left(A(x_1^*) + \frac{C}{\lambda^\alpha}\right)}{pv^{p-1}(x_1^*)} \frac{\left(B(x_2^*) + \frac{C}{\lambda^\beta}\right)}{qu^{q-1}(x_2^*)} V_\lambda(x_2^*) \\ &\le \frac{\left(A(x_1^*) + \frac{C}{\lambda^\alpha}\right)}{pv^{p-1}(x_1^*)} \frac{\left(B(x_2^*) + \frac{C}{\lambda^\beta}\right)}{qu^{q-1}(x_2^*)} V_\lambda(x_1^*). \end{aligned} \tag{3-4}$$

Because $A(x) = A(x')$ and $B(x) = B(x')$ are bounded from below and $V_\lambda(x_1^*) < 0$, therefore, (3-4) is a contradiction for $\lambda > 0$ sufficiently close to zero and the proof of Step 1 is completed.

Step 1 provides a starting point. We start from such a small λ and move the plane T_λ up continuously in the direction of x_n-axis to its limiting position as long as (3-1) holds. Define

$$\lambda_0 = \sup\{\lambda > 0 \mid U_\mu(x) \ge 0,\ V_\mu(x) \ge 0,\ x \in \Sigma_\mu;\ \mu \le \lambda\}. \tag{3-5}$$

Step 2: We prove

$$\lambda_0 = +\infty. \tag{3-6}$$

Before proceeding further, we investigate some properties of $U_{\lambda_0}(x)$ and $V_{\lambda_0}(x)$ for $x \in \Sigma_{\lambda_0}$.

Proposition 3.1. *If $U_{\lambda_0}(x) \equiv 0$, then $V_{\lambda_0}(x) \equiv 0$. If $V_{\lambda_0}(x) \equiv 0$, then $U_{\lambda_0}(x) \equiv 0$.*

Proof. If $U_{\lambda_0}(x) \equiv 0$, then (1-7) becomes

$$0 = (-\Delta)^{\alpha/2} U_{\lambda_0}(x) = p\xi^{p-1}(x) V_{\lambda_0}(x),$$

Obviously, we get $V_{\lambda_0}(x) \equiv 0$. □

Proposition 3.2. *If $U_{\lambda_0}(x) \not\equiv 0$ or $V_{\lambda_0}(x) \not\equiv 0$, then $U_{\lambda_0}(x) > 0$ and $V_{\lambda_0}(x) > 0$ for all $x \in \Sigma_{\lambda_0}$.*

Proof. Since we know that $U_{\lambda_0}(x) \geq 0,\ x \in \Sigma_{\lambda_0}$. If $U_{\lambda_0}(x) > 0$ does not hold, we assume that there exists some point $\tilde{x} \in \Sigma_{\lambda_0}$ such that $U_{\lambda_0}(\tilde{x}) = 0$.

$$\begin{aligned}(-\Delta)^{\alpha/2}U_{\lambda_0}(\tilde{x}) &= C_{n,\alpha}P.V.\int_{\mathbb{R}^n}\frac{U_{\lambda_0}(\tilde{x})-U_{\lambda_0}(y)}{|\tilde{x}-y|^{n+\alpha}}dy\\ &= C_{n,\alpha}P.V.\biggl(\int_{H_{\lambda_0}}\frac{-U_{\lambda_0}(y)}{|\tilde{x}-y|^{n+\alpha}}dy+\int_{\mathbb{R}^n\setminus H_{\lambda_0}}\frac{-U_{\lambda_0}(y)}{|\tilde{x}-y|^{n+\alpha}}dy\biggr)\\ &= C_{n,\alpha}P.V.\biggl(\int_{H_{\lambda_0}}\frac{-U_{\lambda_0}(y)}{|\tilde{x}-y|^{n+\alpha}}dy+\int_{H_{\lambda_0}}\frac{-U_{\lambda_0}(y^{\lambda})}{|\tilde{x}-y^{\lambda}|^{n+\alpha}}dy\biggr)\\ &= C_{n,\alpha}P.V.\int_{H_{\lambda_0}}\biggl(\frac{1}{|\tilde{x}-y^{\lambda}|^{n+\alpha}}-\frac{1}{|\tilde{x}-y|^{n+\alpha}}\biggr)U_{\lambda_0}(y)dy. \qquad (3\text{-}7)\end{aligned}$$

Note that $|\tilde{x}-y^{\lambda}| > |\tilde{x}-y|$, hence,

$$(-\Delta)^{\alpha/2}U_{\lambda_0}(\tilde{x}) < 0. \tag{3-8}$$

On the other hand, according to (1-7),

$$(-\Delta)^{\alpha/2}U_{\lambda_0}(\tilde{x}) = p\xi^{p-1}(\tilde{x})V_{\lambda_0}(\tilde{x}) - A(\tilde{x})U_{\lambda_0}(\tilde{x}) = p\xi^{p-1}(\tilde{x})V_{\lambda_0}(\tilde{x}) \geq 0. \tag{3-9}$$

Evidently, this is contradictory to (3-8). Consequently, we obtain $U_{\lambda_0}(x) > 0$ for all $x \in \Sigma_{\lambda_0}$. Then by Proposition 3.1, we get $V_{\lambda_0}(x) \not\equiv 0$. Similarly to above, we obtain $V_{\lambda_0}(x) > 0$ for all $x \in \Sigma_{\lambda_0}$, completing the proof. □

Now, we start to prove (3-6). If $\lambda_0 < +\infty$, we will show

$$U_{\lambda_0}(x) \equiv 0,\ x \in \Sigma_{\lambda_0}. \tag{3-10}$$

Then by Proposition 3.1, $V_{\lambda_0}(x) \equiv 0,\ x \in \Sigma_{\lambda_0}$. Thus, we obtain

$$u(x', 2\lambda_0) = u(x', 0), \tag{3-11}$$

$$v(x', 2\lambda_0) = v(x', 0). \tag{3-12}$$

But the left-hand side of (3-11) is positive, and the right-hand side of (3-11) is equal to zero. This is contradictory. The same holds for $v(x)$. Hence, (3-6) holds.

In the following, we will prove (3-10). If $U_{\lambda_0}(x) \not\equiv 0,\ x \in \Sigma_{\lambda_0}$, by Proposition 3.2, we have $U_{\lambda_0}(x) > 0,\ V_{\lambda_0}(x) > 0,\ x \in \Sigma_{\lambda_0}$. Hence, for small $\delta > 0$, there exists a positive constant c_0 such that

$$U_{\lambda_0}(x) \geq c_0 > 0, \quad V_{\lambda_0}(x) \geq c_0 > 0, \quad x \in \overline{\Sigma}_{\lambda_0-\delta}.$$

Since $U_{\lambda}(x)$ and $U_{\lambda}(x)$ depends on λ continuously, there exists $\varepsilon > 0$ and $\varepsilon < \delta$ such that for all $\lambda \in (\lambda_0, \lambda_0 + \varepsilon)$,

$$U_{\lambda}(x) \geq 0, \quad V_{\lambda}(x) \geq 0, \quad x \in \overline{\Sigma}_{\lambda_0-\delta}. \tag{3-13}$$

When $x \in \Sigma_\lambda \setminus \overline{\Sigma}_{\lambda_0-\delta}$, we also have $U_\lambda(x) \ge 0,\ V_\lambda(x) \ge 0$. If not, without loss of generality, we assume that there exists a point $\bar{x}_1 \in \Sigma_\lambda \setminus \overline{\Sigma}_{\lambda_0-\delta}$ such that

$$U_\lambda(\bar{x}_1) = \min_{x \in \Sigma_\lambda \setminus \overline{\Sigma}_{\lambda_0-\delta}} U_\lambda(x) < 0.$$

According to Lemma 1.1,

$$V_\lambda(\bar{x}_1) < \frac{A(\bar{x}_1) + \frac{C}{(\delta+\varepsilon)^\alpha}}{p v^{p-1}(\bar{x}_1)} U_\lambda(\bar{x}_1) < 0.$$

Hence, there exists $\bar{x}_2 \in \Sigma_\lambda \setminus \overline{\Sigma}_{\lambda_0-\delta}$ such that

$$V_\lambda(\bar{x}_2) = \min_{x \in \Sigma_\lambda \setminus \overline{\Sigma}_{\lambda_0-\delta}} V_\lambda(x) < 0.$$

By Lemma 1.1 again,

$$U_\lambda(\bar{x}_2) < \frac{B(\bar{x}_2) + \frac{C}{(\delta+\varepsilon)^\beta}}{q u^{q-1}(\bar{x}_2)} V_\lambda(\bar{x}_2) < 0.$$

Similarly to (3-4),

$$U_\lambda(\bar{x}_2) < \frac{\left(A(\bar{x}_1) + \frac{C}{(\delta+\varepsilon)^\alpha}\right)}{p v^{p-1}(\bar{x}_1)} \frac{\left(B(\bar{x}_2) + \frac{C}{(\delta+\varepsilon)^\beta}\right)}{q u^{q-1}(\bar{x}_2)} U_\lambda(\bar{x}_2).$$

Since $A(x) = A(x')$ and $B(x) = B(x')$ are bounded from below and $U_\lambda(\bar{x}_2) < 0$, this is contradictory for δ and ε sufficiently small. Therefore, we obtain

$$U_\lambda(x) \ge 0, \quad V_\lambda(x) \ge 0, \quad x \in \Sigma_\lambda \setminus \overline{\Sigma}_{\lambda_0-\delta}. \tag{3-14}$$

Combining (3-13) and (3-14), we get that for $\lambda \in (\lambda_0, \lambda_0 + \varepsilon)$,

$$U_\lambda(x) \ge 0, \quad V_\lambda(x) \ge 0, \quad x \in \Sigma_\lambda. \tag{3-15}$$

(3-15) indicates that the plane T_{λ_0} can be moved up further. We have reached a contradiction with the definition of λ_0. Hence, we must have $U_{\lambda_0}(x) \equiv 0$.

We have shown that $\lambda_0 = +\infty$ and $U_{\lambda_0}(x) \ge 0$, $V_{\lambda_0}(x) \ge 0$. It indicates that $u(x)$ and $v(x)$ are monotonically increasing in x_n, which completes the proof of Theorem 1.2. □

Proof of Corollary 1.3. We have shown that $u(x)$ and $v(x)$ are monotonically increasing in x_n. In terms of $u(0) = v(0) = 0$ and the condition (1-10), we derive

$$u(x) \equiv 0, \quad v(x) \equiv 0 \quad \text{in } \Omega.$$

Thus we have completed the proof. □

4. The proof of Theorem 1.4

In this section, we will apply Lemma 1.1 to prove Theorem 1.4. We consider the case $A(x) = A$ and $B(x) = B$ in system (1-1), where A, B are constants. In this case, (1-1) becomes (1-11).

Choose any direction to be the x_1 direction. We write $x = (x_1, x') \in \mathbb{R}^n$ with $x' = (x_2, x_3, \ldots, x_n)$. Let

$$\hat{T}_\lambda = \{x \in \mathbb{R}^n \mid x_1 = \lambda, \lambda \in \mathbb{R}, \lambda > -1\}$$

be the moving planes and denote

$$\hat{H}_\lambda = \{x \in \mathbb{R}^n \mid x_1 < \lambda\}, \quad \hat{\Sigma}_\lambda = \{x \in \mathbb{B}_1(0) \mid -1 < x_1 < \lambda\}.$$

For each point $x = (x_1, x') \in \hat{\Sigma}_\lambda$, let $x^\lambda = (2\lambda - x_1, x')$ be the reflection point about the plane $\hat{T}_\lambda$ and $U_\lambda(x)$, $V_\lambda(x)$ defined as before. Then it follows that for $x \in \hat{\Sigma}_\lambda$,

$$(-\Delta)^{\alpha/2} U_\lambda(x) = p\xi^{p-1}(x) V_\lambda(x) - A U_\lambda(x), \tag{4-1}$$

and

$$(-\Delta)^{\beta/2} V_\lambda(x) = q\eta^{q-1}(x) U_\lambda(x) - B V_\lambda(x), \tag{4-2}$$

where $\xi(x)$ is between $v_\lambda(x)$ and $v(x)$ and $\eta(x)$ is between $u_\lambda(x)$ and $u(x)$.

Proof of Theorem 1.4. **Step 1:** We show that for $\lambda > -1$ sufficiently close to -1,

$$U_\lambda(x) \geq 0, \quad V_\lambda(x) \geq 0, \quad x \in \hat{\Sigma}_\lambda. \tag{4-3}$$

The proof is almost the same as Step 1 in the proof of Theorem 1.2.

Step 1 provides a starting point. We start from such a small λ and move the plane $\hat{T}_\lambda$ continuously in the direction of x_1-axis to its limiting position as long as (4-3) holds.

Step 2: Define

$$\lambda_0 = \sup\{\lambda > -1 \mid U_\mu(x) \geq 0, V_\mu(x) \geq 0, x \in \hat{\Sigma}_\mu; \mu \leq \lambda\}. \tag{4-4}$$

We will prove

$$\lambda_0 = 0. \tag{4-5}$$

If $\lambda_0 < 0$, we will show that the plane $\hat{T}_\lambda$ can be moved further right. That is, there exists $\varepsilon > 0$ such that for all $\lambda \in (\lambda_0, \lambda_0 + \varepsilon)$,

$$U_\lambda(x) \geq 0, \quad V_\lambda(x) \geq 0, \quad x \in \hat{\Sigma}_\lambda. \tag{4-6}$$

This is a contradiction with the definition of λ_0. Hence, we must have $\lambda_0 = 0$.

The proof of (4-6) is composed of two parts.

(a): We show that for $\varepsilon > 0$, $\delta > 0$ and $\varepsilon < \delta$, when $\lambda \in (\lambda_0, \lambda_0 + \varepsilon)$,

$$U_\lambda(x) \geq 0, \quad V_\lambda(x) \geq 0, \quad x \in \bar{\hat{\Sigma}}_{\lambda_0-\delta}. \tag{4-7}$$

We know that

$$U_{\lambda_0}(x) \geq 0, \quad V_{\lambda_0}(x) \geq 0, \quad x \in \hat{\Sigma}_{\lambda_0}.$$

In fact, if $\lambda_0 < 0$, we must have

$$U_{\lambda_0}(x) > 0, \quad V_{\lambda_0}(x) > 0, \quad x \in \hat{\Sigma}_{\lambda_0}.$$

If $U_{\lambda_0}(x) > 0$ does not hold, we assume that there exists some point $\tilde{x} \in \hat{\Sigma}_{\lambda_0}$ such that $U_{\lambda_0}(\tilde{x}) = 0$.

$$\begin{aligned}(-\Delta)^{\alpha/2} U_{\lambda_0}(\tilde{x}) &= C_{n,\alpha} P.V. \int_{\mathbb{R}^n} \frac{U_{\lambda_0}(\tilde{x}) - U_{\lambda_0}(y)}{|\tilde{x}-y|^{n+\alpha}}\, dy \\ &= C_{n,\alpha} P.V. \int_{\hat{H}_{\lambda_0}} \frac{-U_{\lambda_0}(y)}{|\tilde{x}-y|^{n+\alpha}}\, dy + \int_{\mathbb{R}^n \setminus \hat{H}_{\lambda_0}} \frac{-U_{\lambda_0}(y)}{|\tilde{x}-y|^{n+\alpha}}\, dy \\ &= C_{n,\alpha} P.V. \int_{\hat{H}_{\lambda_0}} \frac{-U_{\lambda_0}(y)}{|\tilde{x}-y|^{n+\alpha}}\, dy + \int_{\hat{H}_{\lambda_0}} \frac{-U_{\lambda_0}(y^\lambda)}{|\tilde{x}-y^\lambda|^{n+\alpha}}\, dy \\ &= C_{n,\alpha} P.V. \int_{\hat{H}_{\lambda_0}} \left(\frac{1}{|\tilde{x}-y^\lambda|^{n+\alpha}} - \frac{1}{|\tilde{x}-y|^{n+\alpha}} \right) U_{\lambda_0}(y)\, dy.\end{aligned}$$

Note that $|\tilde{x}-y^\lambda| > |\tilde{x}-y|$, hence,

$$(-\Delta)^{\alpha/2} U_{\lambda_0}(\tilde{x}) < 0. \tag{4-8}$$

On the other hand, according to (4-1),

$$(-\Delta)^{\alpha/2} U_{\lambda_0}(\tilde{x}) = p\xi^{p-1}(\tilde{x}) V_\lambda(\tilde{x}) - AU_\lambda(\tilde{x}) = p\xi^{p-1}(\tilde{x}) V_\lambda(\tilde{x}) \geq 0.$$

This is contradictory to (4-8). Consequently, we obtain $U_{\lambda_0}(x) > 0$ for all $x \in \hat{\Sigma}_{\lambda_0}$. Similarly, we can show $V_{\lambda_0}(x) > 0$ for all $x \in \hat{\Sigma}_{\lambda_0}$. Hence, for small $\delta > 0$, there exists a positive constant c_0 such that

$$U_{\lambda_0}(x) \geq c_0 > 0, \quad V_{\lambda_0}(x) \geq c_0 > 0, \quad x \in \bar{\hat{\Sigma}}_{\lambda_0-\delta}.$$

Since $U_\lambda(x)$ and $V_\lambda(x)$ depend on λ continuously, there exists $\varepsilon > 0$ with $\varepsilon < \delta$ such that for all $\lambda \in (\lambda_0, \lambda_0 + \varepsilon)$,

$$U_\lambda(x) \geq 0, \quad V_\lambda(x) \geq 0, \quad x \in \bar{\hat{\Sigma}}_{\lambda_0-\delta}.$$

(b): Using Lemma 1.1 and similarly to the proof of (3-14), we get

$$U_\lambda(x) \geq 0, \quad , V_\lambda(x) \geq 0 \quad x \in \hat{\Sigma}_\lambda \setminus \bar{\hat{\Sigma}}_{\lambda_0-\delta}. \tag{4-9}$$

Together with **(a)** and **(b)**, we prove (4-6) is true for all $\lambda \in (\lambda_0, \lambda_0+\varepsilon)$. Thus, we obtain $\lambda_0 = 0$ and $U_{\lambda_0}(x) \geq 0,\ V_{\lambda_0}(x) \geq 0,\ x \in \hat{\Sigma}_{\lambda_0}$.

Similarly, we move the plane T_λ from 1 to the left and show that

$$U_{\lambda_0}(x) \leq 0,\ V_{\lambda_0}(x) \leq 0, \quad x \in \hat{\Sigma}_{\lambda_0}.$$

Then we obtain that

$$\lambda_0 = 0 \quad \text{and} \quad U_{\lambda_0}(x) \equiv 0,\ V_{\lambda_0}(x) \equiv 0, \quad x \in \hat{\Sigma}_{\lambda_0}.$$

This indicates that $u(x)$ and $v(x)$ are symmetric about T_0. Since the x_1 direction can be chosen arbitrarily, we have actually shown that $u(x)$ and $v(x)$ are radially symmetric about the origin. Thus, we have completed the proof of Theorem 1.4. □

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