

*Pacific  
Journal of  
Mathematics*

CONCORDANCE OF SEIFERT SURFACES

ROBERT MYERS

Volume 298 No. 2

February 2019



## CONCORDANCE OF SEIFERT SURFACES

ROBERT MYERS

**We prove that every oriented nondisk Seifert surface  $F$  for an oriented knot  $K$  in  $S^3$  is smoothly concordant to a Seifert surface  $F'$  for a hyperbolic knot  $K'$  of arbitrarily large volume. This gives a new and simpler proof of the result of Friedl and of Kawauchi that every knot is  $S$ -equivalent to a hyperbolic knot of arbitrarily large volume. The construction also gives a new and simpler proof of the result of Silver and Whitten and of Kawauchi that for every knot  $K$  there is a hyperbolic knot  $K'$  of arbitrarily large volume and a map of pairs  $f : (S^3, K') \rightarrow (S^3, K)$  which induces an epimorphism on the knot groups. An example is given which shows that knot Floer homology is not an invariant of Seifert surface concordance. We also prove that a set of finite volume hyperbolic 3-manifolds with unbounded Haken numbers has unbounded volumes.**

### 1. Introduction

In what follows, the smooth category will always be assumed. This paper concerns two equivalence relations on oriented knots in  $S^3$ , concordance and  $S$ -equivalence. Knots  $K$  and  $K'$  are *concordant* if there is a properly embedded oriented annulus  $A$  in  $S^3 \times [0, 1]$  with  $A \cap (S^3 \times \{0\}) = K$  and  $A \cap (S^3 \times \{1\}) = K'$  such that  $\partial A = K - K'$ . Knots  $K$  and  $K'$  are  *$S$ -equivalent* if they have Seifert surfaces  $F$  and  $F'$  with associated Seifert matrices which are equivalent under integral congruence and elementary expansions and contractions [Trotter 1973].

Concordant knots need not be  $S$ -equivalent, e.g., the trivial knot and a slice knot with nontrivial Alexander polynomial, such as the stevedore's knot  $6_1$ .  $S$ -equivalent knots need not be concordant; Kearton [2004] has shown that every algebraically slice knot is  $S$ -equivalent to a slice knot, but by Casson and Gordon [1978], there are algebraically slice knots which are not slice knots.

The author [Myers 1983] proved that every knot is concordant to a hyperbolic knot, generalizing the result of Kirby and Lickorish [1979] that every knot is concordant to a prime knot. Friedl [2009] and Kawauchi [1989a; 1989b; 1994]

---

*MSC2010:* 57M25.

*Keywords:* knot, Seifert surface, concordance.

have given proofs that every knot is  $S$ -equivalent to a hyperbolic knot, generalizing the result of Kearton [2004] that every knot is  $S$ -equivalent to a prime knot.

Friedl noted, citing [Kearton 1975; 2004; Levine 1970; 1977; Trotter 1973], that two knots are  $S$ -equivalent if and only if they have isometric Blanchfield pairings. He then noted that by combining two results of Kawauchi's imitation theory of knots ([1989a, Theorem 1.1] and [1989b, Properties I and V]) one gets that for every knot  $K$  there is a hyperbolic knot  $K'$  of arbitrarily large volume and a map  $f : (S^3, K') \rightarrow (S^3, K)$  which induces isomorphisms on every quotient of the knot groups by their derived subgroups. Friedl then showed that this result implies that the Blanchfield pairings are isometric. He also added a note in proof that one can combine the existence of such knots and maps with another result of Kawauchi ([1994, Theorem 2.2]) to show  $S$ -equivalence.

It is natural to ask whether for every knot  $K$  there is a hyperbolic knot  $K'$  of arbitrarily large volume to which  $K$  is both  $S$ -equivalent and concordant. It turns out that an affirmative answer is implicit in Kawauchi's construction [1989a]. One can see the concordance by looking at Figure 7 of that paper for the time interval  $0 \leq t \leq 1$ .

Silver and Whitten [2006] proved that given any triple  $(G, \mu, \lambda)$  where  $G$  is a knot group and  $(\mu, \lambda)$  is a meridian-longitude pair for  $G$  there are infinitely many triples  $(\tilde{G}, \tilde{\mu}, \tilde{\lambda})$  where  $\tilde{G}$  is the group of a prime knot and there is an epimorphism  $\phi : (\tilde{G}, \tilde{\mu}, \tilde{\lambda}) \rightarrow (G, \mu, \lambda)$ . In [Silver and Whitten 2005], they strengthened this to the  $\tilde{G}$  being the groups of hyperbolic knots of arbitrarily large volume. In a note in proof they added the comment that Kawauchi informed them that many of the results in the paper can be found in [Kawauchi 1989a; Kawauchi 1992].

The constructions of Kawauchi and of Silver and Whitten mentioned above are rather intricate and the proofs somewhat lengthy. In the present paper, the author gives a simpler and shorter construction and proof. Recall that a *Seifert surface*  $F$  for a knot  $K$  in  $S^3$  is a compact, oriented surface with boundary  $K$ . Both  $F$  and  $K$  will be assumed to be oriented, with  $\partial F = K$ . A Seifert surface  $F'$  for a knot  $K'$  will be said to be *concordant* to  $F$  if there is an embedding  $h : F \times [0, 1] \rightarrow S^3 \times [0, 1]$  such that

$$\begin{aligned} h(F \times \{0\}) &= h(F \times [0, 1]) \cap (S^3 \times \{0\}) = F, \\ h(F \times \{1\}) &= h(F \times [0, 1]) \cap (S^3 \times \{1\}) = F', \end{aligned}$$

and there is an orientation of  $h(F \times [0, 1])$  such that  $(S^3 \times \{0\}) \cap \partial h(F \times [0, 1]) = F$  and  $(S^3 \times \{1\}) \cap \partial h(F \times [0, 1]) = -F'$ . In this case  $K$  and  $K'$  are clearly concordant.

They are also  $S$ -equivalent, which can be seen as follows.

Let  $N$  be a regular neighborhood of  $h(K \times [0, 1])$  in  $S^3 \times [0, 1]$ . Let  $P$  be the closure of  $h(F \times [0, 1]) - N$ . Let  $Q = h(F \times [0, 1]) \cap P$ . Finally let  $R$  be a regular neighborhood of  $Q$  in  $P$ . Then  $R$  is homeomorphic to  $Q \times [-1, +1]$

and hence to  $F \times [-1, +1]$ . Thus one gets a product structure of the form  $k : F \times [0, 1] \times [-1, 1] \rightarrow R$  on  $R$ . By abuse of notation this will be denoted by  $R = F \times [0, 1] \times [-1, 1]$ . Thus we identify  $F$  with  $F \times \{0\} \times \{0\}$  and  $F'$  with  $F \times \{1\} \times \{0\}$ .

Now choose a collection of oriented simple closed curves  $a_i$  on  $F$  which represents a basis for  $H_1(F)$ . Identify  $a_i$  with  $a_i \times \{0\} \times \{0\}$ . Let  $a_i^+ = a_i \times \{0\} \times \{+1\}$ ; this is regarded as  $a_i$  pushed off  $F$  in the positive normal direction. Let  $v_{i,j}$  be the linking number of  $a_i$  and  $a_j^+$ . The resulting matrix is a Seifert matrix  $V$  for  $K$ . Now let  $b_i = a_i \times \{1\} \times \{0\}$  and  $b_i^+ = a_i \times \{1\} \times \{+1\}$ . Letting  $v'_{i,j}$  be the linking number of  $b_i$  and  $b_j^+$  one gets a Seifert matrix  $V'$  for  $K'$ . Let  $A_i$  be the annulus  $a_i \times [0, 1] \times \{0\}$ . Let  $A_i^+$  be the annulus  $a_i \times [0, 1] \times \{+1\}$ . Then  $A_i$  joins  $a_i$  to  $b_i$  and  $A_i^+$  joins  $a_i^+$  to  $b_i^+$ .

Recall that if  $J$  and  $J^+$  are disjoint oriented simple closed curves in  $S^3$  then they bound properly embedded oriented surfaces  $G$  and  $G^+$  in the 4-ball  $B^4$  which can be chosen to meet in a finite number of points. The linking number of  $J$  and  $J^+$  is then equal to the algebraic intersection number of  $G$  and  $G^+$ . See [Rolfsen 1976, page 136].

Regard  $S^3 \times \{1\}$  as  $\partial B^4$ . Choose surfaces  $G_i$  and  $G_i^+$  in  $B^4$  with boundaries  $b_i$  and  $b_i^+$ , respectively. Let  $\widehat{G}_i = A_i \cup G_i$  and  $\widehat{G}_i^+ = A_i^+ \cup G_i^+$ . Since  $A_i \cap A_j^+ = \emptyset$ , the intersection number of  $\widehat{G}_i$  and  $\widehat{G}_j^+$  is equal to that of  $G_i$  and  $G_j^+$ . It follows that the Seifert matrices  $V$  of  $F$  and  $V'$  of  $F'$  with respect to the given bases are the same.

The main result of this paper will now be stated. The notation  $S^3 \setminus K'$  means the compact manifold obtained from  $S^3$  by removing the interior of a regular neighborhood of the knot  $K'$ . Recall that the *Haken number* [1968] of a compact 3-manifold  $M$  is the maximum number of compact, connected, properly embedded, incompressible, boundary incompressible, pairwise nonparallel surfaces in  $M$ .

**Theorem 1.1.** *Let  $F$  be a Seifert surface for a knot  $K$  in  $S^3$ . Assume that  $F$  is not a disk. Then  $F$  is concordant to a Seifert surface  $F'$  for a knot  $K'$  such that*

- (a)  $K'$  is hyperbolic,
- (b)  $S^3 \setminus K'$  has arbitrarily large Haken number,
- (c)  $S^3 - K'$  has arbitrarily large volume, and
- (d) there is a map of pairs  $f : (S^3, K') \rightarrow (S^3, K)$  which induces an epimorphism  $f_* : \pi_1(S^3 - K') \rightarrow \pi_1(S^3 - K)$ .

The paper is organized as follows. Section 2 reviews some basic material. Section 3 proves that every nondisk Seifert surface for a knot can be put in a certain standard position. Section 4 uses standard position to prove (a). Section 5 proves (b). The proof that (b) implies (c) follows from a more general result, that a set of finite volume hyperbolic 3-manifolds with unbounded Haken numbers

has unbounded volumes. This fact appears to be “quasi-known” but the author does not know a precise reference in the literature and so gives a simple proof in the Appendix. Section 6 proves (d). Section 7 considers the question of whether Seifert surface concordance implies the invariance of more than just the union of the sets of invariants of concordance and  $S$ -equivalence. It shows by example that although the Alexander polynomial is an invariant of Seifert surface concordance, its categorification, knot Floer homology, is not.

## 2. Preliminaries

As general references on knot theory and on 3-manifolds see [Lickorish 1997] and [Jaco 1980]. A compact, connected, orientable 3-manifold  $M$  will be called *excellent* if it is irreducible, boundary-irreducible, anannular, atoroidal, and is not a 3-ball.  $M$  will be called *Haken* if it contains a two-sided incompressible surface. By Thurston’s uniformization theorem (see, e.g., [Morgan 1984]), excellent Haken manifolds are hyperbolic.

The following standard technical result will be used to build more complicated hyperbolic 3-manifolds out of simpler pieces. A proof can be found in Section 2 of [Myers 1993].

**Lemma 2.1** (gluing lemma). *Let  $X$  be a compact, connected 3-manifold. Suppose  $F$  is a compact, properly embedded, two-sided 2-manifold in  $X$ . It is not assumed that  $F$  is connected. Let  $Y$  be the 3-manifold obtained by splitting  $X$  along  $F$ . Denote by  $F_1$  and  $F_2$  the two copies of  $F$  in  $\partial Y$  which are identified to obtain  $X$ . If each component of  $Y$  is excellent,  $F_1 \cup F_2$  and the closure of  $Y - (F_1 \cup F_2)$  is incompressible in  $Y$ , and each component of  $F_1 \cup F_2$  has negative Euler characteristic, then  $X$  is excellent.*

An  $n$ -tangle is the disjoint union  $\lambda = \lambda_1 \cup \cdots \cup \lambda_n$  of properly embedded arcs in a 3-ball  $B$ . This is sometimes denoted by the pair  $(B, \lambda)$ . It will always be assumed that  $n \geq 2$ . When the specific number  $n$  of arcs is not at issue or is clear from the context,  $\lambda$  will just be called a *tangle*.

This paper will assume that  $B$  is given a product structure of the form  $[a, b] \times [c, d] \times [e, f]$  with each component  $\lambda_i$  of the tangle  $(B, \lambda)$  joining a point of  $(a, b) \times (c, d) \times \{f\}$  to a point of  $(a, b) \times (c, d) \times \{e\}$ . This is done so that one may compose tangles. The product of the tangles  $(B, \lambda)$  and  $(B, \mu)$  will be obtained by setting  $(B, \lambda)$  on top of  $(B, \mu)$  so that the lower endpoint of each  $\lambda_i$  equals the upper endpoint of each  $\mu_i$ .

The *exterior* of a submanifold of a 3-manifold is the closure of the complement of a regular neighborhood of the submanifold. A knot or tangle will be called *excellent* if its exterior is excellent. In this case by a slight abuse of language the knot or tangle will be called hyperbolic.

### 3. Standard position for Seifert surfaces

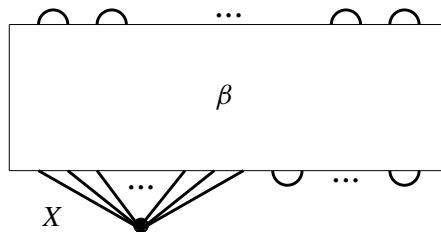
Let  $F$  be a nondisk Seifert surface for a knot  $K$ . This section defines a way of presenting  $F$  by a diagram in the plane, standard position. This presentation will be used to build concordances.

The first step is to define an analogue for certain graphs of a plat presentation of a knot.

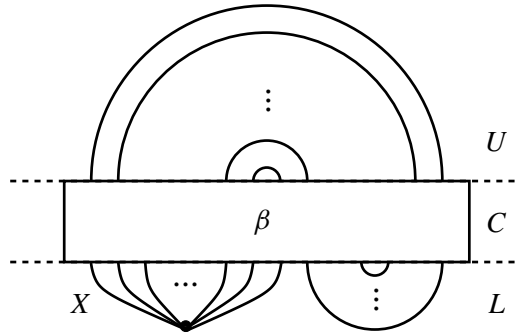
Recall the idea of a plat presentation for a knot or link. Regard  $S^3$  as the union of a 3-ball  $B^+$ , a copy  $B^0$  of  $S^2 \times [-1, 1]$ , and a 3-ball  $B^-$  with  $B^+ \cap B^0 = S^2 \times \{1\}$  and  $B^- \cap B^0 = S^2 \times \{-1\}$ . Choose  $M$  unknotted, unlinked, properly embedded arcs in each of  $B^+$  and  $B^-$ . Arrange them so that the projections  $p^\pm : B^\pm \rightarrow D^\pm$  onto equatorial disks have no crossings and are unnested. Let  $p^0 : B^0 \rightarrow A$  be a projection of  $B^0$  onto an annulus  $A$ . Join the endpoints of the arcs in  $B^+$  to the endpoints of the arcs in  $B^-$  by arcs in  $B^0$  such that projection on  $A$  is given by a braid  $\beta$ . In this paper the braid will be chosen so that it is an element of the braid group of the plane, not the braid group of the sphere. The knot projections will be drawn in  $\mathbb{R}^2$  with regions  $U, C$  and  $L$  representing the projections of regions near the knot in  $B^+, B^0,$  and  $B^-$ , respectively. The map  $p : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  denotes this local projection. The vertical coordinate in  $\mathbb{R}^2$  is denoted by  $y$ . The braid is represented by a box labeled  $\beta$ . The arcs in  $B^+$  and  $B^-$  are represented by arcs on the top and bottom of the box, respectively.

Now suppose that  $M \geq 2$ . Choose  $g$  such that  $2 \leq 2g \leq M$ , and let  $m = M - 2g$ . Replace  $2g$  adjacent arcs in  $L$  with the cone  $X$  on their boundary points having vertex  $v$  in the interior of  $L$ ; require that  $X$  be disjoint from the remaining arcs in  $L$ . This gives a 1-complex in  $\mathbb{R}^3$  consisting of a wedge  $W$  of  $2g$  circles with possibly some additional simple closed curve components. Restrict attention to those  $\beta$  for which there are no such additional components. By changing  $\beta$  one may obtain an isotopic embedding of  $W$  such that  $X$  is to the left of the arcs in  $L$ . This will be called a *plat presentation of a wedge of  $2g$  circles*. See Figure 1.

One may add words to the top and bottom of  $\beta$  to obtain an isotopic embedding of  $W$  so that the arcs in each of  $U$  and  $L$  are concentrically nested. This will be called a *standard presentation of a wedge of  $2g$  circles*. See Figure 2.



**Figure 1.** Plat presentation of a wedge of circles.



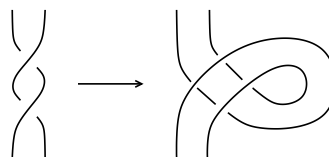
**Figure 2.** Standard presentation of a wedge of circles.

Now take regular neighborhoods in  $U \cup L$  of the arcs and  $X$  and widen the arcs in  $\beta$  to bands which cross in the same manner as the arcs. One gets a *standard presentation of a surface with boundary*. The special case of interest is that of an orientable surface with connected boundary.

**Lemma 3.1.** *Every nondisk Seifert surface for a knot in  $S^3$  has a standard presentation.*

*Proof.* Let  $F$  be a genus  $g \geq 1$  Seifert surface for a knot. Choose a wedge of circles  $W$  in the interior of  $F$  such that the boundary of a regular neighborhood of  $W$  in  $F$  is parallel in  $F$  to  $K$ . Let  $v$  be the vertex of  $W$ . The number of circles is  $2g$ , where  $g$  is the genus of  $F$ . Isotop  $F$  so that the projection of  $W$  onto  $\mathbb{R}^2$  has only transverse double point singularities. In particular,  $p^{-1}(p(v))$  consists of a single point. Isotop  $F$  so that  $p(v)$  has  $y$  coordinate less than or equal to that of any other point of  $p(W)$ . We may assume that for each edge of  $W$  all the critical values of the function  $y \circ p$  are local maxima and minima. Isotop  $F$  so as to move all the minima into  $L$  and all the maxima into  $U$ . One now has a plat presentation of  $W$ .

The only obstruction to widening  $W$  into a plat presentation for  $F$  is that some of the bands may be twisted. Since  $F$  is orientable, the twisting in each band consists of a number of full twists. Each full twist is isotopic to a curl, as illustrated in Figure 3. Isotop the local maxima and local minima of the curls into  $U$  and  $L$ , respectively. The isotopy from plat to standard presentation then preserves the fact that the bands are untwisted.  $\square$



**Figure 3.** Replacing a twist by a curl.

### 4. Constructing a concordance

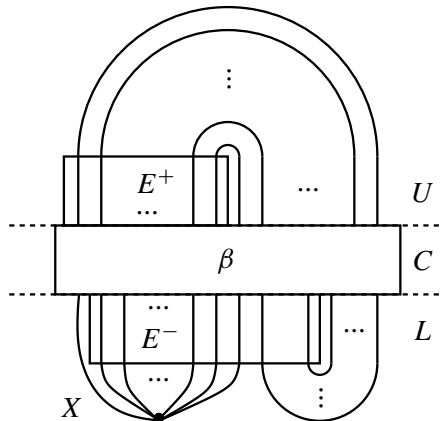
The previous section showed that every nondisk Seifert surface  $F$  for a knot in  $S^3$  has a standard presentation which is obtained by widening a standard presentation of a wedge of circles  $W$ . The basic idea for constructing a concordance from  $F$  to a surface  $F'$  is to construct a concordance from  $W$  to some  $W'$  and widen  $W'$  to get  $F'$ . However, the obvious surface one gets from a projection of  $W'$  might not be concordant to  $F$ . As an example of this phenomenon let  $P$  be a zero crossing projection of the trivial knot and  $P'$  a one crossing projection of the trivial knot. Let  $A$  and  $A'$  be annuli obtained by widening  $P$  and  $P'$  into bands. These annuli cannot be concordant because the components of their boundaries have different linking numbers. The way to fix this is to require that any isotopies of a diagram be *regular*, i.e., they are composed of Reidemeister moves of types II and III together with planar isotopies.

**Lemma 4.1.** *Given a standard presentation of a nondisk Seifert surface  $F$ , there is a concordance of  $F$  with a Seifert surface  $F'$  such that  $S^3 - K'$  is hyperbolic, where  $K' = \partial F'$ .*

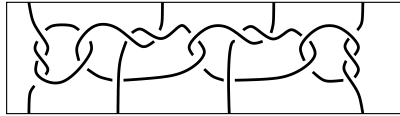
*Proof.* By the gluing lemma, it will be sufficient to show that  $S^3 - F'$  is excellent.

Let  $W$  be the wedge of circles of which  $F$  is a regular neighborhood. Choose a 3-ball  $E^+$  in  $U$  and a 3-ball  $E^-$  in  $L$  as shown in Figure 4.  $E^+$  meets  $C$  in a disk and meets  $W$  in  $M$  straight arcs; these arcs meet  $C$  in the leftmost  $M$  points of  $W \cap U \cap C$ .  $E^-$  meets  $C$  in a disk and meets  $W$  in all but the first endpoint of  $X$  and in the leftmost  $M - 2g$  endpoints of the arcs in  $(W \cap L) - X$ .

The trivial tangles in  $E^+$  and in  $E^-$  are then replaced by concordant hyperbolic tangles  $\alpha$  and  $\gamma$ , respectively. The process starts with a hyperbolic  $n$ -tangle



**Figure 4.**  $W$  before surgery.

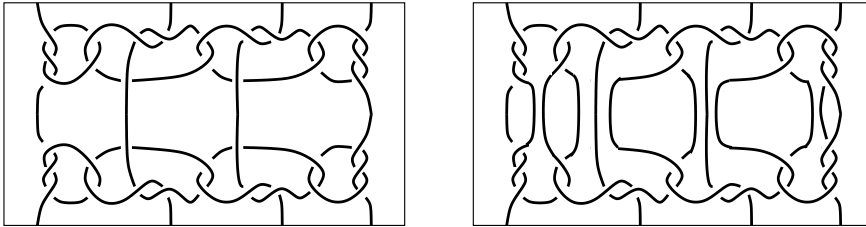


**Figure 5.** A hyperbolic  $n$ -tangle,  $n = 4$ .

constructed in [Myers 1983]. Figure 5 shows the case  $n = 4$ .

This tangle is then composed with its mirror image as in Figure 6, left. By the gluing lemma, the new tangle has an excellent Haken exterior and is thus hyperbolic.

A concordance to the trivial tangle is then constructed by attaching 1-handles, performing type II Reidemeister moves, and then attaching 2-handles. See Figure 6, right, for the first stage.



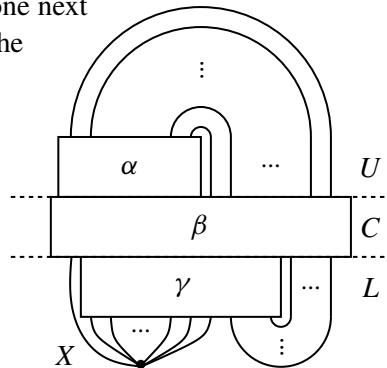
**Figure 6.** A ribbon  $n$ -tangle (left) and a ribbon concordance (right),  $n=4$ .

For future reference the general version of this result is stated below.

**Lemma 4.2.** *Let  $D$  be a disk. Let  $\tau = \tau_1 \cup \dots \cup \tau_n$  be an  $n$ -tangle in  $D \times [0, 1]$  such that each  $\tau_i$  has one boundary point in  $\text{int}(D) \times \{0\}$  and the other in  $\text{int}(D) \times \{1\}$ . Let  $\delta(\tau)$  be the tangle in  $D \times [-1, 1]$  obtained by taking the union of  $\tau$  and its mirror image in  $D \times [-1, 0]$ . Then  $\delta(\tau)$  is concordant to the trivial tangle  $\varepsilon$ .  $\square$*

Continuing with the proof of Lemma 4.1, one next constructs the Seifert surface  $F'$  by replacing the disjoint union of untwisted bands  $F \cap (E^+)$  with the disjoint union of untwisted bands in  $E^+$  whose centerlines form the tangle  $\alpha$ . These bands are chosen so that their intersections with  $\partial E^+$  are the same as those of  $F$  with  $\partial E^+$ . A similar construction in  $\partial E^-$  then completes the construction of  $F'$ .

One now shows that  $S^3 - F'$  is hyperbolic. Figure 7 shows the new wedge of circles  $W'$ . The exterior of  $W' \cap U$  is homeomorphic to the



**Figure 7.**  $W'$  after surgery.

exterior of the tangle  $\alpha$  and is therefore hyperbolic. The exterior of  $W' \cap L$  is homeomorphic to the exterior of the tangle  $\gamma$ . This can be seen as follows. Denote the arcs in  $X$  by  $\gamma_1, \dots, \gamma_k$ , numbered from left to right. Slide the lower endpoint of  $\gamma_2$  along  $\gamma_1$  and into  $C \cap L$ . Continue with  $\gamma_3$  through  $\gamma_{2g-1}$ . The complement in  $L$  of the new set of arcs is homeomorphic to the complement of  $\gamma$  and is therefore hyperbolic. Since the exterior of  $\beta$  in  $C$  is a product, the result follows.  $\square$

### 5. Raising the Haken number and the volume

**Lemma 5.1.** *Given a standard presentation of a nondisk Seifert surface  $F$ , and given an  $N > 0$ , there is a concordance of  $F$  with a Seifert surface  $F'$  such that  $K'$  is hyperbolic and  $S^3 - K'$  has Haken number at least  $N$ .*

*Proof.* In the construction of the previous section, replace each of the tangles  $\alpha$  and  $\gamma$  by  $N$  copies of itself stacked one on top of the other. This gives  $N$  disjoint copies of the exteriors of  $\alpha$  and  $\gamma$ . The boundaries of the exteriors of these tangles are all incompressible and nonparallel in  $S^3 - K'$ . By the gluing lemma, the exterior of the new knot is an excellent Haken manifold and it follows that  $K'$  is hyperbolic.  $\square$

**Lemma 5.2.** *Given a standard presentation of a nondisk Seifert surface  $F$ , and given a  $V > 0$  there is a concordance of  $F$  with a Seifert surface  $F'$  such that  $K'$  is hyperbolic and  $S^3 - K'$  has volume at least  $N$ .*

*Proof.* This follows from Corollary A.3 in the Appendix and Lemma 5.1.  $\square$

### 6. Making a map

**Lemma 6.1.** *Given a standard presentation of a nondisk Seifert surface  $F$ , there is a concordance of  $F$  with a Seifert surface  $F'$  satisfying (a), (b), (c), and (d).*

*Proof.* The proof follows from Lemmas 6.2 and 6.3 below by defining  $f$  to be the identity outside the tangles involved.  $\square$

A tangle  $(B^3, \tau_1 \cup \dots \cup \tau_n)$  is a *boundary tangle* if there are disjoint arcs  $\sigma_1, \dots, \sigma_n$  in  $\partial B^3$  such that  $\partial \sigma_i = \partial \tau_i$  and disjoint, compact, orientable surfaces  $G_i$  in  $B^3$  such that  $\partial G_i = \tau_i \cup \sigma_i$ . Let  $\tau = \tau_1 \cup \dots \cup \tau_n$  and  $G = G_1 \cup \dots \cup G_n$ .

Let  $\tau^* = \tau_1^* \cup \dots \cup \tau_n^*$  be a trivial tangle in  $B^3$  with  $\partial \tau_i^* = \partial \tau_i$  for all  $i$ . Choose disjoint disks  $G_i^*$  in  $B^3$  with  $\partial G_i^* = \partial G_i$ .

**Lemma 6.2.** *There is a map  $g : (B^3, \tau, B^3 - \tau) \rightarrow (B^3, \tau^*, B^3 - \tau^*)$  which is the identity on  $\partial B^3$  and a homeomorphism from  $\tau$  to  $\tau^*$ . In particular,  $g$  induces an epimorphism  $\pi_1(B^3 - \tau) \rightarrow \pi_1(B^3 - \tau^*)$  which carries the meridians of  $\tau$  to the meridians of  $\tau^*$ .*

*Proof.* Let  $Y$  be the exterior of  $\tau$  in  $B^3$ . Let  $H_i = G_i \cap Y$ . Let  $N_i = H_i \times [-1, 1]$  be a regular neighborhood of  $H_i$  in  $Y$ . Let  $N$  be the union of the  $N_i$ . Let  $C$  be a collar

on  $\partial Y$  whose intersection with each  $N_i$  has the form  $A_i \times [-1, 1]$ , where  $A_i$  is a collar on  $\partial H_i$  in  $H_i$ .

In a similar fashion let  $Y^*$  be the exterior of  $\tau^*$  in  $B^3$ , let  $H_i^* = G_i^* \cap Y^*$ ,  $N_i^* = H_i^* \times [-1, 1]$ ,  $N^* = \bigcup N_i^*$ ,  $C^* = C$ , and  $A_i^* = A_i$ .

For each  $t \in [-1, 1]$  define a map from  $N_i$  to  $N_i^*$  by crushing  $(H_i - C) \times \{t\}$  to a point in  $H_i^*$ . This defines the restriction of  $g$  to  $H_i \times [-1, 1]$ .

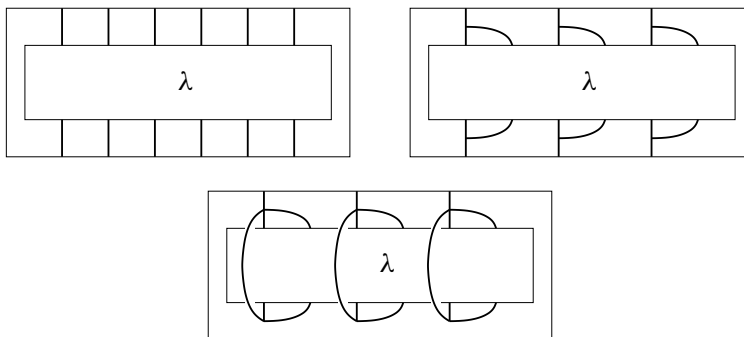
One next defines  $g$  on the closure  $W$  of the complement of  $N$  in  $B^3$  by crushing  $W - C$  to a point. This gives a quotient map onto a 3-ball which may be identified with the closure of  $B^3 - N^*$ .

Putting the two quotient maps together gives a quotient map from  $Y$  to  $Y^*$  which extends to a map  $g : B^3 \rightarrow B^3$  with the required properties.  $\square$

The existence of hyperbolic boundary  $n$ -tangles was proven by Cochran and Orr in a more abstract general setting. In keeping with the desire to make the constructions in this paper as explicit as possible, their procedure is implemented in the following specific construction.

**Lemma 6.3** [Cochran and Orr 1998, Lemma 7.3]. *Hyperbolic boundary  $n$ -tangles exist.*

*Proof.* First choose a hyperbolic  $2n$ -tangle  $\lambda$ . Configure it so that the ambient 3-cell is a rectangular box with each component of the tangle joining the interior of the top of the box to the interior of the bottom of the box. Place it in the interior of a larger box with which it is concentric. Connect the endpoints of  $\lambda$  to the boundary of the larger box by straight arcs as in the first diagram in Figure 8. Then slide the endpoints of every second arc onto the arc preceding it as in the second diagram. Then slide the bottom endpoints of each resulting graph across the front of the larger box onto the top arc as in the third diagram. This does not change the homeomorphism type of the exterior of the graph.



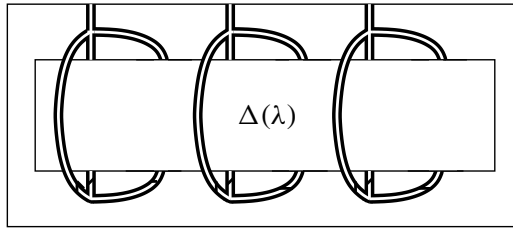
**Figure 8.** Sliding endpoints to obtain a graph.



**Figure 9.** Double  $\Delta(\lambda_n)$  of  $\lambda_n$ ,  $n = 6$ .

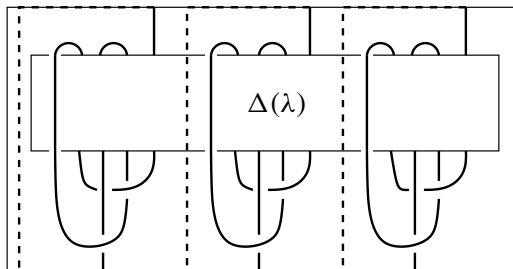
Then one constructs the  $2n$ -tangle  $\Delta(\lambda)$  by replacing each arc of  $\lambda$  by two parallel copies as in Figure 9.

Next one modifies the last diagram of Figure 8 in the following ways to obtain Figure 10. First, one replaces  $\lambda$  in the inner box by  $\Delta(\lambda)$ . Second, one widens the graphs which join the inner box to the boundary of the outer box to obtain surfaces whose unions with the bands inside the inner box are punctured tori. Note the three half-twists inserted into the lower portion of each component of Figure 10 to achieve this. By the gluing lemma, the exterior of this new tangle is hyperbolic since it is obtained from a 3-manifold homeomorphic to the exterior of  $\lambda$  by identifying pairs of incompressible once-punctured tori in its boundary.

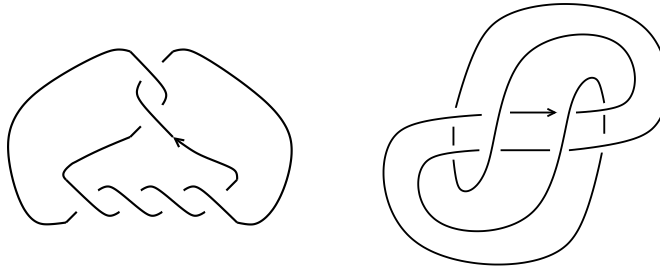


**Figure 10.** A hyperbolic boundary tangle.

Finally one slides the left endpoints of each arc across the front of the larger box (dotted lines) to obtain the final hyperbolic boundary tangle as in Figure 11. The union of this tangle with its mirror image is still a boundary tangle and the rest of the proof proceeds as before.  $\square$



**Figure 11.** The hyperbolic boundary tangle ready for use.



**Figure 12.** Two projections of the stevedore’s knot  $6_1$ .

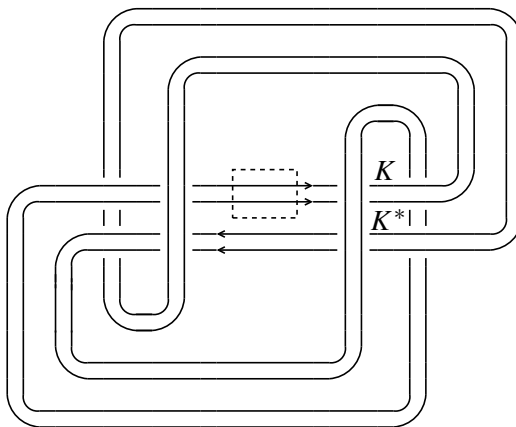
### 7. Noninvariance of knot Floer homology

This section gives an example of a knot  $J$  with Seifert surface  $F$  and a knot  $J'$  with Seifert surface  $F'$  such that  $F$  and  $F'$  are concordant, but  $J$  and  $J'$  have different knot Floer homology.

$J$  is the trefoil knot and  $F$  a genus one Seifert surface.  $J'$  is a certain twisted double of a copy  $K$  of the stevedore’s knot  $6_1$  and  $F'$  is a genus one surface contained in a solid torus  $V$  whose core is  $K$ .

Figure 12 shows two projections of  $K$ . The second will be used since it more clearly displays the fact that  $K$  is a ribbon knot. See [Mizuma 2005] for an isotopy between them.

Figure 13 shows an annulus  $A$  embedded in  $S^3$  with one boundary component being  $K$ . The orientations of  $K$  and the other boundary component  $K^*$  are chosen so that the two curves are homologous in  $A$ . From the diagram one computes that the linking number  $\text{lk}(K, K^*) = 0$ . Let  $\tilde{K} = K \cup K^*$ .



**Figure 13.** A 2-strand cable link  $\tilde{K} = K \cup K^*$  of  $K = 6_1$ .

The knot  $J'$  is obtained by reversing the orientation on  $K^*$  to get a new oriented link  $\widehat{K} = K \cup (-K^*)$  and replacing the trivial tangle in the box in Figure 13 by



This results in the annulus being replaced by a genus one surface  $F'$ . A saddle move on the new diagram followed by a pair of 2-handle additions shows that  $F$  and  $F'$  are concordant.

By a result of Ni [2007], knot Floer homology detects fibered knots.  $J$  is fibered. If  $J'$  were fibered then its companion  $K$  would also be fibered (Proposition 9.11 of [Myers 1980]), but the Alexander polynomial  $2t^{-1} - 5 + 2t$  of  $K$  is not monic, and so  $K$  is not fibered (see, e.g., [Rolfsen 1976, Corollary 10.8]); thus  $J$  is not fibered, and so the knot Floer homologies of  $J$  and  $J'$  must be different.

### Appendix: Pumping up the volume

There are several results in the literature to the effect that a topologically complicated hyperbolic 3-manifold has high volume. See, for example, [Lackenby 2004; Purcell 2007; Shalen 2007].

One measure of the complexity of a compact 3-manifold  $M$  is the *Haken number*  $h(M)$  [Haken 1968; Jaco 1980], the maximum number of compact, connected, properly embedded, incompressible, boundary incompressible, pairwise nonparallel surfaces in  $M$ . Call the union of such a maximal collection of surfaces a *Haken system* for  $M$ .

**Theorem A.1.** *Let  $M_n$  be a sequence of compact, connected, orientable 3-manifolds with complete, finite volume, hyperbolic interiors  $N_n$ .*

$$\text{If } \lim_{n \rightarrow \infty} h(M_n) = \infty, \text{ then } \lim_{n \rightarrow \infty} \text{Vol}(N_n) = \infty.$$

*Proof.* If not, then by passing to a subsequence we may assume that  $\text{Vol}(M_n)$  is bounded above by a positive constant  $V$ . It then follows from Jørgensen’s theorem [Thurston 1980, Theorem 5.12.1, p. 119] that there is a finite set  $\{X_1, \dots, X_r\}$  of compact, connected, orientable 3-manifolds with finite volume, hyperbolic interiors such that for each  $M_n$  there is an  $X_j$  such that  $M_n$  is homeomorphic to the result of Dehn filling along some of the components of  $\partial X_j$ . Let  $H$  be the maximum of the  $h(X_j)$ . Choose an  $n$  such that  $h(M_n) > H$ . By the following lemma, which is stated in greater generality than needed here, we have  $h(M_n) \leq H$ .  $\square$

**Lemma A.2.** *Let  $Q$  and  $Q^*$  be compact, connected, orientable, irreducible,  $\partial$ -irreducible 3-manifolds. Suppose  $Q$  is obtained by Dehn filling along some of the boundary components of  $Q^*$ . Then  $h(Q) \leq h(Q^*)$ .*

*Proof.* Let  $V = V_1 \cup \dots \cup V_p$  be the union of the solid tori attached to  $Q^*$  in order to get  $Q$ . Let  $S = S_1 \cup \dots \cup S_{h(Q)}$  be a Haken system for  $Q$ . Isotop  $S$  so that it

meets  $V$  in a collection of meridinal disks and the number of such disks is minimal. Let  $S^* = S \cap Q^*$ . Then  $S^*$  is properly embedded in  $Q^*$  and has  $h(Q)$  components. It suffices to show that they are incompressible,  $\partial$ -incompressible, and pairwise nonparallel in  $Q^*$ .

*$S^*$  is incompressible in  $Q^*$ :* Suppose  $D^*$  is a compressing disk for  $S^*$  in  $Q^*$ . Then  $\partial D^* = \partial D$  for a disk  $D$  in  $S$ , and  $D \cup D^*$  bounds a 3-ball  $B$  in  $Q$ . Isotoping  $D$  across  $B$  and off  $D^*$  reduces the number of components of  $S \cap V$ , contradicting minimality.

*$S^*$  is  $\partial$ -incompressible in  $Q^*$ :* Suppose  $\Delta$  is a  $\partial$ -compressing bigon for  $S^*$  in  $Q^*$ . Then  $\partial\Delta = \alpha \cup \beta$ , where  $\alpha$  is a properly embedded arc in  $S^*$  and  $\beta$  is a properly embedded spanning arc in an annulus  $A$  in  $\partial V$  such that  $\partial A = \partial D_0 \cup \partial D_1$ , where  $D_0$  and  $D_1$  are components of  $S \cap V$ . Let  $E$  be the 3-ball in  $V$  bounded by  $A \cup D_0 \cup D_1$ . A regular neighborhood of  $\Delta \cup E$  in  $Q$  is a 3-ball across which one can isotop  $S$  to remove  $D_0$  and  $D_1$  from  $S \cap V$ , again contradicting minimality. (Alternatively, one can show from this configuration that  $S^*$  is compressible in  $Q^*$ .)

*The components of  $S^*$  are pairwise nonparallel in  $Q^*$ :* Suppose components  $S_0^*$  and  $S_1^*$  are parallel in  $Q^*$ . These surfaces are the intersections with  $Q^*$  of components  $S_0$  and  $S_1$  of  $S$ . There is an embedding of  $W^* = S_0^* \times [0, 1]$  in  $Q^*$  with  $S_0^* = S_0^* \times \{0\}$ ,  $S_1^* = S_0^* \times \{1\}$ , and  $\partial S_0^* \times [0, 1]$  contained in  $\partial Q^*$ . Each component  $A$  of  $W^* \cap \partial V$  is an annulus for which there exists a 3-ball  $B$  in  $V$  with  $A = B \cap W^*$  such that the closure of  $\partial B - A$  consists of components of  $S \cap V$ . These  $B$  allow one to extend the product structure on  $W^*$  to a product structure  $W = S_0 \times [0, 1]$  in  $Q$  with  $S_0 \times \{0\} = S_0$ ,  $S_0 \times \{1\} = S_1$ , and  $\partial S_0 \times [0, 1]$  contained in  $\partial Q$ , contradicting the fact that  $S$  is a Haken system in  $Q$ . □

**Corollary A.3.** *Let  $Y_n$  be a sequence of compact, connected, orientable 3-manifolds such that the complement  $U_n$  of the torus boundary components of  $Y_n$  is a finite volume hyperbolic manifold with totally geodesic boundary.*

$$\text{If } \lim_{n \rightarrow \infty} h(Y_n) = \infty, \text{ then } \lim_{n \rightarrow \infty} \text{Vol}(U_n) = \infty.$$

*Proof.* Let  $M_n$  be the double of  $Y_n$  along the union  $F$  of its nontorus boundary components. Suppose  $S$  is a Haken system in  $Y_n$ . Let  $\widehat{S}$  be the double of  $S$  along  $S \cap F$ . The incompressibility of  $\widehat{S}$  in  $M_n$  follows from the incompressibility of  $F$  and  $S$  in  $Y_n$  and the  $\partial$ -incompressibility of  $S$  in  $Y_n$ . The fact that  $\partial M_n$  consists of tori then implies that  $\widehat{S}$  is  $\partial$ -incompressible in  $M_n$ . Suppose two components  $\widehat{S}_0$  and  $\widehat{S}_1$  of  $\widehat{S}$  are parallel in  $M_n$  via a product  $\widehat{S}_0 \times [0, 1]$ . This product is invariant under the involution which interchanges the two copies of  $Y_n$ . By [Kim and Tollefson 1977, Theorem A] the restriction of the involution is equivalent to a product involution. Hence the fixed point set consists of annuli. By [Waldhausen 1968, Lemma 3.4] they are isotopic to product annuli. It follows that  $S_0$  and  $S_1$  are

parallel in  $Y_n$ . Thus  $h(M_n) \geq h(Y_n)$ . The result then follows from Theorem A.1 and the fact that the interior  $N_n$  of  $M_n$  is a complete, finite volume hyperbolic 3-manifold with  $\text{Vol}(N_n) = 2 \text{Vol}(U_n)$ .  $\square$

## References

- [Casson and Gordon 1978] A. J. Casson and C. M. Gordon, “On slice knots in dimension three”, pp. 39–53 in *Algebraic and geometric topology, II* (Stanford, CA, 1976), edited by R. J. Milgram, Proc. Sympos. Pure Math. **32**, Amer. Math. Soc., Providence, RI, 1978. MR Zbl
- [Cochran and Orr 1998] T. D. Cochran and K. E. Orr, “Stability of lower central series of compact 3-manifold groups”, *Topology* **37**:3 (1998), 497–526. MR Zbl
- [Friedl 2009] S. Friedl, “Realizations of Seifert matrices by hyperbolic knots”, *J. Knot Theory Ramifications* **18**:11 (2009), 1471–1474. MR Zbl
- [Haken 1968] W. Haken, “Some results on surfaces in 3-manifolds”, pp. 39–98 in *Studies in modern topology*, edited by P. J. Hilton, Studies in Math. **5**, Math. Assoc. Amer., Washington, DC, 1968. MR Zbl
- [Jaco 1980] W. Jaco, *Lectures on three-manifold topology*, CBMS Regional Conference Series in Math. **43**, Amer. Math. Soc., Providence, RI, 1980. MR Zbl
- [Kawauchi 1989a] A. Kawauchi, “Almost identical imitations of (3, 1)-dimensional manifold pairs”, *Osaka J. Math.* **26**:4 (1989), 743–758. MR Zbl
- [Kawauchi 1989b] A. Kawauchi, “An imitation theory of manifolds”, *Osaka J. Math.* **26**:3 (1989), 447–464. MR Zbl
- [Kawauchi 1992] A. Kawauchi, “Almost identical link imitations and the skein polynomial”, pp. 465–476 in *Knots 90* (Osaka, 1990), edited by A. Kawauchi, de Gruyter, Berlin, 1992. MR Zbl
- [Kawauchi 1994] A. Kawauchi, “On coefficient polynomials of the skein polynomial of an oriented link”, *Kobe J. Math.* **11**:1 (1994), 49–68. MR Zbl
- [Kearton 1975] C. Kearton, “Blanchfield duality and simple knots”, *Trans. Amer. Math. Soc.* **202** (1975), 141–160. MR Zbl
- [Kearton 2004] C. Kearton, “S-equivalence of knots”, *J. Knot Theory Ramifications* **13**:6 (2004), 709–717. MR Zbl
- [Kim and Tollefson 1977] P. K. Kim and J. L. Tollefson, “PL involutions of fibered 3-manifolds”, *Trans. Amer. Math. Soc.* **232** (1977), 221–237. MR Zbl
- [Kirby and Lickorish 1979] R. C. Kirby and W. B. R. Lickorish, “Prime knots and concordance”, *Math. Proc. Cambridge Philos. Soc.* **86**:3 (1979), 437–441. MR Zbl
- [Lackenby 2004] M. Lackenby, “The volume of hyperbolic alternating link complements”, *Proc. London Math. Soc.* (3) **88**:1 (2004), 204–224. MR Zbl
- [Levine 1970] J. Levine, “An algebraic classification of some knots of codimension two”, *Comment. Math. Helv.* **45** (1970), 185–198. MR Zbl
- [Levine 1977] J. Levine, “Knot modules, I”, *Trans. Amer. Math. Soc.* **229** (1977), 1–50. MR Zbl
- [Lickorish 1997] W. B. R. Lickorish, *An introduction to knot theory*, Graduate Texts in Math. **175**, Springer, 1997. MR Zbl
- [Mizuma 2005] Y. Mizuma, “An isotopy from the Rolfsen diagram of  $6_1$  to a ribbon diagram”, 2005, Available at [http://katlas.math.toronto.edu/wiki/File:6\\_1\\_Ribbon\\_Sequence.jpg](http://katlas.math.toronto.edu/wiki/File:6_1_Ribbon_Sequence.jpg).

- [Morgan 1984] J. W. Morgan, “On Thurston’s uniformization theorem for three-dimensional manifolds”, pp. 37–125 in *The Smith conjecture* (New York, 1979), edited by J. W. Morgan and H. Bass, Pure Appl. Math. **112**, Academic Press, Orlando, 1984. MR Zbl
- [Myers 1980] R. Myers, “Companionship of knots and the Smith conjecture”, *Trans. Amer. Math. Soc.* **259**:1 (1980), 1–32. MR Zbl
- [Myers 1983] R. Myers, “Homology cobordisms, link concordances, and hyperbolic 3-manifolds”, *Trans. Amer. Math. Soc.* **278**:1 (1983), 271–288. MR Zbl
- [Myers 1993] R. Myers, “Excellent 1-manifolds in compact 3-manifolds”, *Topology Appl.* **49**:2 (1993), 115–127. MR Zbl
- [Ni 2007] Y. Ni, “Knot Floer homology detects fibred knots”, *Invent. Math.* **170**:3 (2007), 577–608. MR Zbl
- [Purcell 2007] J. S. Purcell, “Volumes of highly twisted knots and links”, *Algebr. Geom. Topol.* **7** (2007), 93–108. MR Zbl
- [Rolfsen 1976] D. Rolfsen, *Knots and links*, Math. Lecture Series **7**, Publish or Perish, Berkeley, 1976. MR Zbl
- [Shalen 2007] P. B. Shalen, “Hyperbolic volume, Heegaard genus and ranks of groups”, pp. 335–349 in *Workshop on Heegaard splittings* (Haifa, 2005), edited by C. Gordon and Y. Moriah, Geom. Topol. Monogr. **12**, Geom. Topol., Coventry, 2007. MR Zbl
- [Silver and Whitten 2005] D. S. Silver and W. Whitten, “Hyperbolic covering knots”, *Algebr. Geom. Topol.* **5** (2005), 1451–1469. MR Zbl
- [Silver and Whitten 2006] D. S. Silver and W. Whitten, “Knot group epimorphisms”, *J. Knot Theory Ramifications* **15**:2 (2006), 153–166. MR Zbl
- [Thurston 1980] W. Thurston, “The geometry and topology of three-manifolds”, lecture notes, Princeton University, 1980, Available at <http://library.msri.org/books/gt3m/>.
- [Trotter 1973] H. F. Trotter, “On  $S$ -equivalence of Seifert matrices”, *Invent. Math.* **20** (1973), 173–207. MR Zbl
- [Waldhausen 1968] F. Waldhausen, “On irreducible 3-manifolds which are sufficiently large”, *Ann. of Math. (2)* **87** (1968), 56–88. MR Zbl

Received January 4, 2017. Revised May 18, 2018.

ROBERT MYERS  
DEPARTMENT OF MATHEMATICS  
OKLAHOMA STATE UNIVERSITY  
STILLWATER  
UNITED STATES  
[myersr@math.okstate.edu](mailto:myersr@math.okstate.edu)

# PACIFIC JOURNAL OF MATHEMATICS

Founded in 1951 by E. F. Beckenbach (1906–1982) and F. Wolf (1904–1989)

[msp.org/pjm](http://msp.org/pjm)

## EDITORS

Don Blasius (Managing Editor)  
Department of Mathematics  
University of California  
Los Angeles, CA 90095-1555  
[blasius@math.ucla.edu](mailto:blasius@math.ucla.edu)

Matthias Aschenbrenner  
Department of Mathematics  
University of California  
Santa Barbara, CA 93106-3080  
[matthias@math.ucla.edu](mailto:matthias@math.ucla.edu)

Daryl Cooper  
Department of Mathematics  
University of California  
Santa Barbara, CA 93106-3080  
[cooper@math.ucsb.edu](mailto:cooper@math.ucsb.edu)

Jiang-Hua Lu  
Department of Mathematics  
The University of Hong Kong  
Pokfulam Rd., Hong Kong  
[jhlu@maths.hku.hk](mailto:jhlu@maths.hku.hk)

Paul Balmer  
Department of Mathematics  
University of California  
Los Angeles, CA 90095-1555  
[balmer@math.ucla.edu](mailto:balmer@math.ucla.edu)

Wee Teck Gan  
Mathematics Department  
National University of Singapore  
Singapore 119076  
[matgwt@nus.edu.sg](mailto:matgwt@nus.edu.sg)

Sorin Popa  
Department of Mathematics  
University of California  
Los Angeles, CA 90095-1555  
[popa@math.ucla.edu](mailto:popa@math.ucla.edu)

Paul Yang  
Department of Mathematics  
Princeton University  
Princeton NJ 08544-1000  
[yang@math.princeton.edu](mailto:yang@math.princeton.edu)

Vyjayanthi Chari  
Department of Mathematics  
University of California  
Riverside, CA 92521-0135  
[chari@math.ucr.edu](mailto:chari@math.ucr.edu)

Kefeng Liu  
Department of Mathematics  
University of California  
Los Angeles, CA 90095-1555  
[liu@math.ucla.edu](mailto:liu@math.ucla.edu)

Jie Qing  
Department of Mathematics  
University of California  
Santa Cruz, CA 95064  
[qing@cats.ucsc.edu](mailto:qing@cats.ucsc.edu)

## PRODUCTION

Silvio Levy, Scientific Editor, [production@msp.org](mailto:production@msp.org)

## SUPPORTING INSTITUTIONS

ACADEMIA SINICA, TAIPEI  
CALIFORNIA INST. OF TECHNOLOGY  
INST. DE MATEMÁTICA PURA E APLICADA  
KEIO UNIVERSITY  
MATH. SCIENCES RESEARCH INSTITUTE  
NEW MEXICO STATE UNIV.  
OREGON STATE UNIV.

STANFORD UNIVERSITY  
UNIV. OF BRITISH COLUMBIA  
UNIV. OF CALIFORNIA, BERKELEY  
UNIV. OF CALIFORNIA, DAVIS  
UNIV. OF CALIFORNIA, LOS ANGELES  
UNIV. OF CALIFORNIA, RIVERSIDE  
UNIV. OF CALIFORNIA, SAN DIEGO  
UNIV. OF CALIF., SANTA BARBARA

UNIV. OF CALIF., SANTA CRUZ  
UNIV. OF MONTANA  
UNIV. OF OREGON  
UNIV. OF SOUTHERN CALIFORNIA  
UNIV. OF UTAH  
UNIV. OF WASHINGTON  
WASHINGTON STATE UNIVERSITY

These supporting institutions contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its contents or policies.

---

See inside back cover or [msp.org/pjm](http://msp.org/pjm) for submission instructions.

---

The subscription price for 2019 is US \$490/year for the electronic version, and \$665/year for print and electronic. Subscriptions, requests for back issues and changes of subscriber address should be sent to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163, U.S.A. The Pacific Journal of Mathematics is indexed by Mathematical Reviews, Zentralblatt MATH, PASCAL CNRS Index, Referativnyi Zhurnal, Current Mathematical Publications and Web of Knowledge (Science Citation Index).


---

The Pacific Journal of Mathematics (ISSN 0030-8730) at the University of California, c/o Department of Mathematics, 798 Evans Hall #3840, Berkeley, CA 94720-3840, is published twelve times a year. Periodical rate postage paid at Berkeley, CA 94704, and additional mailing offices. POSTMASTER: send address changes to Pacific Journal of Mathematics, P.O. Box 4163, Berkeley, CA 94704-0163.

---

PJM peer review and production are managed by EditFLOW® from Mathematical Sciences Publishers.

PUBLISHED BY

 **mathematical sciences publishers**  
nonprofit scientific publishing

<http://msp.org/>

© 2019 Mathematical Sciences Publishers

# PACIFIC JOURNAL OF MATHEMATICS

Volume 298 No. 2 February 2019

---

Uniqueness questions for $C^*$ -norms on group rings VADIM ALEKSEEV and DAVID KYED	257
Expected depth of random walks on groups KHALID BOU-RABEE, IOAN MANOLESCU and AGLAIA MYROPOLSKA	267
Signature ranks of units in cyclotomic extensions of abelian number fields DAVID S. DUMMIT, EVAN P. DUMMIT and HERSHY KISILEVSKY	285
Semistable deformation rings in even Hodge–Tate weights LUCIO GUERBEROFF and CHOL PARK	299
Nonholomorphic Lefschetz fibrations with $(-1)$ -sections NORIYUKI HAMADA, RYOMA KOBAYASHI and NAOYUKI MONDEN	375
Tilting modules over Auslander–Gorenstein algebras OSAMU IYAMA and XIAOJIN ZHANG	399
Maximal symmetry and unimodular solvmanifolds MICHAEL JABLONSKI	417
Concordance of Seifert surfaces ROBERT MYERS	429
Resolutions for twisted tensor products ANNE SHEPLER and SARAH WITHERSPOON	445
Iterated automorphism orbits of bounded convex domains in $\mathbb{C}^n$ JOSHUA STRONG	471
Sharp logarithmic Sobolev inequalities along an extended Ricci flow and applications GUOQIANG WU and YU ZHENG	483



0030-8730(201902)298:2;1-Q