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**τ -TILTING FINITE GENTLE ALGEBRAS
ARE REPRESENTATION-FINITE**

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We show that a gentle algebra over a field is τ -tilting finite if and only if it is representation-finite. The proof relies on the “brick- τ -tilting correspondence” of Demonet, Iyama, and Jasso and on a combinatorial analysis.

1. Introduction and main result

The theory of τ -tilting was introduced in [Adachi et al. 2014] as a far-reaching generalization of classical tilting theory for finite-dimensional associative algebras. One of the main classes of objects in the theory is that of τ -rigid modules: a module M over an algebra Λ is τ -rigid if the space of morphisms $\text{Hom}_{\Lambda}(M, \tau M)$ vanishes, where τ is the Auslander–Reiten translation. In [Demonet et al. 2019], conditions were established for an algebra Λ to admit only finitely many isomorphism classes of indecomposable τ -rigid modules. Such an algebra is called τ -tilting finite.

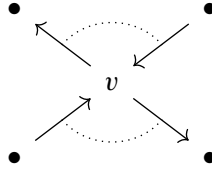
An obvious sufficient condition for an algebra to be τ -tilting finite is for it to be representation-finite. This is not a necessary condition: for instance, if k is a field, then the algebra $k\langle x, y \rangle / (x^2, y^2, xy, yx)$ is representation-infinite (since it is a string algebra in the sense of [Butler and Ringel 1987] and admits at least one band, namely xy^{-1}), but it is τ -tilting finite (since it is local). Our aim in this note is to prove that, for a certain class of algebras called *gentle algebras*, representation-finiteness and τ -tilting finiteness are equivalent conditions.

Gentle algebras form a subclass of the class of string algebras. They enjoy a simple definition in terms of generators and relations: a gentle algebra is a finite-dimensional algebra isomorphic to a quotient of a path algebra of a finite quiver Q by an ideal I generated by paths of length two, satisfying the condition that for every vertex v of Q , the minimal full subquiver with relation of $\bar{Q} = (Q, I)$ containing v and all arrows attached to v is a full subquiver with relations of the one depicted below, where dotted lines indicate relations and where the $\bullet$ vertices may not all be distinct:

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Despite their simple definition, gentle algebras are encountered in many different contexts. They were introduced in [Assem and Skowroński 1987] in the study of iterated tilted algebras of type $\tilde{A}_m$, but have recently appeared in connection with dimer models [Bocklandt 2012; Broomhead 2012], enveloping algebras of some Lie algebras [Huerfano and Khovanov 2006], cluster algebras and categories arising from triangulated surfaces [Labardini-Fragoso 2009; Assem et al. 2010], m -Calabi–Yau tilted algebras [García Elsener 2017; 2018], nonkissing complexes of grids and associated objects [McConville 2017; Garver and McConville 2018; Palu et al. 2017; Brüstle et al. 2019], noncommutative nodal curves [Burban and Drozd 2018], and partially wrapped Fukaya categories [Haiden et al. 2017; Lekili and Polishchuk 2019]. Surface models have been introduced to study the category representations of a gentle algebra and associated categories [Baur and Simões 2018; Oppen et al. 2018; Palu et al. 2018].

In this note, we prove the following theorem on gentle algebras. It is proved for Schurian gentle algebras in [Demonet et al. ≥ 2019] (see also [Demonet 2017]).

Theorem 1.1. *A gentle algebra is τ -tilting finite if and only if it is representation-finite.*

The proof of the theorem uses the “brick- τ -tilting correspondence” of [Demonet et al. 2019] (recalled in Section 2), and applies a reduction of any gentle algebra to two classes of examples (studied in Section 3).

Conventions and a note on terminology. All algebras in this paper are finite-dimensional over a base field k , which is arbitrary. We compose arrows in quivers from left to right: if $1 \xrightarrow{\alpha} 2 \xrightarrow{\beta} 3$ is a quiver, then $\alpha\beta$ is a path, while $\beta\alpha$ is not. For any arrow α of a quiver, we denote its source by $s(\alpha)$ and its target by $t(\alpha)$; we extend this notation naturally to the formal inverse α^{-1} of α .

In order to keep this note short, the notions of strings, bands, and string modules from [Butler and Ringel 1987] will be used freely and without further introduction. We note that the above convention for composition of arrows differs from the one used in [Butler and Ringel 1987].

2. The “brick- τ -tilting correspondence” and two reduction results

We will be using two results, Corollaries 2.3 and 2.4, allowing us to reduce the number of vertices of our bound quivers. They will both follow from the following

consequence of the *brick- τ -tilting correspondence*. Recall that a *brick* is a module whose endomorphism algebra is a division algebra.

Theorem 2.1 [Demonet et al. 2019, Theorem 1.4]. *A finite-dimensional algebra Λ is τ -tilting finite if and only if there are only finitely many isomorphism classes of Λ -modules which are bricks.*

Corollary 2.2. *Let Λ and Λ' be two finite-dimensional algebras, and assume that there exists a fully faithful functor $F : \text{mod } \Lambda \rightarrow \text{mod } \Lambda'$. If Λ' is τ -tilting finite, then so is Λ .*

Proof. If B is a brick over Λ , then FB is a brick over Λ' , since $\text{End}_\Lambda(B)$ is isomorphic to $\text{End}_{\Lambda'}(FB)$. Moreover, two bricks B and B' over Λ are isomorphic if and only if FB and FB' are isomorphic. Therefore, if Λ admits infinitely many bricks, then so does Λ' . The result then follows from Theorem 2.1. $\square$

Corollary 2.3 (first reduction [Demonet et al. 2017, Theorem 5.12(d)]). *If Λ is τ -tilting finite and I is an ideal in Λ , then Λ/I is τ -tilting finite.*

Proof. Apply Corollary 2.2 to $\cdot \otimes_{\Lambda/I} \Lambda/I : \text{mod } \Lambda/I \rightarrow \text{mod } \Lambda$. $\square$

Corollary 2.4 (second reduction). *If Λ is τ -tilting finite and $e \in \Lambda$ is an idempotent, then $e\Lambda e$ is τ -tilting finite.*

Proof. Apply Corollary 2.2 to $\text{Hom}_{e\Lambda e}(\Lambda e, \cdot) : \text{mod } e\Lambda e \rightarrow \text{mod } \Lambda$. Alternatively, apply [Pilaud et al. 2018, Theorem 1.1] for a proof which does not use the brick- τ -tilting correspondence. $\square$

Remark 2.5. In practice, Corollary 2.3 implies that erasing arrows or vertices from a τ -tilting finite algebra yields another τ -tilting finite algebra, and Corollary 2.4 implies that erasing a vertex v and replacing any path of length two without relation of the form

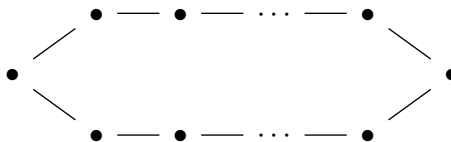
$$u \xrightarrow{\alpha} v \xrightarrow{\beta} w$$

by a “shortcut” arrow $u \xrightarrow{[\alpha\beta]} w$ also preserves τ -tilting finiteness.

3. Two classes of examples

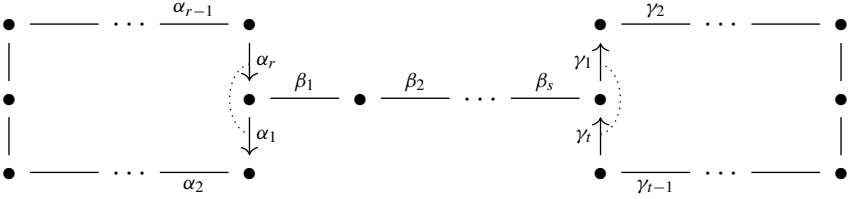
Our strategy to prove Theorem 1.1 will be to reduce any gentle algebra of infinite-representation type to one of the two classes of examples presented in this section.

Example 3.1 (type $\tilde{A}_m$). Let Q be a quiver of type $\tilde{A}_m$, that is to say, an orientation of the following diagram with $m+1$ vertices, where $m \geq 1$:



The representation theory of the path algebra $\Lambda = kQ$ is very well understood in this case; see for instance [Assem et al. 2006, §VIII.2]. In particular, this algebra is τ -tilting infinite.

Example 3.2. The second class of examples that we will consider will be given by the path algebras of quivers Q defined by any orientation of the diagram



modulo the relations $\alpha_r \alpha_1$ and $\gamma_t \gamma_1$ (note that the orientations of α_1 , α_r , γ_1 , and γ_t are imposed, while those of other arrows can be arbitrary). We allow $r = 1$ (in which case $\alpha_1 = \alpha_r$ is a loop whose square vanishes); we allow the same for t . We also allow $s = 0$; in this case, we require that the cycle on the left and the cycle on the right are not both oriented cycles; otherwise, the algebra would be infinite-dimensional.

Proposition 3.3. *The algebras defined in Example 3.2 are τ -tilting infinite.*

Proof. Let Λ be an algebra in the class defined in Example 3.2.

We first deal with the case where $s = 0$. In this case, let v be the vertex common to both cycles. Let $e = 1 - e_v$. Then the algebra $e\Lambda e$ is of type $\tilde{A}_m$, so it is τ -tilting infinite. By Corollary 2.4, Λ is τ -tilting infinite.

Assume, therefore, that $s \geq 1$. We will construct an infinite family of bricks for Λ ; by the brick- τ -tilting correspondence (see Theorem 2.1), this will imply that Λ is τ -tilting infinite.

Let $b' = \alpha_1 \alpha_2^{\delta_2} \cdots \alpha_{r-1}^{\delta_{r-1}} \alpha_r$ be the string corresponding to the cycle on the left, $b'' = \gamma_1 \gamma_2^{\zeta_2} \cdots \gamma_{t-1}^{\zeta_{t-1}} \gamma_t$ be the one corresponding to the cycle on the right, and $\omega = \beta_1^{\varepsilon_1} \cdots \beta_s^{\varepsilon_s}$ be the middle string followed from left to right, where the δ_i , ε_i , and ζ_i are the appropriate signs.

Define $b = (b')^{\varepsilon_1} \omega (b'')^{\varepsilon_1} \omega^{-1}$. We claim that the string module defined by b is a brick. To prove this, we need to prove that the only substring of b appearing both on top of and at the bottom of b is b itself, so that the endomorphism ring of the string module is isomorphic to the base field k (using the description of all morphisms between string modules obtained in [Crawley-Boevey 1989]). Here, we say that a substring σ' of a string σ is *on top of* σ if the arrows in σ adjacent to σ' are leaving σ' , and that it is *at the bottom of* σ if the arrows in σ adjacent to σ' are entering σ' .

Note first that the middle copy of ω is neither on top of nor at the bottom of b . Indeed, if $\varepsilon_1 = 1$, then ω is not at the bottom, since the first arrow of $(b'')^{\varepsilon_1}$ is direct, and ω is not on top, since the last arrow of $(b')^{\varepsilon_1}$ is direct; if $\varepsilon_1 = -1$, then ω is

not at the bottom, since the last arrow of $(b')^{\varepsilon_1}$ is inverse, and ω is not on top, since the last arrow of $(b'')^{\varepsilon_1}$ is inverse.

Next, let us deal with the substrings of length 0 of b . The starting point of b is on top if $\varepsilon_1 = 1$ or at the bottom if $\varepsilon_1 = -1$. It appears twice more in b : at the end of b' and the end of b . At the end of b' it is neither on top nor at the bottom, and at the end of b it is on top if $\varepsilon_1 = 1$ or at the bottom if $\varepsilon_1 = -1$. Therefore, this vertex does not occur both on top and at the bottom of b . The other vertices appearing several times in b are the vertices of ω and the starting/ending points of b'' . The former appear either twice at the top or twice at the bottom of b . The latter cannot be both on top and at the bottom, since it appears in the middle of paths of length two of the following form: if $\varepsilon_1 = 1$, then these paths are $\beta_s^{\varepsilon_s} \gamma_1$ and $\gamma_t \beta_s^{-\varepsilon_s}$, and if $\varepsilon_1 = -1$, then they are $\beta_s^{\varepsilon_s} \gamma_t^{-1}$ and $\gamma_1^{-1} \beta_s^{-\varepsilon_s}$. In both cases the middle vertices are either on top of b or at the bottom of b , but not both. Thus, no substring of length zero appears both at the top and the bottom of b .

Assume that there is a substring ρ of length at least one, different from b , which appears both on top and at the bottom of b . Since the only arrows of b that are used twice are those of ω , ρ has to be a substring of ω and of ω^{-1} . Since ω does not go twice through the same vertex, the only substring both on top and at the bottom of ω is ω itself. Hence, $\rho = \omega$. But we saw above that the middle substring ω is neither on top nor at the bottom of b . This is a contradiction.

Thus, the string module defined by b is a brick.

Using the above arguments, one can also check that all powers of b define bricks as well. Thus, Λ admits infinitely many pairwise nonisomorphic bricks, and by the brick- τ -tilting correspondence (see [Theorem 2.1](#)), it is τ -tilting infinite. $\square$

4. Proof of the main theorem

We now prove [Theorem 1.1](#). Let $\bar{Q} = (Q, I)$ be a gentle bound quiver. Assume that the algebra $\Lambda = kQ/I$ is of infinite representation type. Let us show that it is τ -tilting infinite.

By [[Butler and Ringel 1987](#)], there exists a band b on $\bar{Q}$. Our strategy will be to reduce to one of the two cases in the following lemma.

Lemma 4.1. *The algebra Λ is τ -tilting infinite if $\bar{Q}$ admits a band b satisfying one of the following conditions:*

- (1) *b does not go through the same vertex twice (except for the starting and ending points of b) or*
- (2) *b has the form $b = b' \omega b'' \omega^{-1}$, where*
 - *b' and b'' are strings such that $s(b') = t(b')$ and $s(b'') = t(b'')$,*
 - *$(b')^2$ and $(b'')^2$ are not strings,*

- ω is a possibly trivial string,
- none of b' , b'' , and ω go through the same vertex twice (except for the endpoints of b' and b''), and
- the only vertices that b' , b'' , and ω may have in common are their ending points.

Proof. Let I be the ideal generated by all arrows and vertices through which b does not go. If we are in case (1), then Λ/I is isomorphic to the path algebra of a quiver of type $\tilde{A}_m$. These algebras are τ -tilting infinite, so the result follows from [Corollary 2.3](#). If we are in case (2), then Λ/I is isomorphic to an algebra in the class defined in [Example 3.2](#). By [Proposition 3.3](#), these are τ -tilting infinite, so the result follows again from [Corollary 2.3](#). $\square$

To prove [Theorem 1.1](#), it is therefore sufficient to show that a band as in [Lemma 4.1](#) always exists.

Let b be a band of minimal length on $\bar{Q}$. If b does not go through the same vertex twice (except at its endpoints), then by [Lemma 4.1](#)(1), the theorem is proved.

Assume, therefore, that there is a vertex u through which b passes twice. Up to cyclic reordering of b , we can assume that this vertex is the starting point of b . Up to choosing another such vertex u , we can also assume that $b = b'b''$, with b' and b'' nontrivial strings starting and ending at u and such that b' does not go through the same vertex twice (except at its endpoints). Note that, by minimality of b , the string b' cannot be a band; that is, $(b')^2$ cannot be a string. The same is true for b'' .

Lemma 4.2. *Let $b = b'b''$ be as above. Then b' and b'' cannot have a vertex in common apart from their starting and ending points.*

Proof. Assume that b' and b'' have another vertex in common, and let v be such a vertex. Write $b' = \alpha_1^{\delta_1} \cdots \alpha_r^{\delta_r}$ and $b'' = \beta_1^{\varepsilon_1} \cdots \beta_s^{\varepsilon_s}$, where the α_i and β_i are arrows and the δ_i and ε_i are ± 1 , and assume that $s(\alpha_j^{\delta_j}) = v = t(\beta_k^{\varepsilon_k})$, with $i \neq 1, r$ and $j \neq 1, s$. By minimality of b , $\alpha_j^{\delta_j} \alpha_{j+1}^{\delta_{j+1}} \cdots \alpha_r^{\delta_r} \beta_1^{\varepsilon_1} \cdots \beta_k^{\varepsilon_k}$ cannot be a band. This implies that $\delta_j = \varepsilon_k$ and that the composition of $\alpha_j^{\delta_j}$ and $\beta_k^{\varepsilon_k}$ is a relation in $\bar{Q}$. But then $\alpha_{j-1}^{-\delta_{j-1}} \alpha_{j-2}^{-\delta_{j-2}} \cdots \alpha_1^{-\delta_1} \beta_1^{\varepsilon_1} \cdots \beta_k^{\varepsilon_k}$ is a band, since $\beta_k^{\varepsilon_k}$ cannot be in a relation both with $\alpha_j^{\delta_j}$ and with $\alpha_{j-1}^{\delta_{j-1}}$. This again contradicts the minimality of b . $\square$

Now, if b'' does not go twice through the same vertex (except for its endpoints), then we are in case (2) of [Lemma 4.1](#) with ω trivial, and the theorem is proved. Assume thus that b'' does go twice through a vertex v outside its endpoints (or three times through v if it is the endpoint of b''). Up to choosing another vertex v , we can assume that there is a substring $b''' = \beta_j^{\varepsilon_j} \beta_{j+1}^{\varepsilon_{j+1}} \cdots \beta_k^{\varepsilon_k}$ of b'' which starts and ends in v , does not go through the same vertex twice outside its endpoints, and is such that $\omega := \beta_1^{\varepsilon_1} \cdots \beta_{j-1}^{\varepsilon_{j-1}}$ does not go through the same vertex twice.

If β_k and β_{j-1} form a relation of $\bar{Q}$, then β_j and β_k cannot form a relation, since there is at most one involving β_k and β_{j-1} , β_j are distinct. Thus, b''' is a band with

no repeated vertices (except for its endpoints), and we have reduced to case (1) of [Lemma 4.1](#), proving the theorem.

If, on the other hand, β_k and β_{j-1} do not form a relation, then $b'\omega b'''\omega^{-1}$ is a band satisfying the conditions of case (2) of [Lemma 4.1](#).

This finishes the proof of [Theorem 1.1](#).

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An A_∞ version of the Poincaré lemma	385
CAMILO ARIAS ABAD, ALEXANDER QUINTERO VÉLEZ and SEBASTIÁN VÉLEZ VÁSQUEZ	
Wonderful compactification of character varieties	413
INDRANIL BISWAS, SEAN LAWTON and DANIEL RAMRAS	
What do Frobenius's, Solomon's, and Iwasaki's theorems on divisibility in groups have in common?	437
ELENA K. BRUSYANSKAYA, ANTON A. KLYACHKO and ANDREY V. VASIL'EV	
On homogeneous and inhomogeneous Diophantine approximation over the fields of formal power series	453
YANN BUGEAUD and ZHENLIANG ZHANG	
Torsion of rational elliptic curves over the maximal abelian extension of $\mathbb{Q}$	481
MICHAEL CHOU	
Local estimates for Hörmander's operators of first kind with analytic Gevrey coefficients and application to the regularity of their Gevrey vectors	511
MAKHLOUF DERRIDJ	
Boundedness of singular integrals with flag kernels on weighted flag Hardy spaces	545
YONGSHENG HAN, CHIN-CHENG LIN and XINFENG WU	
Exceptional sequences and spherical modules for the Auslander algebra of $k[x]/(x^t)$	599
LUTZ HILLE and DAVID PLOOG	
The topological biquandle of a link	627
EVA HORVAT	
An endpoint estimate of the Kunze–Stein phenomenon on $SU(n, m)$	645
TAKESHI KAWAZOE	
Mabuchi metrics and properness of the modified Ding functional	659
YAN LI and BIN ZHOU	
A generalization of Maloo's theorem on freeness of derivation modules	693
CLETO B. MIRANDA-NETO and THYAGO S. SOUZA	
τ -tilting finite gentle algebras are representation-finite	709
PIERRE-GUY PLAMONDON	
Yamabe equation on some complete noncompact manifolds	717
GUODONG WEI	
Weighted infinitesimal unitary bialgebras on rooted forests and weighted cocycles	741
YI ZHANG, DAN CHEN, XING GAO and YAN-FENG LUO	