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A HECKE ALGEBRA ISOMORPHISM
OVER CLOSE LOCAL FIELDS**

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The proof of Lemma 2.5 of the author’s article “A Hecke algebra isomorphism over close local fields” (*Pacific J. Math.* 319:2 (2022), 307–332) is incorrect. We use a slight variant of the original approach to correct the proof. This leads to some modifications to some parts of Section 3 of the original article, and these are given in Section 2 of this note. With these modifications, Theorem 4.1 of the original article holds.

We retain the notation in [Ganapathy 2022, Section 2]. Let T be a torus over F . Then T is determined by the Γ_F -module $X_*(T)$. Let \mathcal{T}^{ft} be the Néron–Raynaud model of T and \mathcal{T} its identity component. Let $m \geq 1$ be such that T splits over an at most m -ramified Galois extension of F . Then the action of Γ_F on $X_*(T)$ factors through Γ_F/I_F^m . For any field F' that is at least m -close to F , we obtain a torus T' over F' via the action of $\Gamma_{F'} \rightarrow \Gamma_{F'}/I_{F'}^m \xrightarrow{\cong} \Gamma_F/I_F^m$ on $X_*(T)$. This torus splits over an at most m -ramified extension of F' . Let \mathcal{T}'^{ft} be the Néron–Raynaud model of T' and \mathcal{T}' its identity component.

Theorem 0.1 [Chai and Yu 2001, Section 9]. *Let $m \geq 1$ and let h be as in [Chai and Yu 2001, Section 8]. Assume $e \geq m + 3h$. Then for any nonarchimedean local field F' that is e -close to F , the group schemes $\mathcal{T}^{\text{ft}} \times_{\mathfrak{O}_F} \mathfrak{O}_F/\mathfrak{p}_F^m$ and $\mathcal{T}'^{\text{ft}} \times_{\mathfrak{O}_{F'}} \mathfrak{O}_{F'}/\mathfrak{p}_{F'}^m$ are isomorphic. In particular,*

$$\mathcal{T}^{\text{ft}}(\mathfrak{O}_F/\mathfrak{p}_F^m) \cong \mathcal{T}'^{\text{ft}}(\mathfrak{O}_{F'}/\mathfrak{p}_{F'}^m)$$

as groups. This isomorphism continues to hold when we replace \mathcal{T}^{ft} by \mathcal{T} .

In [Ganapathy 2022, Section 2C], we had constructed a group-theoretic section of the Kottwitz homomorphism $\kappa_{T,F} : T(F) \rightarrow X_*(T)_{I_F}^\sigma$ and had used Theorem 0.1 for the neutral component \mathcal{T} to give a proof of Lemma 2.5 in the same article. If T splits over an unramified extension of F or is an induced torus over F , the results in [Ganapathy 2022, Section 2] go through. However, the Kottwitz homomorphism for

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a general torus need not admit a group-theoretic section, as the following example illustrates.

Example 0.2. Let \check{F} be the completion of the maximal unramified subextension of \mathbb{Q}_2 . Let $L = \check{F}(\sqrt{-1})$. Then L is a wildly ramified quadratic extension of \check{F} . Let $T = \text{Nm}_{L/\check{F}}^1 \mathbb{G}_m$ denote the norm-1 torus. Let γ be the nontrivial element of $\text{Gal}(L/\check{F})$. Then $X_*(T)_{I_F} \cong \mathbb{Z}/2\mathbb{Z}$. Note that $\kappa_{T,\check{F}}$ has a group-theoretic section if and only if $-1 \in T(\check{F})$ does not lie in $T(\check{F})_1$. Note that

$$T(\check{F})_1 = \{y \in L^\times \mid x\gamma(x)^{-1} = y \text{ for some } x \in L^\times\}.$$

Since $-1 = (\sqrt{-1})\gamma(\sqrt{-1})^{-1}$, -1 indeed lies in $T(\check{F})_1$. We conclude that $\kappa_{T,\check{F}}$ does not admit a group-theoretic section.

The error in [Ganapathy 2022, Section 2] is that Lemma 2.3 is false in general (the $n_{\check{\lambda}}$ defined in the line above Lemma 2.3 may not be well-defined). Consequently, Lemma 2.4 cannot be salvaged to yield a well-defined set of representatives for the torsion elements of $X_*(T)_{I_F}$ that forms a group and is σ -stable.

1. Proof of [Ganapathy 2022, Lemma 2.5]

Let T be a torus over F and let \check{F} be the splitting extension of $T_{\check{F}}$ in the completion of F_s . Fix a uniformizer $\varpi_{\check{F}}$ of \check{F} . Consider the Kottwitz homomorphism $\kappa_{T,\check{F}} : T(\check{F}) \rightarrow X^*(T)_{I_F}$. Let $X_*(T)_{I_F}/\text{tor}$ denote the quotient of $X_*(T)_{I_F}$ by its torsion subgroup. Note that $X_*(T)_{I_F}/\text{tor}$ is isomorphic to $\text{Hom}_{\mathbb{Z}}(X^*(T)^{I_F}, \mathbb{Z})$. This leads to the valuation homomorphism $\omega_{T,\check{F}} : T(\check{F}) \rightarrow \text{Hom}_{\mathbb{Z}}(X^*(T)^{I_F}, \mathbb{Z})$. Note that $\text{Ker}(\omega_{T,\check{F}}) = T(\check{F})_b = \mathcal{T}^{\text{ft}}(\mathcal{D}_{\check{F}})$ is the maximal bounded subgroup of $T(\check{F})$ and it contains $T(\check{F})_1$. We will construct a group-theoretic section of the valuation homomorphism. We will then use Theorem 0.1 for \mathcal{T}^{ft} to prove [Ganapathy 2022, Lemma 2.5] over \check{F} . We will show that this isomorphism over \check{F} is σ -equivariant to obtain the required isomorphism over F (see Lemmas 1.2 and 1.3).

1A. A group-theoretic section of the valuation homomorphism and its consequences. Let $\check{\lambda}_1, \dots, \check{\lambda}_n \in X_*(T)_{I_F}$ be such that their images $\check{\lambda}_1^t, \dots, \check{\lambda}_n^t$ form a basis of $X_*(T)_{I_F}/\text{tor}$. Fix $\tilde{\lambda}_1, \dots, \tilde{\lambda}_n \in X_*(T)$ such that $\text{pr}(\tilde{\lambda}_i) = \check{\lambda}_i$, where $\text{pr} : X_*(T) \rightarrow X_*(T)_{I_F}$ is the natural surjection. Define $n_{\check{\lambda}_i} = \tilde{\lambda}_i(\varpi_{\check{F}})$. Define $n_{\check{\lambda}_i}^t = n_{\check{\lambda}_i} = \text{Nm}_{\check{F}/\check{F}} n_{\check{\lambda}_i}$. For $\check{\lambda}^t \in X_*(T)_{I_F}/\text{tor}$, write $\check{\lambda}^t = \sum_i c_i \check{\lambda}_i^t$ and define $n_{\check{\lambda}^t} = \prod_i n_{\check{\lambda}_i}^{c_i}$. Note that $n_0 = 1$ by construction.

Lemma 1.1. *The set $\mathcal{S} := \{n_{\check{\lambda}^t} \mid \check{\lambda}^t \in X_*(T)_{I_F}/\text{tor}\}$ is a subgroup of $T(\check{F})$. The map $\nabla_{T,\check{F}} : X_*(T)_{I_F}/\text{tor} \rightarrow \mathcal{S}$, $\check{\lambda}^t \mapsto n_{\check{\lambda}^t}$, is a group isomorphism.*

Proof. It is clear that \mathcal{S} is a subgroup of $T(\check{F})$. It is also clear that $\nabla_{T,\check{F}}$ is a surjective group homomorphism. We just need to see that it is injective. Suppose $n_{\check{\lambda}^t} = 1$.

We need to show that $\check{\lambda}^t = 0$. Write $\check{\lambda}^t = \sum_i c_i \check{\lambda}_i^t$. The natural pairing between $X_*(T)$ and $X^*(T)$ induces a perfect pairing $\langle \cdot, \cdot \rangle : X_*(T)_{I_F}/\text{tor} \times X^*(T)^{I_F} \rightarrow \mathbb{Z}$. Let $\check{\lambda}_1, \dots, \check{\lambda}_n \in X^*(T)^{I_F}$ be such that $\langle \check{\lambda}_j, \check{\lambda}_k \rangle = \delta_{j,k}$, $1 \leq j, k \leq n$. Now $n_{\check{\lambda}^t} = \prod_i \text{Nm}_{\check{F}/\check{F}} n_{\check{\lambda}_i}^{c_i} = 1$. This implies that $1 = \check{\chi}_j(n_{\check{\lambda}^t}) = \text{Nm}_{\check{F}/\check{F}} \check{\chi}_j(n_{\check{\lambda}_j})^{c_j} = (\text{Nm}_{\check{F}/\check{F}} \varpi_{\check{F}}^{c_j})$. This forces $c_j = 0$. Since j was arbitrary, this shows that $\check{\lambda}^t = 0$. \square

Lemma 1.2. *Let T be a torus over F . Let \mathcal{T}^{ft} be as above and for $m \geq 1$, let $\check{T}_m = \text{Ker}(\mathcal{T}^{\text{ft}}(\mathfrak{O}_{\check{F}}) \rightarrow \mathcal{T}^{\text{ft}}(\mathfrak{O}_{\check{F}}/\mathfrak{p}_{\check{F}}^m))$. Let $e \geq m + 4h$. If \check{F} and \check{F}' are e -close, we have an isomorphism*

$$\check{\mathcal{T}}_m : T(\check{F})/\check{T}_m \rightarrow T'(\check{F}')/\check{T}'_m.$$

Proof. Since $\mathcal{T}^{\text{ft}}(\mathfrak{O}_{\check{F}}) = T(\check{F})_b$, we have by Theorem 0.1 (which also holds over \check{F} ; see [Chai and Yu 2001]) an isomorphism

$$(1-1) \quad T(\check{F})_b/\check{T}_m \rightarrow T'(\check{F}')_b/\check{T}'_m.$$

Since T splits over an at most m -ramified extension of F , the action of Γ_F on $X_*(T)$ factors through Γ_F/I_F^m . Since the action of Γ_F/I_F^m on $X_*(T)$ is Del_m -equivariant, we have $X_*(T)_{I_F} \cong X_*(T)_{I_{F'}}$ and $X_*(T)_{I_F}/\text{tor} \cong X_*(T)_{I_{F'}/\text{tor}}$ via Del_m . We identify these groups via these isomorphisms. Let $\varpi_{\check{F}'}$ be a uniformizer of \check{F}' such that $\varpi_{\check{F}} \bmod \mathfrak{p}_{\check{F}}^{rm} \mapsto \varpi_{\check{F}'} \bmod \mathfrak{p}_{\check{F}'}^{rm}$ where $r = [\check{F} : \check{F}']$. For $1 \leq i \leq n$, define $n'_{\check{\lambda}_i} = \tilde{\lambda}_i(\varpi_{\check{F}'})$, $n'_{\check{\lambda}_i} = \text{Nm}_{\check{F}'/\check{F}} n_{\check{\lambda}_i}$. Form the subgroup $\mathcal{S}' \subset T(\check{F}')$ as before. Since $\nabla_{T, \check{F}}, \nabla_{T', \check{F}'}$ are group isomorphisms, we get

$$T(\check{F})/\check{T}_m \cong X_*(T)_{I_F}/\text{tor} \times T(\check{F})_b/\check{T}_m,$$

and similarly over \check{F}' . These observations, combined with (1-1), finish the proof of the lemma. \square

Lemma 1.3. *The isomorphism $\check{\mathcal{T}}_m : T(\check{F})/\check{T}_m \rightarrow T'(\check{F}')/\check{T}'_m$ of Lemma 1.2 is σ -equivariant. It induces a group isomorphism $\mathcal{T}_m : T(F)/T_m \rightarrow T'(F')/T'_m$.*

Proof. We know that the isomorphism in (1-1) is σ -equivariant. We need to see that for $\check{\lambda}^t \in \mathcal{S}$, $\sigma(n_{\check{\lambda}}) \bmod \check{T}_m \mapsto \sigma'(n'_{\check{\lambda}}) \bmod \check{T}'_m$. It suffices to see this for $\check{\lambda}_i^t$, $1 \leq i \leq n$. Fix i and let $\check{\lambda}^t = \check{\lambda}_i^t$. Write

$$(1-2) \quad \sigma(\check{\lambda}^t) = \sum_j c_j \check{\lambda}_j^t.$$

Let $\tilde{\sigma}$ be any lift of σ to Γ_F/I_F^m and we denote its action on $X_*(T)$ as $\tilde{\sigma}$. We know

$$\sigma(n_{\check{\lambda}_i}) = \text{Nm}_{\check{F}/\check{F}} \tilde{\sigma}(n_{\check{\lambda}}) = \text{Nm}_{\check{F}/\check{F}} \tilde{\sigma}(\tilde{\lambda})(\tilde{\sigma}(\varpi_{\check{F}}))$$

and

$$n_{\sigma(\check{\lambda}^t)} = \prod_j \text{Nm}_{\check{F}/\check{F}} n_{\check{\lambda}_j}^{c_j} = \prod_j \text{Nm}_{\check{F}/\check{F}} \tilde{\lambda}_j(\varpi_{\check{F}})^{c_j}.$$

Equation (1-2) implies that $\tilde{\sigma}(\tilde{\lambda}) - \sum_j c_j \tilde{\lambda}_j \in X_*(T)(I_F)$, so

$$\tilde{\sigma}(\tilde{\lambda}) - \sum_j c_j \tilde{\lambda}_j = \sum_k d_k (\gamma_k(\tilde{\mu}_k) - \tilde{\mu}_k),$$

for suitable $\gamma_k \in I_F/I_F^m$ and $\tilde{\mu}_k \in X_*(T)$. Now,

$$\tilde{\sigma}(\tilde{\lambda})(\tilde{\sigma}(\varpi_{\tilde{F}})) = \prod_j \tilde{\lambda}_j (\tilde{\sigma}(\varpi_{\tilde{F}})^{c_j}) \cdot \prod_k (\gamma_k(\mu_k) - \mu_k) (\tilde{\sigma}(\varpi_{\tilde{F}})^{d_k})$$

Define

$$u_{\tilde{\lambda}, \tilde{\sigma}} = \prod_j \tilde{\lambda}_j ((\tilde{\sigma}(\varpi_{\tilde{F}})\varpi_{\tilde{F}}^{-1})^{c_j}) \prod_k \mu_k (\gamma_k^{-1}(\tilde{\sigma}(\varpi_{\tilde{F}}))(\tilde{\sigma}(\varpi_{\tilde{F}})^{-1})^{d_k})$$

and define $u_{\tilde{\lambda}, \sigma} = \text{Nm}_{\tilde{F}/\check{F}} u_{\tilde{\lambda}, \tilde{\sigma}}$. Then we have $\sigma(n_{\check{\lambda}^t}) = u_{\tilde{\lambda}, \sigma} \cdot n_{\sigma(\check{\lambda}^t)}$.

By construction of $\check{\mathcal{T}}_m$, we have $n_{\sigma(\check{\lambda}^t)} \bmod \check{T}_m \mapsto n'_{\sigma(\check{\lambda}^t)} \bmod \check{T}'_m$. Further $u_{\tilde{\lambda}, \tilde{\sigma}} \in T(\tilde{F})_1$. Recall that $r = [\tilde{F} : \check{F}]$. With $\varpi_{\tilde{F}}$ and $\varpi_{\check{F}}$ as above, the map $X_*(T) \rightarrow T(\tilde{F})$, $\tilde{\lambda} \mapsto \tilde{\lambda}(\varpi_{\tilde{F}})$, is a group-theoretic section of the Kottwitz homomorphism over \tilde{F} , and using the Chai–Yu isomorphism $T(\tilde{F})_1/\tilde{T}_{rm} \cong T'(\tilde{F}')_1/\tilde{T}'_{rm}$ we obtain that

$$\check{\mathcal{T}}_{rm} : T(\tilde{F})/\tilde{T}_{rm} \cong T(\tilde{F}')/\tilde{T}'_{rm}$$

as groups. Since under the isomorphism $\mathfrak{O}_{\tilde{F}}/\mathfrak{p}_{\tilde{F}}^{rm} \cong \mathfrak{O}_{\check{F}'}/\mathfrak{p}_{\check{F}'}^{rm}$, we have

$$\begin{aligned} \tilde{\sigma}(\varpi_{\tilde{F}})\varpi_{\tilde{F}}^{-1} \bmod \mathfrak{p}_{\tilde{F}}^{rm} &\mapsto \tilde{\sigma}'(\varpi_{\check{F}'})\varpi_{\check{F}'}^{-1} \bmod \mathfrak{p}_{\check{F}'}^{rm}, \\ \gamma_k^{-1}(\tilde{\sigma}(\varpi_{\tilde{F}}))(\tilde{\sigma}(\varpi_{\tilde{F}}))^{-1} \bmod \mathfrak{p}_{\tilde{F}}^{rm} &\mapsto \gamma_k'^{-1}(\tilde{\sigma}'(\varpi_{\check{F}'}))(\tilde{\sigma}'(\varpi_{\check{F}'}))^{-1} \bmod \mathfrak{p}_{\check{F}'}^{rm} \end{aligned}$$

we have that $u_{\tilde{\lambda}, \tilde{\sigma}} \bmod \tilde{T}_{rm} \mapsto u_{\tilde{\lambda}', \tilde{\sigma}'} \bmod \tilde{T}'_{rm}$ via $\check{\mathcal{T}}_{rm}$. By the functoriality of the Chai–Yu isomorphism [2001, Section 9.2], we have the commutative diagram

$$\begin{array}{ccc} T(\tilde{F})_1/\tilde{T}_{rm} & \xrightarrow{\text{Nm}} & T(\check{F})_1/\check{T}_m \\ \cong \downarrow & & \downarrow \cong \\ T'(\tilde{F}')_1/\tilde{T}'_{rm} & \xrightarrow{\text{Nm}} & T'(\check{F}')_1/\check{T}'_m \end{array}$$

It follows that $u_{\tilde{\lambda}, \sigma} \bmod \check{T}_m \mapsto u'_{\tilde{\lambda}', \sigma'} \bmod \check{T}'_m$. We have proved that $\sigma(n_{\check{\lambda}^t}) \bmod \check{T}_m \mapsto \sigma'(n_{\check{\lambda}^t}) \bmod \check{T}'_m$ for all $\check{\lambda}^t = \check{\lambda}_i^t$, $1 \leq i \leq n$. Hence this same claim holds for all $\check{\lambda}^t \in X_*(T)_{I_F}/\text{tor}$. This implies that $\check{\mathcal{T}}_m$ is σ -equivariant. The claim that $\check{\mathcal{T}}_m$ restricts to an isomorphism $\check{\mathcal{T}}_m : T(F)/T_m \rightarrow T'(F')/T'_m$ follows from the fact that $H^1(\sigma, \check{T}_m) = 1$ (see [Serre 1979, Chapter XII, §3, Lemma 3]). \square

1B. Some remarks. Assume $e \geq m + 4h$. We have σ -equivariant isomorphisms $\check{\mathcal{T}}_m$ and $\check{\mathcal{T}}_{m+h}$ constructed above (we also have $\check{\mathcal{T}}_{rm}$ and $\check{\mathcal{T}}_{r(m+h)}$). Let $t \in T(\check{F})_b$ with

$\kappa_{T, \check{F}}(t) = \check{\mu}$. Write $t = \text{Nm}_{\check{F}/\check{F}} \tilde{t}$, with $\tilde{t} \in T(\check{F})$. By functoriality of the Chai–Yu isomorphism (for $T_{\check{F}} \hookrightarrow R_{\check{F}} = \text{Res}_{\check{F}/\check{F}} T_{\check{F}}$), we have the commutative diagram

$$\begin{CD} T(\check{F})_b/\check{T}_{m+h} @>i>> T(\check{F})_b/\check{T}_{r(m+h)} \\ @V\check{\mathcal{J}}_{m+h}VV @VV\check{\mathcal{J}}_{r(m+h)}V \\ T'(\check{F}')_b/\check{T}'_{m+h} @>i>> T'(\check{F}')_b/\check{T}'_{r(m+h)} \end{CD}$$

As explained in [Aubert and Varma 2024, Theorem 2.5.3], it follows from the arguments in [Chai and Yu 2001, Section 8] that $T(\check{F})_b \cap \check{T}_{r(m+h)} \subset \check{T}_m$. Let $\tilde{t}' \in T(\check{F}')$ be such that $\check{\mathcal{J}}_{r(m+h)}(\tilde{t} \bmod \check{T}_{r(m+h)}) = \tilde{t}' \bmod \check{T}'_{r(m+h)}$. Using the Galois equivariance of $\check{\mathcal{J}}_{r(m+h)}$ and the commutativity of the above diagram, we have $\check{\mathcal{J}}_{m+h}(t \bmod T(\check{F})_b \cap \check{T}_{r(m+h)}) = t' \bmod T'(\check{F}')_b \cap \check{T}'_{r(m+h)}$ where $t' = \text{Nm}_{\check{F}'/\check{F}'} \tilde{t}'$. Hence $\check{\mathcal{J}}_m(t \bmod \check{T}_m) = \check{\mathcal{J}}_{m+h}(t \bmod \check{T}_m) = t' \bmod \check{T}'_m$. By Diagram (7.3.1) in [Kottwitz 1997], $\kappa_{T', \check{F}'}(t') = \check{\mu}$.

Now, let $t \in T(\check{F})$. Write $t = t_1 n_{\check{\mu}'}$ for suitable $t_1 \in T(\check{F})_b$ and $\check{\mu}' \in X_*(T)_{I_F}/\text{tor}$. Then $\kappa_{T, \check{F}}(t) = \kappa_{T, \check{F}}(t_1) + \check{\mu}$. Also $t \bmod \check{T}_m \mapsto (t'_1 \bmod \check{T}'_m)(n'_{\check{\mu}'}) \bmod \check{T}'_m$ for a suitable $t'_1 \in T'(\check{F}')_b$. Then $\kappa_{T', \check{F}'}(t'_1 n'_{\check{\mu}'}) = \kappa_{T', \check{F}'}(t'_1) + \check{\mu}$. By the preceding paragraph, we see that $\kappa_{T, \check{F}}(t_1) = \kappa_{T', \check{F}'}(t'_1)$. Hence $\check{\mathcal{J}}_m$ is compatible with the Kottwitz homomorphism $\kappa_{T, \check{F}}$. Also $\check{\mathcal{J}}_m$ is compatible with $\kappa_{T, F}$.

2. Modifications to [Ganapathy 2022, Section 3]

2A. Modifications to [Ganapathy 2022, Section 3A]. The correction given in Section 1 leads to some corrections in [Ganapathy 2022, Section 3]. One important modification is that we need to replace the set of representatives $\{n_{\check{\lambda}} \mid \check{\lambda} \in X_*(T)_{I_F}\}$ and $\{n_{\check{\lambda}_{\text{ad}}} \mid \check{\lambda} \in X_*(T_{\text{ad}})_{I_F}\}$ used in the proofs in [Ganapathy 2022, Section 3A] with the set of representatives given in Lemma 2.1. Let M, M^*, A, S, T, B and σ be as in [Ganapathy 2022, Section 3]. So M^* is an inner form of a quasisplit connected, reductive group M with $M_{\text{ad}} \cong \text{Res}_{L/F} \text{PGL}_n$ for a finite separable extension L/F . Let $\check{F} \supset L\check{F}$ be the splitting extension of $T_{\check{F}}$. Let $e = [L : L \cap \check{F}]$ and $f = [L \cap \check{F} : F]$. Fix a uniformizer $\varpi_{\check{F}}$ of \check{F} .

Lemma 2.1. *Let $\omega_{T, \check{F}} : T(\check{F}) \rightarrow X_*(T)_{I_F}/\text{tor}$ and $\omega_{T_{\text{ad}}, \check{F}} = \kappa_{T_{\text{ad}}, \check{F}} : T_{\text{ad}}(\check{F}) \rightarrow X_*(T_{\text{ad}})_{I_F}$ be the valuation homomorphisms on T and T_{ad} , respectively. There exist group-theoretic sections $\nabla_{T, \check{F}} : X_*(T)_{I_F}/\text{tor} \rightarrow T(\check{F})$ and $\nabla_{T_{\text{ad}}, \check{F}} : X_*(T_{\text{ad}})_{I_F} \rightarrow T_{\text{ad}}(\check{F})$ of $\omega_{T, \check{F}}$ and $\omega_{T_{\text{ad}}, \check{F}}$, respectively, such that $\nabla_{T, \check{F}}$ and $\nabla_{T_{\text{ad}}, \check{F}}$ agree on the subset $X_*(T_{\text{sc}})_{I_F}$.*

Proof. Let us begin by noting that $X_*(T_{\text{ad}})$ has a \mathbb{Z} -basis permuted by Γ_F and $X_*(T_{\text{ad}})_{I_F}$ is torsion-free and admits a \mathbb{Z} -basis permuted by σ . Note that $M_{\text{ad}, \check{F}} = \prod_{1 \leq i \leq e, 1 \leq j \leq f} M_{\text{ad}, \check{F}}^{(i, j)}$ where each $M_{\text{ad}, \check{F}}^{(i, j)} \cong \text{PGL}_n / \check{F}$. Following the notation of

[Bourbaki 2002], for $1 \leq i \leq e, 1 \leq j \leq f$, let

$$\tilde{\lambda}_{\text{ad},n-1}^{(i,j)} = \epsilon_1^{(i,j)} - \frac{1}{n}(\epsilon_1^{(i,j)} + \epsilon_2^{(i,j)} + \dots + \epsilon_n^{(i,j)}),$$

and, for $1 \leq k \leq n - 2$,

$$\tilde{\lambda}_{\text{ad},k}^{(i,j)} = \epsilon_k^{(i,j)} - \epsilon_{k+1}^{(i,j)}.$$

The set

$$\{\tilde{\lambda}_{\text{ad},k}^{(i,j)} \mid 1 \leq k \leq n - 1, 1 \leq i \leq e, 1 \leq j \leq f\}$$

yields a \mathbb{Z} -basis of $X_*(T_{\text{ad}})$. Let $\text{pr} : X_*(T_{\text{ad}}) \rightarrow X_*(T_{\text{ad}})_{I_F}$ be the natural projection. For $1 \leq k \leq n - 1$ and $1 \leq j \leq f$, let $\check{\lambda}_{\text{ad},k}^{(j)} = \text{pr}(\tilde{\lambda}_{\text{ad},k}^{(1,j)})$. Then the set

$$\{\check{\lambda}_{\text{ad},k}^{(j)} \mid 1 \leq k \leq n - 1, 1 \leq j \leq f\}$$

yields a \mathbb{Z} -basis of $X_*(T_{\text{ad}})_{I_F}$. Let

$$n_{\check{\lambda}_{\text{ad},k}^{(j)}} = \text{Nm}_{\check{F}/\check{F}} \tilde{\lambda}_{\text{ad},k}^{(1,j)}(\varpi_{\check{F}}), \quad 1 \leq k \leq n - 2, \quad \text{and} \quad n_{\check{\lambda}_{\text{ad},n-1}^{(j)}} = \text{Nm}_{\check{F}/\check{F}} \tilde{\lambda}_{\text{ad},n-1}^{(1,j)}(\varpi_{\check{F}}).$$

The elements $n_{\check{\lambda}_{\text{ad},k}^{(j)}}$, $1 \leq k \leq n - 1, 1 \leq j \leq f$, are used to obtain a set of representatives

$$\{n_{\check{\lambda}_{\text{ad}}} \mid \check{\lambda}_{\text{ad}} \in X_*(T_{\text{ad}})_{I_F}\}$$

that form a group; see Lemma 1.1. Let $\nabla_{T_{\text{ad}},\check{F}} : X_*(T_{\text{ad}})_{I_F} \rightarrow T_{\text{ad}}(\check{F})$, $\check{\lambda}_{\text{ad}} \mapsto n_{\check{\lambda}_{\text{ad}}}$, denote this group-theoretic section of $\omega_{T_{\text{ad}},\check{F}}$.

Next note that $X_*(T_{\text{sc}})_{I_F} \subset X_*(T)_{I_F}/\text{tor}$. Hence the elements $\check{\lambda}_{\text{ad},k}^{(j)}$, $1 \leq k \leq n - 2, 1 \leq j \leq f$, lie in $X_*(T)_{I_F}/\text{tor}$. Also, $j(X_*(T)_{I_F}/\text{tor})$ is of finite index in $X_*(T_{\text{ad}})_{I_F}$, so there exists a nonnegative integer r , which we may choose as small as possible, such that for each $1 \leq j \leq f$, $r \cdot \check{\lambda}_{\text{ad},n-1}^{(j)} = j(\check{\lambda}_{n-1}^{(j)})$ for a $\check{\lambda}_{n-1}^{(j)} \in X_*(T)_{I_F}/\text{tor}$. For the same r , there exists $\tilde{\lambda}_{n-1}^{(1,j)} \in X_*(T)$ such that $j(\tilde{\lambda}_{n-1}^{(1,j)}) = r \cdot \tilde{\lambda}_{\text{ad},n-1}^{(1,j)}$ and $\text{pr}(\tilde{\lambda}_{n-1}^{(1,j)}) = \check{\lambda}_{n-1}^{(j)}$. For $1 \leq k \leq n - 2$,

$$\tilde{\lambda}_{\text{ad},k}^{(1,1)} \in X_*(T), \quad \text{pr}(\tilde{\lambda}_{\text{ad},k}^{(1,1)}) = \check{\lambda}_{\text{ad},k}^{(1)} \quad \text{and} \quad j(\tilde{\lambda}_{\text{ad},k}^{(1)}) = \check{\lambda}_{\text{ad},k}^{(1)}.$$

Set

$$n_{\check{\lambda}_{\text{ad},k}^{(j)}} = \text{Nm}_{\check{F}/\check{F}} \tilde{\lambda}_{\text{ad},k}^{(1,j)}(\varpi_{\check{F}}), \quad 1 \leq k \leq n - 2, \quad \text{and} \quad n_{\check{\lambda}_{n-1}^{(j)}} = \text{Nm}_{\check{F}/\check{F}} \tilde{\lambda}_{n-1}^{(1,j)}(\varpi_{\check{F}}).$$

Now, the set $\{\check{\lambda}_{\text{ad},k}^{(j)} \mid 1 \leq k \leq n - 2, 1 \leq j \leq f\} \cup \{\check{\lambda}_{n-1}^{(j)} \mid 1 \leq j \leq f\}$ is \mathbb{Z} -linearly independent. Further, it may be extended to a basis of $X_*(T)_{I_F}/\text{tor}$. For the remaining basis elements of $X_*(T)_{I_F}/\text{tor}$, we choose representatives as in Section 1A. This then yields a set of representatives $\{n_{\check{\lambda}} \mid \check{\lambda} \in X_*(T)_{I_F}/\text{tor}\}$ that forms a group. Let $\nabla_{T,\check{F}} : X_*(T)_{I_F}/\text{tor} \rightarrow T(\check{F})$, $\check{\lambda} \rightarrow n_{\check{\lambda}}$ denote this group-theoretic section of $\omega_{T,\check{F}}$. By construction, we have $\nabla_{T,\check{F}}$ and $\nabla_{T_{\text{ad}},\check{F}}$ agree on $X_*(T_{\text{sc}})_{I_F}$. This finishes the proof of the lemma. \square

Lemmas 3.1 and 3.2 in [Ganapathy 2022] are not affected.

Let $\Omega_{\check{M}}$ and $\Omega_{\check{M},\text{ad}}$ be as in [Ganapathy 2022, Section 3A]. We fix a σ -stable alcove \check{a} in $\mathcal{A}(S, \check{F})$ and identify $\Omega_{\check{M}}$ with $\Omega_{\check{a}}$ and $\Omega_{\check{M},\text{ad}}$ with $\Omega_{\check{a},\text{ad}}$. Let $\check{v}_{\text{ad}} = t_{\check{\eta}_{\text{ad}}}\check{z}$ be as in [Ganapathy 2022, Section 3A]. With notation as in Lemma 2.1, $\check{\eta}_{\text{ad}} = \check{\lambda}_{\text{ad},n-1}^{(1)}$. Let $\check{z} = \check{z}^{(1)} = s_1^{(1)} \cdots s_{n-1}^{(1)}$. Let $n_{\check{\lambda}_{\text{ad},n-1}^{(1)}} \in T_{\text{ad}}(\check{F})$ be as in Lemma 2.1. We fix a system of pinings $\{x_{\check{a}} \mid \check{a} \in \Phi(M, S)\}$ that is σ -stable as in [Ganapathy 2022, Section 3A]. Let $n_{\check{z}^{(1)}} = n_{s_1^{(1)}} \cdots n_{s_{n-1}^{(1)}}$. Let $\sigma^* = \text{Ad}(n_{\check{v}_{\text{ad}}}) \circ \sigma$ where $n_{\check{v}_{\text{ad}}} = n_{\check{\lambda}_{\text{ad},n-1}^{(1)}} n_{\check{z}^{(1)}}$, and let $M^* = M_{\check{F}}^{\sigma^*}$. Let $\Omega_M = \Omega_M^{\sigma}$ and $\Omega_{M^*} = \Omega_M^{\sigma^*}$. Similarly define $\Omega_{M,\text{ad}}$ and $\Omega_{M^*,\text{ad}}$. By [Ganapathy 2022, Lemma 3.2] we have $\Omega_M = \Omega_{M^*}$ and $\Omega_{M,\text{ad}} = \Omega_{M^*,\text{ad}} \cong \mathbb{Z}/n\mathbb{Z}$. The group $j(\Omega_M) \subset \Omega_{M,\text{ad}}$ is cyclic. Assume $[\Omega_{M,\text{ad}} : j(\Omega_M)] = r$ and that $j(\Omega_M) \neq 0$. Let $\check{\tau}_0 \in \Omega_M \subset \Omega_M^{\sigma}$ be such that $j(\check{\tau}_0)$ is a generator of $j(\Omega_M)$. Then $j(\check{\tau}_0) = \check{v}_{\text{ad}}^r \sigma(\check{v}_{\text{ad}})^r \cdots \sigma^{k-1}(\check{v}_{\text{ad}})^r$. Write $\check{\tau}_0 = t_{\check{\lambda}_0} \check{y}_0$, where $\check{\lambda}_0 \in X_*(T)_{I_F}^{\sigma}$ and $\check{y}_0 \in W(M, S)$. Note that $\check{y}_0 = (\check{z}^{(1)})^r \sigma(\check{z}^{(1)})^r \cdots \sigma^{f-1}(\check{z}^{(1)})^r$. We may and do assume that $\check{\lambda}_0 \in (X_*(T)_{I_F}/\text{tor})^{\sigma}$. Let $n_{\check{\lambda}_0} \in T(\check{F})$ be as in Lemma 2.1. Note that $n_{\check{\lambda}_0}$ may not be fixed by σ . Let $n_{\check{y}_0} := n_{\check{z}^{(1)}}^r \sigma(n_{\check{z}^{(1)}}^r) \cdots \sigma^{f-1}(n_{\check{z}^{(1)}}^r)$.

Lemma 2.2. *Let $\check{\tau}_0$ be as in the preceding paragraph. There exists $v \in T(\check{F})_1$ such that $n_{\check{\tau}_0} = vn_{\check{\lambda}_0} n_{\check{y}_0} \in M^*(F)$ and $\kappa_{M^*,F}(n_{\check{\tau}_0}) = \check{\tau}_0$.*

Proof. Recall that we have fixed representatives $\{n_{\check{\lambda}} \mid \check{\lambda} \in X_*(T)_{I_F}/\text{tor}\}$ that forms a group. Note that $\sigma(\check{\lambda}_0) = \check{\lambda}_0$ and $\sigma(\check{y}_0) = \check{y}_0$. Let us compute $\sigma^*(n_{\check{\lambda}_0} n_{\check{y}_0})$. Using the definition of $n_{\check{y}_0}$, we have $\sigma(n_{\check{y}_0}) = n_{\check{y}_0}$. Using [Ganapathy 2022, Lemma 3.1(b)], we have

$$\sigma^*(n_{\check{\lambda}_0} n_{\check{y}_0}) = \sigma^*(n_{\check{\lambda}_0}) n_{\check{\lambda}_{\text{ad},n-1}^{(1)} - \check{y}_0(\check{\lambda}_{\text{ad},n-1}^{(1)})} n_{\check{y}_0}.$$

Now,

$$u = \sigma^*(n_{\check{\lambda}_0}) n_{\sigma^*(\check{\lambda}_0)}^{-1} \in T(\check{F})_1$$

since its image under $\kappa_{T,\check{F}}$ is 0. Since $H^1(\sigma^*, T(\check{F})_1) = 1$, there exists $v \in T(\check{F})_1$ such that $\sigma^*(v)v^{-1} = u^{-1}$. Now $\sigma^*(vn_{\check{\lambda}_0}) = vu^{-1}\sigma^*(n_{\check{\lambda}_0}) = vn_{\sigma^*(\check{\lambda}_0)}$. Then

$$\sigma^*(vn_{\check{\lambda}_0} n_{\check{y}_0}) = vn_{\sigma^*(\check{\lambda}_0)} n_{\check{\lambda}_{\text{ad},n-1}^{(1)} - \check{y}_0(\check{\lambda}_{\text{ad},n-1}^{(1)})} n_{\check{y}_0} = vn_{\sigma^*(\check{\lambda}_0) + \check{\lambda}_{\text{ad},n-1}^{(1)} - \check{y}_0(\check{\lambda}_{\text{ad},n-1}^{(1)})} n_{\check{y}_0} = vn_{\check{\lambda}_0} n_{\check{y}_0}.$$

The second equality follows from Lemma 2.1 and that $\check{\lambda}_{\text{ad},n-1}^{(1)} - \check{y}_0(\check{\lambda}_{\text{ad},n-1}^{(1)}) \in X_*(T_{\text{sc}})_{I_F} \subset X_*(T)_{I_F}/\text{tor}$. To get the third equality, note that from the proof of [Ganapathy 2022, Lemma 3.2], $\sigma^*(\check{\lambda}_0) - \sigma(\check{\lambda}_0) = \check{\lambda}_{\text{ad},n-1}^{(1)} - (\text{Ad}(z^{(1)})(\check{y}_0))(\check{\lambda}_{\text{ad},n-1}^{(1)})$ but $\sigma(\check{\lambda}_0) = \check{\lambda}_0$ and $\text{Ad}(z^{(1)})(\check{y}_0) = \check{y}_0$. This finishes the proof of the lemma. \square

Now, given $\check{\tau} = t_{\check{\lambda}} \check{w} \in \Omega_M$ with $t_{\check{\lambda}} \in X_*(T)_{I_F}$ and $\check{w} \in W(M, S)$, we have $j(\check{\tau}) = sj(\check{\tau}_0)$ for a unique nonnegative integer s with $0 \leq s < n/r$. Let $\check{\mu} = \check{\tau} - s\check{\tau}_0$. Write $\check{\mu} = t_{\check{\mu}_0} \check{w}_0 \in \Omega_M$. Then $j(\check{\tau}) = sj(\check{\tau}_0)$ implies that $\check{w} = \check{y}_0^s$, so $\check{w}_0 = 1$ and the element $\check{\mu}$ is just given by the translation $t_{\check{\mu}_0} \in X_*(T)_{I_F}$. We identify $\check{\mu}$ and $\check{\mu}_0$. Since σ fixes $\check{\tau}$ and $\check{\tau}_0$, we have $\sigma(\check{\mu}) = \check{\mu}$. We claim that $\sigma^*(\check{\mu}) = \check{\mu}$. To

see this, note that since $j(\check{\mu}) = 0$, we have that $j(\text{Ad}(\check{z}^{(1)})(\check{\mu}) - \check{\mu}) = 0$, but since $\text{Ad}(\check{z}^{(1)})(\check{\mu}) - \check{\mu} \in X_*(T_{\text{sc}})_{I_F}$, and since j acts as identity on $X_*(T_{\text{sc}})_{I_F}$, it follows that $\text{Ad}(\check{z}^{(1)})(\check{\mu}) - \check{\mu} = 0$. This then implies that $\sigma^*(\check{\mu}) = \text{Ad}(\check{z}^{(1)})(\check{\mu}) = \check{\mu}$. So $\check{\mu} \in X_*(T)_{I_F}^{\sigma^*}$. Set $n_{\check{\tau}} = n_{\check{\mu}} n_{\check{\tau}_0}^s$ with $n_{\check{\mu}} \in T^*(F)$ satisfies $\kappa_{T^*, F}(n_{\check{\mu}}) = \check{\mu}$.

Proposition 2.3. *Let $\check{\tau} \in \Omega_{M^*} = \Omega_M$. Then $\sigma^*(n_{\check{\tau}}) = n_{\check{\tau}}$. In particular, $n_{\check{\tau}} \in M^*(F)$ and $\check{p} : \Omega_{M^*} \rightarrow M^*(F)$, $\check{\tau} \mapsto n_{\check{\tau}}$, is a (set-theoretic) section of $\kappa_{M^*, F}$.*

Proof. It suffices to prove that $\sigma^*(n_{\check{\tau}_0}) = n_{\check{\tau}_0}$, but this is Lemma 2.2. □

2B. Modifications to [Ganapathy 2022, Section 3B]. Via Del_m , we have isomorphisms $X_*(T) \cong X_*(T')$ and $X_*(T_{\text{ad}}) \cong X_*(T'_{\text{ad}})$ that are Γ_F/I_F^m -equivariant, and $\Omega_{\check{M}} \cong \Omega_{\check{M}'}$ and $\Omega_{\check{M}_{\text{ad}}} \cong \Omega_{\check{M}'_{\text{ad}}}$. We identify these groups via these isomorphisms. We construct $\nabla_{T', \check{F}'} : X_*(T)_{I_F}/\text{tor} \rightarrow T'(\check{F}')$, $\check{\lambda} \mapsto n'_{\check{\lambda}}$, and $\nabla_{T'_{\text{ad}}, \check{F}'} : X_*(T_{\text{ad}})_{I_F} \rightarrow T'_{\text{ad}}(\check{F}')$, $\check{\lambda} \mapsto n_{\check{\lambda}}$, exactly as in Lemma 2.1, but with $\varpi_{\check{F}}$ replaced with $\varpi_{\check{F}'}$ where $\varpi_{\check{F}} \bmod \mathfrak{p}_{\check{F}}^m \mapsto \varpi_{\check{F}'} \bmod \mathfrak{p}_{\check{F}'}^m$ as in Lemma 1.2. Let $\check{\tau}_0$ be as in Lemma 2.2. Let $n'_{\check{\lambda}_0}, n'_{\sigma^*(\check{\lambda}_0)} \in T'(\check{F}')$ be such that under $\check{\mathcal{T}}_m$, $n_{\check{\lambda}_0} \bmod \check{T}_m \mapsto n'_{\check{\lambda}_0} \bmod \check{T}'_m$, and similarly for $n'_{\sigma^*(\check{\lambda}_0)}$. Then, since $\check{\mathcal{T}}_m$ is σ^* -equivariant, we have $u \bmod \check{T}_m \mapsto u' \bmod \check{T}'_m$, where $u' = \sigma'^*(n'_{\check{\lambda}_0}) n'^{-1}_{\sigma^*(\check{\lambda}_0)}$. By the proof of the fact that $H^1(\sigma^*, T(\check{F})_1) = 1$ [Serre 1979, Chapter XII, §3, Lemma 3], it follows that we may choose $v' \in T(\check{F})_1$ such that $\sigma^*(v')v'^{-1} = u'^{-1}$ and such that $v \bmod \check{T}_m \mapsto v' \bmod \check{T}'_m$. Let $n'_{\check{y}_0} = n'^r_{\check{z}(1)} \sigma'(n'_{\check{z}(1)})^r \cdots \sigma^{f-1}(n'_{\check{z}(1)})^r$. Set $n'_{\check{\tau}_0} = v' n'_{\check{\lambda}_0} n'_{\check{y}_0}$. Given $\check{\tau} \in \Omega_M$, we may write $\check{\tau} = \check{\mu} + s\check{\tau}_0$ for a unique $0 \leq s < n/r$ as in the paragraph preceding Proposition 2.3. Set $n'_{\check{\tau}} = n'_{\check{\mu}} n'^s_{\check{\tau}_0}$ where $n'_{\check{\mu}} \in T'^*(F')$ with $\mathcal{T}_m^*(n_{\check{\mu}} \bmod T_m^*) \mapsto n'_{\check{\mu}} \bmod T'^*$. Note that $\kappa_{T'^*, F'}(n'_{\check{\mu}}) = \check{\mu}$ by Section 1B. By Proposition 2.3, $n'_{\check{\tau}} \in M'^*(F')$.

Proposition 2.4 [Ganapathy 2022, Proposition 3.4]. *Let $m \geq 1$ and let $e \geq m + 4h$. If the fields F and F' are e -close, then we have an isomorphism $M^*(F)/M_m^* \cong M'^*(F')/M_m'^*$.*

Proof. The proof given in [Ganapathy 2022, Proposition 3.4] works with straightforward modifications.

Consider the set theoretic section $\check{p} : \Omega_{M^*} \rightarrow M^*(F)$ in Proposition 2.3 and let p be its composition with the natural projection $M^*(F) \rightarrow M^*(F)/M_m^*$. Similarly, we get $\check{p}' : \Omega_{M'^*} \xrightarrow{\check{p}'} M'^*(F')$ and p' .

It suffices to prove that the sections p and p' satisfy (a) and (b) of [Ganapathy 2022, Proposition 3.4].

To see (a), it suffices to prove that

$$\begin{array}{ccc} M^*(F)_1/M_m^* & \xrightarrow{\cong} & M'^*(F')_1/M_m'^* \\ \downarrow \text{Inn}(n_{\check{\tau}}) & & \downarrow \text{Inn}(n'_{\check{\tau}}) \\ M^*(F)_1/M_m^* & \xrightarrow{\cong} & M'^*(F')_1/M_m'^* \end{array}$$

is commutative for $\check{\tau} \in \Omega_{M^*}$. Let \check{P} be the Iwahori subgroup of $M(\check{F}) (= M^*(\check{F}))$ attached to the σ -stable alcove \check{a} and let \check{P}' be the corresponding Iwahori subgroup of $M'(\check{F}')$. Then by [Ganapathy 2019, Theorem 4.5], we have that $\check{P}/\check{P}_m \cong \check{P}'/\check{P}'_m$. Since $\check{\nu}_{\text{ad}} \in \Omega_{\check{a}, \text{ad}}$, the alcove \check{a} is also σ^* -stable. By Propositions 4.10 and 6.2 in [Ganapathy 2019], the isomorphism $\check{P}/\check{P}_m \cong \check{P}'/\check{P}'_m$ is σ - and σ^* -equivariant. This implies that $\check{P} \cap M^*(F) = M^*(F)_1$, $\check{P}_m \cap M^*(F) = M_m^*$ and similarly that $\check{P}' \cap M'^*(F') = M'^*(F')_1$, $\check{P}'_m \cap M'^*(F') = M_m'^*$. Since $\check{\tau} \in \Omega_{M^*} = \Omega_{\check{a}}^{\sigma^*} \subset \Omega_{\check{a}}$, we see that $n_{\check{\tau}}$ normalizes \check{P} and \check{P}_m . To finish the proof of (a), it suffices to observe that the following diagram is commutative:

$$\begin{CD} \check{P}/\check{P}_m @>\cong>> \check{P}'/\check{P}'_m \\ @VV\text{Inn}(n_{\check{\tau}})V @VV\text{Inn}(n'_{\check{\tau}})V \\ \check{P}/\check{P}_m @>\cong>> \check{P}'/\check{P}'_m \end{CD}$$

This follows by arguing as in the proof of [Ganapathy 2019, Proposition 6.2].

Let us prove (b). The element $n_{\check{y}_0}^{n/r}$ equals $\check{a}^\vee(-1) \in M^*(F)_1$ for a suitable $\check{a} \in \check{\Phi}(M, S)$.

Let $\check{\tau}_1, \check{\tau}_2 \in \Omega_{M^*}$. As in the proof of Proposition 2.3, write $\check{\tau}_i = \check{\mu}_i + s_i \check{\tau}_0$, and $\check{\tau}_1 + \check{\tau}_2 = \check{\mu} + s \check{\tau}_0$. Note that $s \pmod{(n/r)} \equiv s_1 + s_2 \pmod{(n/r)}$.

Recall that $n_{\check{\mu}}, n_{\check{\mu}_1}, n_{\check{\mu}_2} \in T^*(F)$ and $n_{\check{\mu}} \pmod{T_m^*} \mapsto n'_{\check{\mu}} \pmod{T_m'^*}$ and for $i = 1, 2$, $n_{\check{\mu}_i} \pmod{T_m^*} \mapsto n'_{\check{\mu}_i} \pmod{T_m'^*}$. Write $n_{\check{\tau}_0}^s = t_s n_{\check{y}_0}^s$ where $t_s \in T(\check{F})$. Similarly write $n_{\check{\tau}_0}^{s'} = t'_s n_{\check{y}_0}^{s'}$. Then it is straightforward to see that $t_s \pmod{\check{T}_m} \mapsto t'_s \pmod{\check{T}_m'}$ via $\check{\mathcal{T}}_m$. The same claim holds for $t_{s_i}, i = 1, 2$. Also, $\check{a}^\vee(-1) \pmod{\check{T}_m} \mapsto \check{a}'^\vee(-1) \pmod{\check{T}_m'}$. Finally, we note that $n_{\check{\tau}_1 + \check{\tau}_2}^{-1} n_{\check{\tau}_1}^{-1} n_{\check{\tau}_2}^{-1} \in M^*(F)_1 \cap T(\check{F})$ and by [Ganapathy 2019, Proof of Proposition 6.2 and Corollary 6.3], we see that on the subgroup $M^*(F)_1 \cap T(\check{F})$ the isomorphism of [Ganapathy 2019, Corollary 6.3] restricts to $\check{\mathcal{T}}_m^*$. Hence the sections p, p' satisfy (b). □

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
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