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GARY SEITZ (1943–2023): IN MEMORIAM

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In memoriam

Gary Seitz

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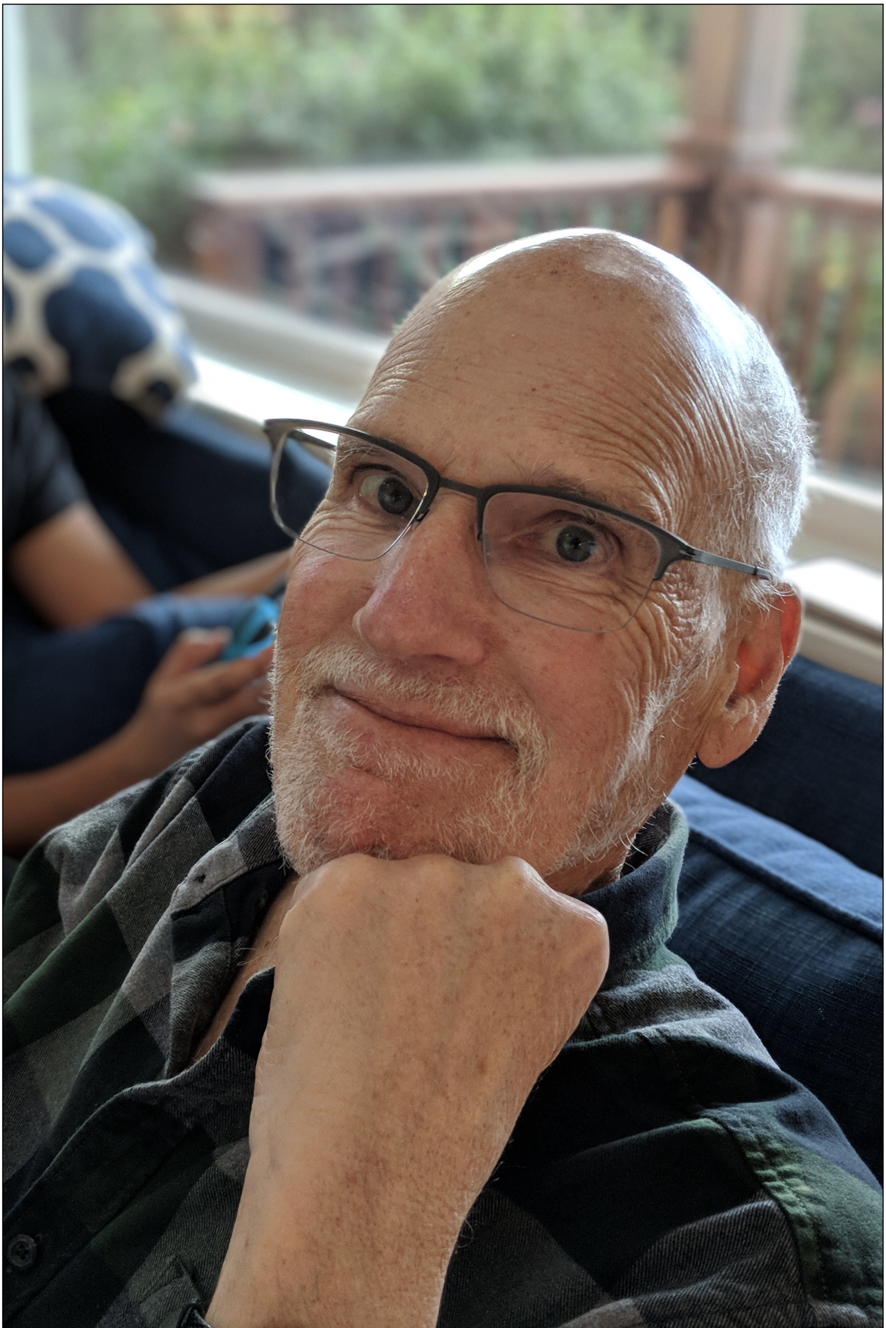

Gary Seitz (1943–2023)

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GARY SEITZ (1943–2023): IN MEMORIAM

Gary Seitz was born in Santa Monica, California, and grew up in Los Angeles. His father, David Seitz, was in the casino business, and his mother Sarah worked in a hair salon. As a youth, Gary was into body-building and bowling, and at one point had to make a career decision between going into mathematics or becoming a professional bowler. Fortunately for us, he decided on the former, and did his Bachelor's and Master's degrees at Berkeley. While at Berkeley he met Sheila Coutin, and they married while still undergraduates in 1964. They had two sons, Aaron and Steve, both of whom went on to have academic careers.

After Berkeley, Gary moved to the University of Oregon in Eugene for his PhD with advisor Charles Curtis, which he completed in 1968. He then held a postdoctoral research position at the University of Illinois at Chicago Circle until 1970, when he returned as a faculty member to Eugene, where he remained until his retirement. He served the University of Oregon with enormous distinction, both academically and administratively, as Head of Department 1994–2001, CAS Distinguished Professor from 2000, and Associate Dean of Natural Sciences 2002–2005.

Gary was a leading figure in algebra for over 50 years, publishing about 100 articles and books on a wide variety of topics, mainly centering around group theory: finite groups, algebraic groups, representation theory, maximal subgroups, and applications to other areas such as number theory and algebraic combinatorics. He was extremely collaborative in his research, publishing with 29 different coauthors, and holding visiting appointments at Caltech, Notre Dame, IHES, Bar Ilan, Tel Aviv, the Technion, IAS Princeton, Aarhus, Utrecht, Essen, Tokyo, Warwick and Imperial College London. He was named a Fellow of the American Mathematical Society in 2013.

Let us discuss some of the themes of Gary's research in a little more detail. In his PhD thesis and several subsequent papers, he proved deep results about a wide class of finite solvable groups known as M-groups. He then moved to Chicago, which at the time was a tremendous centre for finite group theory, particularly surrounding the finite simple groups and the attempt to classify them. Jacques Tits had recently introduced his theory of BN-pairs for finite groups, and their associated buildings, and

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he had shown that any simple group with a BN-pair of rank at least 3 is necessarily a group of Lie type, providing a geometric setting for these families of simple groups, and also a powerful method for identifying them. Tits' methods were largely geometric, and for ranks 1 and 2 the geometric structure was not strong enough — for example, the groups with a BN-pair of rank 1 are just the 2-transitive permutation groups. It was important to fill this gap, and together with Christoph Hering and Bill Kantor, also based in Chicago at the time, Gary succeeded in classifying the split BN-pairs of rank 1; soon thereafter, with Paul Fong, Gary classified the split BN-pairs of rank 2.

These results provided an essential tool in the ongoing programme to classify the finite simple groups, which was proceeding apace at the time. Gary made further key contributions to this programme with his work, partly with Michael Aschbacher, on standard subgroups. A standard subgroup of a finite group G is a quasisimple subgroup that is embedded in a very specific way in an involution centralizer in G. One key part of the classification programme was to determine the finite groups G that have a standard subgroup A belonging to one of the known families of quasisimple groups. Many authors were involved in this project; Gary and Michael handled the case where $C_G(A)$ has 2-rank at least 2, and in several further papers Gary dealt with the case of 2-rank 1 when A is a group in the family Lie(2) of groups of Lie type in characteristic 2. Along the way, Gary and Michael found it necessary to develop a complete theory of involution classes and centralizers for groups in Lie(2), and wrote a much-cited paper on this that proved to be a precursor of Gary's later fundamental work on unipotent elements in algebraic groups, on which more later.

The completion of the classification of finite simple groups was first announced in the early 1980s. However, not much was known about the subgroup structure of these groups, and in particular their maximal subgroups. The study of these maximal subgroups formed one of the themes of Gary's work for the next 30 years. The finite groups of Lie type are intimately related to the corresponding simple algebraic groups over algebraically closed fields, and in the 1950s Dynkin had solved the maximal subgroup problem for classical groups over $\mathbb{C}$; a major part of his solution was the determination of all triples (G, H, V) with V a finite-dimensional complex vector space and $G < H < \mathrm{Cl}(V)$, where $\mathrm{Cl}(V)$ is a classical group on V and G, H are connected algebraic groups acting irreducibly on V. Gary took on the formidable project of generalizing this result to algebraically closed fields of arbitrary characteristic. He gave part of this problem to his then PhD student Donna Testerman, and between them they solved it completely, publishing their results in two *Memoirs of the AMS*, totalling about 500 pages. This work has been used many times both within finite and algebraic group theory, and in its applications.

For the simple algebraic groups of exceptional Lie types $(G_2, \ldots, E_8)$, one can hope to determine all the connected maximal subgroups. In further pioneering work, Gary achieved this in another *Memoir* published in 1991, assuming the characteristic

p of the underlying field is not too small ($p > 7$ suffices in all cases). In a later *Memoir* with Martin Liebeck, he extended the result to all characteristics, and also to disconnected subgroups. These results had implications for maximal subgroups of the finite exceptional groups of Lie type, and in a long series of papers with Martin, and also some with Arjeh Cohen, Jan Saxl and others, Gary built an edifice of theory on the subgroup structure of these families of simple groups.

We mentioned before Gary's work with Michael Aschbacher classifying involution classes in the groups in Lie(2). He continued to work on many different aspects of unipotent elements in finite and algebraic groups G of Lie type. In a 1983 paper, he built a theory of root groups relative to arbitrary maximal tori of the finite groups, and used this to determine the subgroups containing such tori. In other papers published in the 1990s, he classified the subgroups generated by root elements, and at the other extreme, by regular or semiregular unipotent elements.

In 2000 Gary achieved a breakthrough, solving the "saturation" problem of J-P. Serre for arbitrary unipotent elements of order p (the characteristic of the underlying field, assumed good for the simple algebraic group G): he proved that any such unipotent element u is contained in a unique 1-dimensional unipotent subgroup of a particularly nice A_1 subgroup (called a "good" A_1 by Gary), unique up to conjugacy in G by $C_G(u)$. This enables one to answer many questions about unipotents by studying the good A_1's, a beautiful class of subgroups. Gary published numerous further papers on this topic, culminating in his book with Martin Liebeck, which presents a complete theory of unipotent classes and centralizers in simple algebraic groups, and nilpotent classes in the corresponding Lie algebras.

Of course much of Gary's work already discussed involves heavy use of the representation theory of finite and algebraic groups. Gary also published many articles that are purely on this topic. An early one was a much-cited 1974 paper with Vicente Landazuri giving lower bounds for the dimensions of irreducible representations of groups in Lie(p) over fields of characteristic coprime to p; this work has been built on by many authors to classify all the low-dimensional representations of these groups, an important theory with many applications. Another highlight is Gary's 1992 paper with Jens Jantzen on the innocent-looking problem of determining which irreducible representations (in arbitrary characteristic) of the symmetric group S_n remain irreducible on restriction to S_{n-1}. The results and conjectures posed in this paper formed the first step in a theory of modular branching rules for representations of S_n developed by Alexander Kleshchev and others, now a fundamental tool of representation theory.

Another topic on which Gary made decisive contributions is the theory of G-complete reducibility (G-cr): this was introduced by Serre as a way of interpreting concepts of representation theory in the more general setting of maps between algebraic groups. In a 1996 *Memoir* with Martin Liebeck, Gary proved that arbitrary reductive subgroups of exceptional algebraic groups are completely reducible provided the underlying characteristic p is not too small ($p > 7$ suffices in all cases). Together with results of Jantzen and McNinch, and Serre himself, on classical groups, these results formed the basis of G-cr theory, which has since been taken much further by many authors.

Gary was involved in several projects applying group theory to other areas of mathematics. The most striking of these was his proof with Yoav Segev and Andrei Rapinchuk that all finite quotients of the multiplicative group of a finite dimensional division algebra are solvable. A consequence was the solution of the Margulis–Platonov conjecture on the normal subgroup structure of algebraic groups over number fields. The method behind their proof was based on the remarkable idea, pioneered by Segev, that the commuting graph of such a finite quotient must have strong connectivity properties; in several substantial papers, they proved that commuting graphs of nonsolvable groups could not have such properties.

Let us finally mention the topic on which Gary was working for most of the last ten years of his life: the theory of multiplicity-free representations. The project was to classify the triples (G, H, V), where $H < G < \mathrm{GL}(V)$ are connected reductive algebraic groups over an algebraically closed field of characteristic zero, and V is an irreducible G-module whose restriction to H is multiplicity-free (i.e., each composition factor appears with multiplicity 1). A great deal of classical work, going back to Dynkin, Howe, Kac, Stembridge, Weyl and others, can be set in this context. In his final *Memoir* with Martin and Donna, Gary determined all such triples in cases where H and G are both simple algebraic groups of type A, showing that there are many beautiful families of such representations.

There are large parts of Gary's output that we have not mentioned, but we hope we have conveyed some of the profound influence of his work across many areas and over many years.

Gary was the advisor of eleven PhD students, almost all of whom continued into the academic profession. Three of them, George McNinch, Gerhard Röhrle and Donna Testerman, have contributed articles to this volume.

We three had the privilege of learning continually from Gary's enormous depth of knowledge and ideas, as well as collaborating with him over many years. But we valued above all his warm, generous friendship; his wisdom in matters mathematical and non-mathematical; his boundless energy; and his wonderful company, full of laughter and fun. We miss him deeply.

Martin Liebeck
Gerhard Röhrle
Donna Testerman

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