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HOWE DUALITY FOR THE DUAL PAIR $\mathrm{SL}_2(\mathbb{R}) \times F_{4,1}$: A PING PONG OF K-TYPES

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We prove Howe duality for an exceptional theta correspondence. To that end, we relate the K-types of corresponding representations by exploiting a pair of see-saw identities.

1. Introduction

Let $\mathbb{O}$ be the algebra of Cayley octonions over the field of real numbers $\mathbb{R}$. Let J be the 27-dimensional space of 3×3 hermitian symmetric matrices with coefficients in $\mathbb{O}$. Let $N_J : J \to \mathbb{R}$ be the cubic form (the norm of J), essentially the determinant of 3×3 matrices. For every $e \in J$ such that $N_J(e) \neq 0$ there is a structure of exceptional Jordan algebra on J such that e is the identity of J. Let $G = \mathrm{Aut}(J, e)$ be the group of automorphisms of the resulting Jordan algebra, which is the same as the group of linear transformations of J preserving N_J and the point e. It is a simple Lie group of absolute type F_4. See [6] for all of this. If we pick e to be

$$\begin{pmatrix} +1 & & \\ & +1 & \\ & & +1 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} -1 & & \\ & -1 & \\ & & +1 \end{pmatrix},$$

then G is compact for the first choice of e and of split rank one for the second [11]. The Jordan algebra, by way of the Koecher–Tits construction [8], gives rise to a simply connected group $G(J)$, of the exceptional type E_7 and split rank 3 over $\mathbb{R}$ (the same group for both choices of e). The group $G(J)$ comes along with the dual pair (see [7])

$$\mathrm{SL}_2(\mathbb{R}) \times G \subset G(J).$$

These dual pairs are completely analogous to $\mathrm{SL}_2(\mathbb{R}) \times \mathrm{O}(p, q)$ in $\mathrm{Sp}_{2n}(\mathbb{R})$ where $n = p + q$. Indeed, if we take J to be the space of $n \times n$ symmetric matrices with coefficients in $\mathbb{R}$, then orthogonal groups are stabilizers of generic points in J, and $G(J)$ is $\mathrm{Sp}_{2n}(\mathbb{R})$.

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The group $G(J)$ has a minimal (holomorphic) representation Π that appears as a local component of a global representation [5]. In [3], Π was restricted to the dual pair $\mathrm{SL}_2(\mathbb{R}) \times G$, with G compact, and the following decomposition was obtained:

$$\Pi = \bigoplus_{n \geq 0} \delta(2n+12) \otimes E_n.$$

Here $\delta(2n+12)$ is the holomorphic representation of the lowest weight $2n+12$ and E_n is the irreducible representation of G of the highest weight $n\varpi_4$ where ϖ_4 is the fourth fundamental weight for F_4. It is the highest weight of the 26-dimensional irreducible representation of G (the complement of the line through e in J).

Here we study the restriction of Π to the dual pair with G noncompact. Let K be the maximal compact subgroup of G. We emphasize that we do not work with continuous representations of noncompact groups, but with the corresponding $(\mathfrak{g}, K)$-modules, where $\mathfrak{g}$ is the complex Lie algebra of G. Thus, if π is a $(\mathfrak{g}, K)$-module of finite length, we define

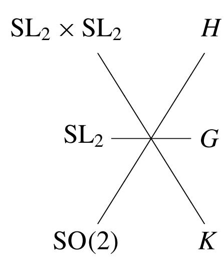
$$\Theta(\pi) = (\Pi \otimes \pi^\vee)_{\mathfrak{g}}.$$

Here $\pi^\vee$ is the contragredient of π, and the subscript $\mathfrak{g}$ is saying that we are taking co-invariants with respect to the action of $\mathfrak{g}$ on the tensor product. If π is irreducible then $\Theta(\pi) \otimes \pi$ is the maximal π-isotypic quotient of Π (see [1]). In other words, the above definition generalizes the usual definition of the theta lift.

We can analogously define $\Theta(\sigma)$ for an $(\mathfrak{sl}_2, \mathrm{SO}(2))$-module σ of finite length. Observe that $\Theta(\pi)$ and $\Theta(\sigma)$ are naturally $(\mathfrak{sl}_2, \mathrm{SO}(2))$ and $(\mathfrak{g}, K)$-modules, respectively. We shall show that $\Theta(\pi)$ and $\Theta(\sigma)$ are finite length modules, and that they have unique irreducible quotients, if π and σ are irreducible. The main input is the structure of lifts of types. More precisely, if τ is a K-type, then

$$\Theta(\tau) = (\Pi \otimes \tau^\vee)_K$$

is also an $(\mathfrak{sl}_2, \mathrm{SO}(2))$-module that we determine explicitly. Similarly, we have a description of the lift for $\mathrm{SO}(2)$-types. This is done in the last section using a strategy of Howe [4], involving the following see-saw diagram of dual pairs in $G(J)$:



Here H is a simply connected, hermitian symmetric group of absolute type E_6. The centralizer of G sits diagonally in $\mathrm{SL}_2 \times \mathrm{SL}_2$. Thus $\Theta(\tau)$ is naturally an $(\mathfrak{sl}_2 + \mathfrak{sl}_2, \mathrm{SO}(2) \times \mathrm{SO}(2))$-module. We compute it, and then restrict it to the diagonal $\mathfrak{sl}_2$. A similar strategy is used for lifts of SO(2)-types.

With the lift of types computed, we can play a game of ping pong with types: if $\sigma \otimes \pi$ is a quotient of Π and τ is a type of π then, by a see-saw identity, σ must have an SO(2)-type determined by τ and vice versa. More details in the next section where main results are obtained. A similar strategy (and the name ping-pong) was used in [2] to establish Howe duality for exceptional p-adic dual pairs.

2. Main results

The correspondence with compact G establishes a correspondence of infinitesimal characters in the noncompact case. The reader can consult [10] for more details on this. Let us write down the correspondence. Using the standard realization of the F_4 root system, the infinitesimal character of E_n (the representation with the highest weight $n\varpi_4$) is

$$\tfrac{1}{2}(2n+11, 5, 3, 1).$$

On the other hand, the infinitesimal character of $\delta(2n+12)$ is $2n+11$, which we recognize as the first entry above. This means that if σ has infinitesimal character x, then $\Theta(\sigma)$ has infinitesimal character $\frac{1}{2}(x, 5, 3, 1)$. More generally, if σ is annihilated by an ideal in the center of $U(\mathfrak{sl}_2)$ of finite codimension, then $\Theta(\pi)$ is also annihilated by an ideal in the center of $U(\mathfrak{g})$ of finite codimension. Hence, for σ of finite length, in order to prove that $\Theta(\sigma)$ has finite length, it suffices to prove that it is admissible. The same goes for $\Theta(\pi)$.

The maximal compact subgroup of $\mathrm{SL}_2(\mathbb{R})$ is SO(2). Its irreducible representations are one-dimensional characters parameterized with integers n. Let (n) denote the corresponding one-dimensional representation. Since the center of $\mathrm{SL}_2(\mathbb{R})$ is also the center of the simply connected $G(J)$, only even $n = 2m$ characters appear in Π.

The maximal compact subgroup of G is denoted by K. It is a simple group of type B_4. The group K can be picked to be the intersection of G with the compact form of G, where the two groups are the stabilizers of the two choices for e, as in the introduction. Let $\mathfrak{g}$ be the complex simple Lie algebra of G, and $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$ the corresponding Cartan decomposition. Here $\mathfrak{p}$ is the 16-dimensional spin representation of K. Let μ be its highest weight, and let λ be the highest weight of the standard 9-dimensional irreducible representation of K. Let $\tau(m, n)$ be the irreducible representation of K of the highest weight

$$m\lambda + n\mu.$$

Applying the branching rule [9] to the representations E_n, we see that only these representations of K lie in Π. Let

$$\Theta(\tau(m,n)) = (\Pi \otimes \tau(m,n)^{\vee})_K$$

be the lift of $\tau(m,n)$. It is naturally an $\mathfrak{sl}_2$-module. We have, by Proposition 3.1,

$$\Theta(\tau(m,n)) \cong U(\mathfrak{sl}_2) \otimes_{U(\mathfrak{so}(2))} \otimes(2m+4).$$

A power of this identity is demonstrated by the following lemma:

Lemma 2.1. *Let σ be a finite length $(\mathfrak{sl}_2, \mathrm{SO}(2))$-module. Then*

$$\mathrm{Hom}_K(\Theta(\sigma), \tau(m,n)) \cong \mathrm{Hom}_{\mathfrak{sl}_2}(\Theta(\tau(m,n)), \sigma) \cong \mathrm{Hom}_{\mathrm{SO}(2)}((2m+4), \sigma).$$

Proof. The first isomorphism is a see-saw identity, obtained by switching the order of taking $\mathfrak{sl}_2$ and K co-invariants. The second isomorphism follows from the Frobenius reciprocity, since $\Theta(\tau(m,n))$ is isomorphic to $U(\mathfrak{sl}_2) \otimes_{U(\mathfrak{so}(2))} \otimes(2m+4)$. □

Now we have the following consequence, Santa Claus is coming to town:

Proposition 2.2. *Let σ be a finite length $(\mathfrak{sl}_2, \mathrm{SO}(2))$-module. Then $\Theta(\sigma) \neq 0$ if and only if σ has a type $2m+4$ for some $m \geq 0$. $\Theta(\sigma)$ has finite length. If σ is irreducible, $\Theta(\sigma)$ has multiplicity free K-types, consisting of all $\tau(m,n)$ such that $2m+4$ is a type of σ.*

Proof. This is all trivial from the lemma; only the finite length of $\Theta(\sigma)$ perhaps merits some explanation. It is a combination of admissibility (from the lemma) and the fact that $\Theta(\sigma)$ is annihilated by an ideal in $Z(\mathfrak{g})$ of finite codimension. □

Now we go in the opposite direction. For a character $2m+4$ of $\mathrm{SO}(2)$ consider $\Theta(2m+4)$. By Proposition 3.2, it is a quotient of

$$U(\mathfrak{g}) \otimes_{U(\mathfrak{k})} F_m$$

where $F_m = \mathbb{C}$ if $m \leq 0$, otherwise

$$F_m = \tau(0,0) \oplus \tau(1,0) \oplus \cdots \oplus \tau(m,0).$$

Lemma 2.3. *Let π be a finite length $(\mathfrak{g}, K)$-module. Then*

$$\mathrm{Hom}_{\mathrm{SO}(2)}(\Theta(\pi), (2m+4)) \cong \mathrm{Hom}_{\mathfrak{g}}(\Theta(2m+4), \pi) \subseteq \mathrm{Hom}_K(F_m, \pi).$$

Proof. The isomorphism is again a see-saw identity. The inclusion follows from the Frobenius reciprocity, since $\Theta(2m+4)$ is a quotient of $U(\mathfrak{g}) \otimes_{U(\mathfrak{k})} F_m$. □

We now record an easy consequence.

Proposition 2.4. *Let π be a finite length $(\mathfrak{g}, K)$-module. Then $\Theta(\pi) \neq 0$ only if π contains a type $\tau(m,0)$ for some m. $\Theta(\pi)$ is of finite length.*

We are now ready to state and prove the main result of this paper.

Theorem 2.5. *Let σ be an irreducible $(\mathfrak{sl}_2, \mathrm{SO}(2))$-module. Assume that σ contains the type $(2m+4)$, for $m \geq 0$, and no smaller types $2n+4$, with $n \geq 0$. Then $\Theta(\sigma)$ has a unique irreducible quotient. It contains the type $\tau(m,0)$ with multiplicity one, and no types $\tau(n,0)$ with $n < m$. Conversely, let π be an irreducible $(\mathfrak{g}, K)$-module containing the type $\tau(m,0)$, and no smaller such types. If $\Theta(\pi)$ is nonzero, then $\Theta(\pi)$ has a unique irreducible quotient. It contains the type $(2m+4)$, and no smaller types $2n+4$, with $n \geq 0$.*

Proof. Assume π is a quotient of $\Theta(\sigma)$. We do not assume that π is irreducible. By Lemma 2.1, we have the sequence of inclusions

$$\begin{aligned}\mathrm{Hom}_K(\pi, \tau(m,0)) \subseteq \mathrm{Hom}_K(\Theta(\sigma), \tau(m,0)) &\cong \mathrm{Hom}_{\mathfrak{sl}_2}(\Theta(\tau(m,0)), \sigma)\\ &\cong \mathrm{Hom}_{\mathrm{SO}(2)}((2m+4), \sigma).\end{aligned}$$

We can ran this sequence with any $2n+4$ in place of $2m+4$. If $n < m$, by the assumption, the last space is trivial, hence $\tau(n,0)$ is not a type of π. We shall use this in a moment. Since π is a quotient of $\Theta(\sigma)$ and σ is irreducible, $\pi \otimes \sigma$ is a quotient of Π. But this implies that σ is a quotient of $\Theta(\pi)$, and by Lemma 2.3 we have a second sequence of inclusions (note that we are starting with the space of the same dimension as as the space we ended with in the first sequence):

$$\begin{aligned}\mathrm{Hom}_{\mathrm{SO}(2)}(\sigma, (2m+4)) \subseteq \mathrm{Hom}_{\mathrm{SO}(2)}(\Theta(\pi), (2m+4)) &\cong \mathrm{Hom}_{\mathfrak{g}}(\Theta(2m+4), \pi)\\ &\subseteq \mathrm{Hom}_K(F_m, \pi).\end{aligned}$$

Since π has no type $\tau(n,0)$ with $n < m$, we ended with $\mathrm{Hom}_K(\tau(m,0), \pi)$, which has the same dimension as $\mathrm{Hom}_K(\pi, \tau(m,0))$, the starting space in the first sequence of inclusions. Thus all inclusions in the two sequences are isomorphisms, and all spaces are one-dimensional, since $\mathrm{Hom}_{\mathrm{SO}(2)}((2m+4), \sigma)$ is one-dimensional.

However, we did not assume that π is irreducible. If $\pi = \pi_1 \oplus \pi_2$ and if we run the above argument for each π_1 and π_2, then we can write the chain

$$\begin{aligned}1+1 &= \dim \mathrm{Hom}_K(\pi_1, \tau(m,0)) + \dim \mathrm{Hom}_K(\pi_2, \tau(m,0))\\ &= \dim \mathrm{Hom}_K(\pi, \tau(m,0)) = 1,\end{aligned}$$

a contradiction. Thus $\Theta(\sigma)$ has a unique irreducible quotient. It contains $\tau(m,0)$, with multiplicity one.

In the other direction, now π is irreducible and σ is a quotient of $\Theta(\pi)$, we start with the second sequence. That sequence ends with

$$\mathrm{Hom}_K(F_m, \pi) \cong \mathrm{Hom}_K(\tau(m,0), \pi),$$

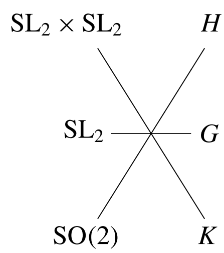
since π does not contain K-types $\tau(n,0)$ with $n < m$. Next, we run the first sequence. The conclusion is that all spaces have the same dimension d, equal to the multiplicity of $\tau(m,0)$ in π which is not 0. Again, we did not assume that σ is irreducible. So, if $\sigma=\sigma_1\oplus\sigma_2$ and we run the same argument for each σ_1 and σ_2, since

$$\dim \mathrm{Hom}_{\mathrm{SO}(2)}(\sigma_1, (2m+4)) + \dim \mathrm{Hom}_{\mathrm{SO}(2)}(\sigma_2, (2m+4)) = \dim \mathrm{Hom}_{\mathrm{SO}(2)}(\sigma, (2m+4)),$$

we arrive at $d+d=d$, a contradiction. □

3. Computing lifts of types

In this section we verify the expressions for $\Theta(\tau(m,n))$ and $\Theta(2m+4)$ used in the proof of the main result. As indicated in the introduction, we use the following see-saw diagram in $G(J)$:



Here H is a simply connected, hermitian symmetric group of absolute type E_6. Our SL_2, the centralizer of G, sits diagonally in $\mathrm{SL}_2\times\mathrm{SL}_2$, the centralizer of K. A word of caution here. If we pick a different SL_2 in $\mathrm{SL}_2\times\mathrm{SL}_2$, the one consisting of all (g, hgh^{-1}), where $h=\left(\begin{smallmatrix}1&0\\0&-1\end{smallmatrix}\right)$, then G and H in the above are replaced by their compact forms. In other words, it is important how we identify groups isomorphic to $\mathrm{SL}_2(\mathbb{R})$.

Let (e,h,f) be an $\mathfrak{sl}_2$-triple such that $\mathbb{C}\cdot h$ is the Lie algebra of SO(2). For an integer $n>0$, let $\delta(n)$ be the irreducible lowest weight n module. Let v_n be a nonzero lowest weight vector. Let $\bar\delta(m)$ be the complex conjugate of $\delta(m)$. It is the irreducible highest weight $-m$ module. Observe that there is a natural map

$$U(\mathfrak{sl}_2)\otimes_{U(\mathfrak{so}(2))}(n-m)\to\delta(n)\otimes\bar\delta(m)$$

where $1\in\mathbb{C}\cong(n-m)$ is mapped to $v_n\otimes v_m$. Since $\delta(n)$ is a free $\mathbb{C}[e]$-module generated by v_n and $\bar\delta(m)$ is a free $\mathbb{C}[f]$-module generated by v_m, the above map is easily checked to be an isomorphism.

Proposition 3.1. *Let $\tau(m,n)$ be the irreducible K-type as previously. Then*

$$\Theta(\tau(m,n))\cong U(\mathfrak{sl}_2)\otimes_{U(\mathfrak{so}(2))}\otimes(2m+4).$$

Proof. Since the centralizer of K is $\mathrm{SL}_2 \times \mathrm{SL}_2$, $\Theta(\tau(m, n))$ is naturally an $\mathrm{SL}_2 \times \mathrm{SL}_2$-module. By [13, Proposition 3.3.3] (careful with SL_2's) we have

$$\Theta(\tau(m, n)) \cong \delta(2m + n + 8) \otimes \bar{\delta}(n+4).$$

In view of the discussion above, and $(2m+n+8)-(n+4)=2m+4$, the proposition follows. □

It remains to discuss $\Theta(2m+4)$. Let L be a maximal compact subgroup of H. We can assume that $K \subset L$. Let $\mathfrak{h}$ and $\mathfrak{l}$ be the complex Lie algebras of H and L. Since H/L is a hermitian symmetric space, $\mathfrak{l}$ is a Levi subalgebra such that $[\mathfrak{l}, \mathfrak{l}]$ is a simple Lie algebra of type D_5. We have a Cartan decomposition

$$\mathfrak{h} = \bar{\mathfrak{u}} + \mathfrak{l} + \mathfrak{u}$$

such that $\mathfrak{q} = \mathfrak{l} + \mathfrak{u}$ is a parabolic subalgebra. If F is a finite-dimensional $\mathfrak{l}$-module, we can define a highest weight module

$$U(\mathfrak{h}) \otimes_{U(\mathfrak{q})} F \cong U(\bar{\mathfrak{u}}) \otimes F.$$

We now restrict this module to $\mathfrak{g} = \mathfrak{k} + \mathfrak{p}$. Recall that $\mathfrak{p}$ is 16-dimensional spin-module. On the other hand, $\bar{\mathfrak{u}}$ and $\mathfrak{u}$ are two 16-dimensional spin modules for $[\mathfrak{l}, \mathfrak{l}]$, the simple algebra of type D_5. Hence $\mathfrak{p}$ must embed diagonally into $\bar{\mathfrak{u}} + \mathfrak{u}$. Now it is not difficult to check that the natural map

$$U(\mathfrak{g}) \otimes_{U(\mathfrak{k})} F \to U(\mathfrak{h}) \otimes_{U(\mathfrak{q})} F$$

given by the identity on $1 \otimes F$ is an isomorphism. We are ready to prove the following:

Proposition 3.2. *For m integer, $\Theta(2m+4)$ is a quotient of $U(\mathfrak{g}) \otimes_{U(\mathfrak{k})} F_m$ where $F_m = \mathbb{C}$ if $m \leq 0$; otherwise*

$$F_m = \tau(0, 0) \oplus \tau(1, 0) \oplus \cdots \oplus \tau(m, 0).$$

Proof. $\Theta(2m+4)$ is an $(\mathfrak{h}, L)$-module, determined in [12, Section 6]. It is a quotient of the Verma module $U(\mathfrak{h}) \otimes_{U(\mathfrak{q})} F_m$ where F_m is a one dimensional representation of L if $m \leq 0$. Otherwise F_m, restricted to $[L, L]$, is irreducible with the highest weight $(m, 0, 0, 0, 0)$. The restriction of this representation to K is the claimed sum. □

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