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p-NUCLEARITY OF REDUCED GROUP L^p-OPERATOR ALGEBRAS

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Let $p \in (1, \infty)$. G. An, J.-J. Lee, and Z.-J. Ruan introduced p-nuclearity for L^p-operator algebras. They proved that the reduced group L^p-operator algebra $F^p_\lambda(G)$, where G is a discrete group, is p-nuclear and the p-pseudomeasure algebra $PM_p(G)$ is p-semidiscrete if G is amenable. In this paper, we show that the following are equivalent: (i) G is amenable; (ii) the reduced group L^p-operator algebra $F^p_\lambda(G)$ is p-nuclear; (iii) the p-pseudomeasure algebra $PM_p(G)$ is p-semidiscrete. This solves an open problem raised by N. C. Phillips concerning the p-nuclearity for reduced group L^p-operator algebras.

1. Introduction

For $p \in [1, \infty)$, we say that a Banach algebra A is an *L^p-operator algebra* if it is isometrically isomorphic to a norm-closed subalgebra of the algebra $\mathcal{B}(E)$ of all bounded linear operators on some L^p-space E. Clearly, L^p-operator algebras are a natural generalization of operator algebras on Hilbert spaces (and in particular C^*-algebras) by replacing Hilbert spaces with L^p-spaces.

The study of L^p-operator algebras traces back to C. Herz's influential work on harmonic analysis of group algebras in the 1970's [22; 23; 24]. For a locally compact group G, Herz introduced the Banach algebra $PF_p(G)$, defined as the operator norm closure of the image of the left regular representation $\lambda_p : L^1(G) \to \mathcal{B}(L^p(G))$. The Banach algebra $PF_p(G)$ is called p-pseudofunctions of G by C. Herz. This algebra is also called the reduced group L^p-operator algebra of G and it is denoted by $F^p_\lambda(G)$ in [18]. If $p = 2$, then $F^2_\lambda(G)$ is the reduced group C^*-algebra of G, which is usually denoted by $C^*_\lambda(G)$. We adopt the notation $F^p_\lambda(G)$ throughout the following of this paper.

Associated with $F^p_\lambda(G)$ there are two other natural algebras, the p-pseudomeasure algebra $PM_p(G)$ and the algebra of p-convolvers $CV_p(G)$. The p-pseudomeasure algebra $PM_p(G)$ is the weak* closure of $F^p_\lambda(G)$ in $\mathcal{B}(L^p(G))$. Let

$$\rho_p : G \to \mathcal{B}(l^p(G))$$

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be the right regular representation. The algebra of p-convolvers $CV_p(G)$ is the commutant of $\rho_p(G)$. If $p = 2$, then $PM_2(G)$ and $CV_2(G)$ are the group von Neumann algebra, which is denoted by $L(G)$. The reader is referred to [11; 12] for more research concerning them, especially on the problem of determining whether $PM_p(G) = CV_p(G)$.

Recently, interest in L^p-operator algebras has been renewed due to the work of N. C. Phillips. In the last ten years, Phillips introduced and studied L^p-operator algebras [31; 32; 33; 34; 35; 36; 37]. These studies encourage many authors to participate in the research of L^p-operator algebras. This includes the work on group L^p-operator algebras [18]; groupoid L^p-operator algebras [16]; L^p-operator crossed products [19; 42; 43] and the l^p-Toeplitz algebra [41]. Although most previous investigations have been very largely focused on various examples, some recent works were undertaken in a more abstract and systematic way [3; 6; 17]. Surprisingly, when $p \in [1, \infty) \setminus \{2\}$, the research on L^p-operator algebra has many wonderful results for rigidity problems [6; 7; 20]. The reader is referred to [15] for more historical comments and recent developments in L^p-operator algebras.

Nuclearity is an important property for C^*-algebras. This property was introduced by Takesaki [40]. A C^*-algebra A is called *nuclear* if for any C^*-algebra B there is a unique norm on the algebraic tensor product $A \otimes B$. By the remarkable work of Lance [28], Choi and Effros [5] and Kirchberg [26], the nuclearity is equivalent to *completely positive approximation property*, that is, there exist nets of contractive completely positive maps $\varphi_\alpha : A \to M_{n(\alpha)}$ and $\psi_\alpha : M_{n(\alpha)} \to A$ such that

$$\|\psi_\alpha \circ \varphi_\alpha(a) - a\| \to 0$$

for all $a \in A$. Also it is well known that the nuclearity is equivalent to the amenability for C^*-algebras [8; 21], which was originally introduced by B. E. Johnson [25] for Banach algebras.

The semidiscreteness of von Neumann algebras is close related to nuclearity of C^*-algebras. A von Neumann algebra M is *semidiscrete* if there exist nets of weak* continuous contractive completely positive maps $\varphi_\alpha : M \to M_{n(\alpha)}$ and $\psi_\alpha : M_{n(\alpha)} \to M$ such that

$$\langle \psi_\alpha \circ \varphi_\alpha(a) - a, f \rangle \to 0$$

for all $a \in M$ and $f \in M_*$, where M_* is the predual of M. The following theorem is a classical result concerning the nuclear reduced group C^*-algebras and semidiscrete group von Neumann algebras.

Theorem 1.1 [4, Theorem 2.6.8]. *Let G be a discrete group. The following statements are equivalent*:

(i) *G is amenable.*

(ii) *The reduced group C^*-algebra $C^*_\lambda(G)$ is nuclear.*

(iii) *The group von Neumann algebra $L(G)$ is semidiscrete.*

In [1, Proposition 5.1(a)], G. An, J.-J. Lee and Z.-J. Ruan studied p-nuclearity of reduced group L^p-operator algebra $F^p_\lambda(G)$ and p-semidiscreteness of p-pseudomeasure algebra $PM_p(G)$. They proved the following proposition.

Proposition 1.2 [1, Proposition 5.1]. *Let $p > 1$ and let G be a discrete amenable group.*

(i) *The reduced group L^p-operator algebra $F^p_\lambda(G)$ is p-nuclear.*

(ii) *The p-pseudomeasure algebra $PM_p(G)$ is p-semidiscrete.*

Since $F^1_\lambda(G)$ is always 1-nuclear for all discrete group G (see [1, Theorem 6.4]), we only consider the following problem for $p \in (1, \infty)$.

Problem 1.3 [35, Problem 10.4]. Let $p \in (1, \infty)$. If G is a discrete group and $F^p_\lambda(G)$ is p-nuclear, does it follow that G is amenable?

In this paper, we solve N. C. Phillips' problem by proving the following theorem.

Theorem 1.4. *Let $p \in (1, \infty)$ and let G be a discrete group. The following statements are equivalent*:

(i) *G is amenable.*

(ii) *$F^p_\lambda(G)$ is p-nuclear.*

(iii) *$PM_p(G)$ is p-semidiscrete.*

(iv) *There exists an isomorphism $\Phi : F^p_\lambda(G) \overset{\vee p}{\otimes} F^p_\lambda(G) \to F^p_\lambda(G) \overset{\wedge p}{\otimes} F^p_\lambda(G)$, where $\overset{\vee p}{\otimes}$ and $\overset{\wedge p}{\otimes}$ are the p-operator space injective and projective tensor products, respectively.*

(v) *The canonical linear map $h : F^p_\lambda(G) \otimes F^p_\lambda(G) \to \mathcal{B}(l^p(G))$ given by*

$$h(\lambda_p(s) \otimes \lambda_p(t)) = \lambda_p(s)\rho_p(t)$$

is continuous with respect to the p-operator space injective tensor norm, where $\otimes$ is the algebraic tensor product.

(vi) *For any $f \in C_c(G)$, we have $\|\lambda_p(f)\| \geq \left|\sum_{t\in G} f(t)\right|$.*

(vii) *For any finite subset $E \subset G$, we have $|E| = \left\|\sum_{t\in E} \lambda_p(t)\right\|$.*

Remark 1.5. Condition (vi) implies that the trivial representation 1_G extends to a representation of $F^p_\lambda(G)$. This is also equivalent to the amenability of G (see [13, Theorem 5.2]).

In the reduced group C^*-algebra case, one way to prove (ii) $\Rightarrow$ (i) of Theorem 1.1 is based on the Arveson's extension theorem (see [4, Theorem 2.6.8]). In the proof of (iii) $\Rightarrow$ (i) of Theorem 1.1, Arveson's extension theorem is also applied to construct unital completely positive map from $\mathcal{B}(l^2(G))$ to $L(G)$ that restricts to the identity on $L(G)$ (i.e., semidiscreteness implies injectivity). Arveson's extension theorem states that every contractive completely positive map $\varphi : B \to \mathcal{B}(H)$ can be extended to a contractive completely positive map $\overline{\varphi} : A \to \mathcal{B}(H)$, where A is a C^*-algebra, B is an operator subsystem of A, and H is a complex Hilbert space. However, J-J. Lee gave an example that the Arveson–Wittstock–Hahn–Banach theorem does not hold for p-operator space [30], that is, there are p-operator spaces $V \subset W$, an SQ_p-space E, and a p-completely contractive map $\varphi : V \to \mathcal{B}(E)$ such that φ does not extend to a p-completely contractive map on W. The lack of a valid Arveson–Wittstock–Hahn–Banach theorem for p-operator spaces forces us to adopt alternative approaches. Our proof of Theorem 1.4 is inspired by the method of C. Anantharaman-Delaroche (see [2, Proposition 3.5]). Her proof is based on the functorial property [2, Proposition 2.6] of spatial and maximal tensor products and weak containment of unitary representations [2, Proposition 3.5]. Our proof relies on

- the functorial properties of p-operator spaces projective and injective tensor products (see Lemma 2.4, 2.5 and the proof of (ii) $\Rightarrow$ (iv), (iii) $\Rightarrow$ (v) in Theorem 1.4);
- the uniform convexity of $l^p(G)$ that is motivated by G. Pisier (see [38, Theorem 3.30] and the proof of (vii) $\Rightarrow$ (i)).

The paper is organized as follows. In Section 2, we make some preparations for the proof of Theorem 1.4. In Section 3, we give a proof of Theorem 1.4.

2. Preliminaries

In this section, we recall some notation, definitions and lemmas for the proof of Theorem 1.4.

2.1. ***Reduced group L^p-operator algebras and p-pseudomeasure algebras.*** Let $p \in (1, \infty)$. For a discrete group G, we let $\lambda_p : G \to \mathcal{B}(l^p(G))$ denote the *left regular representation*, that is $\lambda_p(s)(\delta_t) = \delta_{st}$ for all $s, t \in G$, where $\{\delta_t\}_{t\in G}$ is the canonical basis of $l^p(G)$.

Definition 2.1. The reduced group L^p-operator algebra of G, denoted $F^p_\lambda(G)$, is the completion of $C_c(G)$ with respect to the norm $\|\lambda_p(f)\|$.

There are many equivalent definitions for amenable groups. We will use the following definition in the proof of the Theorem 1.4.

Definition 2.2 [9, Definition 11.2.3]. Let $p \in [1, \infty)$ and let G be a discrete group. The group G is amenable if there exists a net $f_\alpha \in l^p(G)$ such that $f_\alpha \geq 0$, $\|f_\alpha\|_p = 1$ and $\|\lambda_p(s) f_\alpha - f_\alpha\|_p \to 0$ for all $s \in G$.

For $p \in (1, \infty)$, and we denote by p' its conjugate exponent, which satisfies $\frac{1}{p} + \frac{1}{p'} = 1$. Let $\mathcal{N}(l^p(G)) = l^{p'}(G) \widehat{\otimes} l^p(G)$ denote the space of nuclear operators on $l^p(G)$, where $\widehat{\otimes}$ is the projective tensor product. Then $\mathcal{B}(l^p(G))$ is the dual space of $\mathcal{N}(l^p(G))$ by way of dual paring $\langle T, \xi \otimes \eta\rangle = \langle \xi, T\eta\rangle$ where $\xi \in l^{p'}(G)$ and $\eta \in l^p(G)$. We say that a net (T_α) in $\mathcal{B}(l^p(G))$ converges weak* to an operator T in $\mathcal{B}(l^p(G))$ if $\langle T_\alpha, f\rangle \to \langle T, f\rangle$ for all $f \in l^{p'}(G) \widehat{\otimes} l^p(G)$.

Definition 2.3. The p-pseudomeasure algebra of G, denoted $PM_p(G)$, is the weak* closure of $F^p_\lambda(G)$ in $\mathcal{B}(l^p(G))$.

When $p = 2$, we have $PM_2(G) = L(G)$, where $L(G)$ is the group von Neumann algebra of G.

The p-pseudomeasure algebra has a predual $A_p(G)$, that is, $PM_p(G) = A_p(G)'$ [1]. The algebra $A_p(G)$ is called the Figà–Talamanca–Herz algebra and will be introduced next. Let $\Lambda_p : l^{p'}(G) \widehat{\otimes} l^p(G) \to C_0(G)$ be given by

$$\Lambda_p(\xi \otimes \eta)(s) = \langle \xi, \lambda_p(s)\eta\rangle$$

for all $s \in G$, $\eta \in l^p(G)$, $\xi \in l^{p'}(G)$. Since $C_c(G)$ is dense in $l^p(G)$ and $l^{p'}(G)$, it follows that Λ_p maps into $C_0(G)$. Then $A_p(G)$ is defined to be the *coimage* of Λ_p, i.e., the space of $f \in C_0(G)$ for which there are $(\xi_n) \subset l^{p'}(G)$ and $(\eta_n) \subset l^p(G)$ such that

$$f(s) = \sum_{n=1}^{\infty} \xi_n * \check{\eta}_n(s) = \sum_{n=1}^{\infty} \langle \xi_n, \lambda_p(s)\eta_n\rangle$$

with norm

$$\|f\|_{A_p(G)} = \inf\left\{ \sum_{n=1}^{\infty} \|\xi_n\| \|\eta_n\| : f = \sum_{n=1}^{\infty} \xi_n * \check{\eta}_n \right\} < \infty,$$

where $\check{\eta}_n(s) = \eta_n(s^{-1})$ and $\xi_n * \check{\eta}_n(s) = \sum_{t \in G} \xi(t)\check{\eta}_n(t^{-1}s)$. It follows from [22] that $A_p(G)$ is a commutative Banach algebra with pointwise multiplication.

It follows from the definition that $A_p(G)$ can be identified with the quotient of nuclear space $\mathcal{N}(l^p(G))$. In fact, we have $A_p(G) = \mathcal{N}(l^p(G))/PM_p(G)_\perp$, where $PM_p(G)_\perp = \ker \Lambda_p$ and $PM_p(G)_\perp$ is called the pre-annihilator of $PM_p(G)$ in $\mathcal{N}(l^p(G))$. Therefore we have the isometric isomorphism $PM_p(G) = A_p(G)'$.

2.2. ***p*-operator spaces.** The notion of p-operator spaces is closely related to that of L^p-operator algebras. Let $p \in (1, \infty)$. For each positive integer n, let $l^p_n = L^p(\{1, 2, \dots, n\}, \nu)$, where ν is the counting measure on $\{1, 2, \dots, n\}$. We denote $M^p_n = \mathcal{B}(l^p_n)$. Let m be a positive integer, and we denote $M^p_{n,m} = \mathcal{B}(l^p_m, l^p_n)$. A *p-operator space* is defined to be a Banach space together with a matrix norm, i.e., a norm $\|\cdot\|_n$ on each matrix space $M_n(V)$, which satisfies the following two conditions:

(i) $\mathcal{D}_\infty : \|x \oplus y\|_{n+m} = \max\{\|x\|_n, \|y\|_m\}$ for $x \in M_n(V)$ and $y \in M_m(V)$.

(ii) $\mathcal{M}_p : \|\alpha x \beta\|_n \leq \|\alpha\| \|x\|_n \|\beta\|$ for $x \in M_n(V)$ and $\alpha, \beta \in M_n^p$.

Let V and W be p-operator spaces. We say that a linear map $\varphi : V \to W$ is p-completely bounded if

$$\|\varphi\|_{pcb} = \sup_{n \in \mathbb{Z}_{>0}} \{\|\varphi_n\|\} < \infty,$$

where $\varphi_n : [x_{ij}] \in M_n(V) \to [\varphi(x_{ij})] \in M_n(W)$ is the induced map from $M_n(V)$ to $M_n(W)$. We say that φ is a p-complete contraction (respectively, a p-complete isometry) if $\|\varphi\|_{pcb} \leq 1$ (respectively, φ_n is an isometry for each $n \in \mathbb{Z}_{>0}$).

Let E be an L^p-space and let n be a positive integer. Then

$$E^n = l^p(\{1, 2, \dots, n\}, E)$$

with the norm $\|[x_i]\| = \left(\sum_{i=1}^n \|x_i\|^p\right)^{\frac{1}{p}}$ is again an L^p-space. We can obtain a norm $\|\cdot\|_n$ on the matrix space $M_n(\mathcal{B}(E))$ by the canonical identification $M_n(\mathcal{B}(E)) \cong \mathcal{B}(E^n)$. Then it follows from [27] that $\mathcal{B}(E)$ is a p-operator space. On the other hand, Le Merdy proves that every p-operator space is p-completely isometrically isomorphic to a norm-closed subspace of $\mathcal{B}(E)$ for some $E \in SQ_p$ (see [29, Theorem 4.1]), where SQ_p is the collection of subspaces of quotients of L^p-spaces. The reader is referred to [1; 10; 30] for more research on p-operator spaces.

The p-operator space projective tensor norm $\|\cdot\|_{\wedge_p,n}$ on $M_n(V \otimes W)$ is defined by

$$\|u\|_{\wedge_p,n} = \inf\bigl\{\|\alpha\| \, \|v\| \, \|w\| \, \|\beta\| : u = \alpha(v \otimes w)\beta \\ \text{for } \alpha \in M^p_{n,kl},\ v \in M_k(V),\ w \in M_l(W) \text{ and } \beta \in M^p_{kl,n}\bigr\}.$$

We let $V \overset{\wedge p}{\otimes} W$ denote the completion of $V \otimes W$ with respect to this matrix norm, and call $V \overset{\wedge p}{\otimes} W$ the p-operator space projective tensor product of V and W.

The following lemma is a functorial property of the p-operator space projective tensor product.

Lemma 2.4 [1, p. 938]. *Let V_1, V_2, W_1 and W_2 be p-operator spaces. If*

$$u_i : V_i \to W_i, \quad i = 1, 2,$$

are p-complete contractions, then the corresponding mapping

$$u_1 \otimes u_2 : V_1 \otimes V_2 \to W_1 \otimes W_2$$

extends to a p-complete contraction

$$u_1 \overset{\wedge p}{\otimes} u_2 : V_1 \overset{\wedge p}{\otimes} V_2 \to W_1 \overset{\wedge p}{\otimes} W_2.$$

We let $\mathcal{CB}_p(V, W)$ denote the space of p-completely bounded maps from V to W. It follows from Le Merdy's characterization theorem that $\mathcal{CB}_p(V, W)$ is a p-operator space with the matrix norm given by

$$M_n(\mathcal{CB}_p(V, W)) = \mathcal{CB}_p(V, M_n(W)).$$

In particular, the dual space $V' = \mathcal{CB}_p(V, \mathbb{C})$ has a natural p-operator space structure given by

$$M_n(V') = \mathcal{CB}_p(V, M_n^p).$$

Let V and W be p-operator spaces. There exists an injective embedding

$$\theta : x \otimes y \in V \otimes W \hookrightarrow \theta(x \otimes y) \in \mathcal{CB}_p(V', W)$$

given by $\theta(x \otimes y)(f) = f(x)y$ for $f \in V'$. The completion $V \overset{\vee_p}{\otimes} W$ of $V \otimes W$ in $\mathcal{CB}_p(V', W)$ is a p-operator subspace of $\mathcal{CB}_p(V', W)$. We call $V \overset{\vee_p}{\otimes} W$ the p-operator space injective tensor product of V and W. Let $M_m(V')_1$ and $M_k(W')_1$ denote the closed unit ball of $M_m(V')$ and $M_k(W')$, respectively. It follows from [1] that for each $u \in M_n(V \otimes W)$, the p-operator space injective tensor norm $\|u\|_{\vee_p, n}$ can be expressed by

$$\|u\|_{\vee_p, n} = \sup\bigl\{ \|(\varphi \otimes \psi)_n(u)\| : \varphi \in M_m(V')_1, \psi \in M_k(W')_1, m, k \in \mathbb{Z}_{>0} \bigr\}.$$

The following lemma is a functorial property of the p-operator space injective tensor product.

Lemma 2.5 [1, p. 942]. *Let V_1, V_2, W_1 and W_2 be p-operator spaces. If*

$$u_i : V_i \to W_i, \quad i = 1, 2,$$

are p-complete contractions, then the corresponding mapping

$$u_1 \otimes u_2 : V_1 \otimes V_2 \to W_1 \otimes W_2$$

extends to a p-complete contraction

$$u_1 \overset{\vee_p}{\otimes} u_2 : V_1 \overset{\vee_p}{\otimes} V_2 \to W_1 \overset{\vee_p}{\otimes} W_2.$$

2.3. ***Spatial L^p-operator tensor products and p-completely bounded maps of L^p-operator algebras.*** Let $p \geq 1$. Let (X, μ) and (Y, ν) be two measure spaces, there is an L^p-tensor product $L^p(X, \mu) \otimes_p L^p(Y, \nu)$ which can be canonical identified with $L^p(X \times Y, \mu \times \nu)$ via $\xi \otimes \eta(x, y) = \xi(x)\eta(y)$ for all $\xi \in L^p(X, \mu)$ and $\eta \in L^p(Y, \nu)$. If $a \in \mathcal{B}(L^p(X, \mu))$ and $b \in \mathcal{B}(L^p(Y, \nu))$, then there is a corresponding tensor product operator $a \otimes b \in \mathcal{B}(L^p(X \times Y, \mu \times \nu))$. Let $A \subset \mathcal{B}(L^p(X, \mu))$ and $B \subset \mathcal{B}(L^p(Y, \nu))$ be two norm-closed subalgebras. Define an algebra

$$A \otimes_p B \subset \mathcal{B}(L^p(X \times Y, \mu \times \nu))$$

to be the closed linear span of all $a \in A$ and $b \in B$. Then $A \otimes_p B$ is an L^p-operator algebra, and it is called the spatial L^p-operator tensor product of A and B.

Remark 2.6. Let $A \subset \mathcal{B}(L^p(X, \mu))$ and $B \subset \mathcal{B}(L^p(Y, \nu))$ be L^p-operator algebras. Then it follows from [1, Theorem 3.3] that $A \overset{\vee p}{\otimes} B$ is p-completely isometric to $A \otimes_p B$.

Given a norm-closed subalgebra A of $\mathcal{B}(L^p(X, \mu))$, the spatial tensor product $M_n^p \otimes_p A$ is the L^p-matrix algebra. Clearly, each element of $M_n^p \otimes_p A$ is of form $[a_{i,j}]_{1\le i,j\le n}$ with $a_{i,j} \in A$, which is also written as $\sum_{i,j=1}^n e_{i,j} \otimes a_{i,j}$, where $\{e_{i,j}\}_{1\le i,j\le n}$ are the canonical matrix units of M_n^p.

Definition 2.7. Let A be a closed subalgebra of $\mathcal{B}(L^p(X, \mu))$, B be a closed subalgebra of $\mathcal{B}(L^p(Y, \nu))$ and φ be a linear map $\varphi : A \to B$. We denote by φ_n the map from $M_n^p \otimes_p A$ to $M_n^p \otimes_p B$ defined by

$$\varphi_n\Big(\sum_{i,j=1}^n e_{i,j} \otimes a_{i,j}\Big) = \sum_{i,j=1}^n e_{i,j} \otimes \varphi(a_{i,j})$$

for $\sum_{i,j=1}^n e_{i,j} \otimes a_{i,j} \in M_n^p \otimes_p A$. We denote

$$\|\varphi\|_{pcb} = \sup_{n\in\mathbb{Z}_{>0}} \|\varphi_n\|.$$

We say that φ is *p-completely bounded* if $\|\varphi\|_{pcb} \le C$ for some positive constant C, say that φ is *p-completely contractive* if $\|\varphi\|_{pcb} \le 1$, and say that φ is *p-completely isometric* if φ_n is isometric for all positive integer n.

2.4. *p-nuclearity and p-semidiscreteness.*

We now define p-nuclearity.

Definition 2.8 [1, Proposition 5.1(a)]. Let $(X, \mathcal{B}, \mu)$ be a measure space, and let $A \subset \mathcal{B}(L^p(X, \mu))$ be a norm-closed subalgebra. We say that A is p-nuclear if there exist nets of p-completely contractive maps $\varphi_\alpha : A \to M_{n(\alpha)}^p$ and $\psi_\alpha : M_{n(\alpha)}^p \to A$ such that

$$\|\psi_\alpha \circ \varphi_\alpha(a) - a\| \to 0$$

for all $a \in A$.

Definition 2.9. The p-pseudomeasure algebra $PM_p(G)$ is p-semidiscrete if there exist nets of weak* continuous p-completely contractive maps $\varphi_\alpha : PM_p(G) \to M_{n(\alpha)}^p$ and $\psi_\alpha : M_{n(\alpha)}^p \to PM_p(G)$ such that

$$\langle \psi_\alpha \circ \varphi_\alpha(a) - a, f\rangle \to 0$$

for all $a \in PM_p(G)$ and $f \in A_p(G)$.

When $p = 2$ and A is a C^*-algebra, R. R. Smith proved that p-nuclearity is equivalent to nuclearity (see [39, Theorem 1.1]).

Remark 2.10. The p-nuclearity is not equivalent to the amenability of L^p-operator algebras. The reader is referred to [43, Remark 1.4(iii)] for some examples.

Example 2.11 [1; 43]. Let $p \in [1, \infty)$. The following are examples of p-nuclear L^p-operator algebras:

(i) $C(X)$, where X is a compact metric space;

(ii) M_n^p and $\overline{\bigcup_{n=1}^{\infty} M_n^p}$;

(iii) the reduced group L^p-operator algebra $F_\lambda^p(G)$, where G is a discrete amenable group;

(iv) the L^p-Cuntz algebra $\mathcal{O}_d^p$;

(v) the rotation L^p-operator algebras $F^p(\mathbb{Z}, F^p(\mathbb{Z}), \beta_\theta)$ and $F^p(\mathbb{Z}, S^1, \alpha_\theta)$.

3. Proof of Theorem 1.4

We show (i) $\Rightarrow$ (ii) $\Rightarrow$ (iv) $\Rightarrow$ (v) $\Rightarrow$ (vi) $\Rightarrow$ (vii) $\Rightarrow$ (i), then (i) $\Rightarrow$ (iii) $\Rightarrow$ (v); this will prove Theorem 1.4.

(i) $\Rightarrow$ (ii): This follows from [1, Proposition 5.1].

(ii) $\Rightarrow$ (iv): Let $\varphi_\alpha : F_\lambda^p(G) \to M_{n(\alpha)}^p$ and $\psi_\alpha : M_{n(\alpha)}^p \to F_\lambda^p(G)$ be the nets of p-completely contractive maps such that

$$\|\psi_\alpha \circ \varphi_\alpha(a) - a\| \to 0$$

for all $a \in F_\lambda^p(G)$. Since $M_{n(\alpha)}^p$ has p-OAP, it follows from [1, Theorem 3.12] that there exists an isomorphism Ψ from $M_{n(\alpha)}^p \overset{\vee p}{\otimes} F_\lambda^p(G)$ to $M_{n(\alpha)}^p \overset{\wedge p}{\otimes} F_\lambda^p(G)$. By Lemmas 2.4 and 2.5, we have

$$\begin{tikzcd}
F_\lambda^p(G) \overset{\vee p}{\otimes} F_\lambda^p(G) \arrow[r, "\varphi_\alpha \overset{\vee p}{\otimes} \mathrm{Id}_{F_\lambda^p(G)}"] & M_{n(\alpha)}^p \overset{\vee p}{\otimes} F_\lambda^p(G) \arrow[d, "\Psi"] \\
F_\lambda^p(G) \overset{\wedge p}{\otimes} F_\lambda^p(G) & M_{n(\alpha)}^p \overset{\wedge p}{\otimes} F_\lambda^p(G) \arrow[l, "\psi_\alpha \overset{\wedge p}{\otimes} \mathrm{Id}_{F_\lambda^p(G)}"']
\end{tikzcd}$$

Define $\Phi_\alpha = (\psi_\alpha \overset{\wedge p}{\otimes} \mathrm{Id}_{F_\lambda^p(G)}) \circ \Psi \circ (\varphi_\alpha \overset{\vee p}{\otimes} \mathrm{Id}_{F_\lambda^p(G)})$. Then Φ_α is a bounded linear map from $F_\lambda^p(G) \overset{\vee p}{\otimes} F_\lambda^p(G)$ to $F_\lambda^p(G) \overset{\wedge p}{\otimes} F_\lambda^p(G)$. We denote by $\mathrm{Id}_{F_\lambda^p(G)} \otimes \mathrm{Id}_{F_\lambda^p(G)} : F_\lambda^p(G) \otimes F_\lambda^p(G) \to F_\lambda^p(G) \otimes F_\lambda^p(G)$ the algebraic tensor product map. Since $\|\Phi_\alpha(x) - \mathrm{Id}_{F_\lambda^p(G)} \otimes \mathrm{Id}_{F_\lambda^p(G)}(x)\| \to 0$ for all $x \in F_\lambda^p(G) \otimes F_\lambda^p(G)$, it follows that $\mathrm{Id}_{F_\lambda^p(G)} \otimes \mathrm{Id}_{F_\lambda^p(G)}$ is a bounded linear map from the p-operator space injective tensor norm on $F_\lambda^p(G) \otimes F_\lambda^p(G)$ to the p-operator space projective tensor norm on $F_\lambda^p(G) \otimes F_\lambda^p(G)$. Hence it extends to a bounded linear map $\Phi : F_\lambda^p(G) \overset{\vee p}{\otimes} F_\lambda^p(G) \to F_\lambda^p(G) \overset{\wedge p}{\otimes} F_\lambda^p(G)$. Since $\|\cdot\|_{\wedge_p}$ is the largest p-operator space norm [10, Proposition 4.8], it follows that Φ is an isomorphism. This proves (iv).

(iv) $\Rightarrow$ (v): Recall that $\rho_p : G \to \mathcal{B}(l^p(G))$ is the right regular representation. We denote by $\lambda_p \cdot \rho_p$ the biregular representation $(s,t) \to \lambda_p(s)\rho_p(t)$ of $G \times G$ on $l^p(G)$. Since $\|\cdot\|_{\wedge_p}$ is the largest p-operator space norm [10, Proposition 4.8], it follows that the canonical linear map $h : F^p_\lambda(G) \otimes F^p_\lambda(G) \to B(l^p(G))$ defined by

$$h(\lambda_p(s) \otimes \lambda_p(t)) = \lambda_p(s)\rho_p(t)$$

has a continuous extension on $F^p_\lambda(G) \overset{\wedge p}{\otimes} F^p_\lambda(G)$ and it is denoted by $(\lambda_p \cdot \rho_p)_\lambda$. By (iv), there exists a bounded linear map from $F^p_\lambda(G) \overset{\vee p}{\otimes} F^p_\lambda(G)$ to $B(l^p(G))$, which is denoted by $\widetilde{(\lambda_p {\cdot} \rho_p)}_\lambda$. This proves (v).

(v) $\Rightarrow$ (vi): We recall that σ_p is the conjugacy representation $s \mapsto \lambda_p(s)\rho_p(s)$ of G on $l^p(G)$. By (iv), the following diagram is commutative:

$$\begin{tikzcd}
F^p(G) \arrow[r, "\sigma_p"] \arrow[d, "\lambda_p"] & B(l^p(G)) \\
F^p_\lambda(G) \arrow[r, "\iota"] & F^p_\lambda(G) \overset{\vee p}{\otimes} F^p_\lambda(G). \arrow[u, "\widetilde{(\lambda_p \cdot \rho_p)}_\lambda"']
\end{tikzcd}$$

Here $\iota(s) = \lambda_p(s) \otimes \lambda_p(s)$ for each $s \in G$. Let $\theta = \widetilde{(\lambda_p {\cdot} \rho_p)}_\lambda \circ \iota$. Then $\sigma_p = \theta \circ \lambda_p$.

Claim 1: $\|\theta\| \le 1$. To see this, recall that $\rho_p(s)\delta_t = \delta_{ts^{-1}}$ for all $s \in G$. Then

$$\|\lambda_p(f)\| = \|\rho_p(f)\|$$

for all $f \in C_c(G)$. In fact, let $V : l^p(G) \to l^p(G)$ be the invertible isometry given by $V\delta_t = \delta_{t^{-1}}$. Then one can check that $V\lambda_p(f)V^{-1}\delta_t = \rho_p(f)\delta_t$. This shows that $\|\lambda_p(f)\| = \|\rho_p(f)\|$.

For any $f \in C_c(G)$ with $\|\lambda_p(f)\| \le 1$ and $\xi \in l^p(G)$, we have

$$\|\theta(f)\xi\| = \|\lambda_p(f)\rho_p(f)\xi\| \le \|\lambda_p(f)\| \cdot \|\rho_p(f)\| \cdot \|\xi\| = \|\lambda_p(f)\|^2 \cdot \|\xi\|.$$

Hence $\|\theta(f)\| \le \|\lambda_p(f)\|^2 \le 1$, and therefore $\|\theta\| \le 1$. This proves Claim 1.

Now we will prove (vi). Since $\sigma_p = \theta \circ \lambda_p$ and $\|\theta\| \le 1$, it follows that

$$\|\sigma_p(f)\| = \|\theta \circ \lambda_p(f)\| \le \|\lambda_p(f)\|.$$

It is easy to check that $\sigma_p(s)\delta_e = \delta_e$. Hence

$$\|\lambda_p(f)\| \ge \|\sigma_p(f)\| \ge \|\sigma_p(f)\delta_e\| = \Big|\sum_{t \in G} f(t)\Big|.$$

This proves (vi).

(vi) $\Rightarrow$ (vii): For any finite subset $E \subset G$, by (vi), we have $\big\|\sum_{t \in E} \lambda_p(t)\big\| \ge |E|$. Obviously, $\big\|\sum_{t \in E} \lambda_p(t)\big\| \le |E|$. This proves (vii).

(vii) $\Rightarrow$ (i): For any finite subset $E \subset G$, we can assume that $e \in E$, where e is the unit of G. By (v), we have $\big\|\sum_{t \in E} \lambda_p(t)/|E|\big\| = 1$. Then there exists a sequence

(ξ_i) in $l^p(G)$ such that $\xi_i \geq 0$, $\|\xi_i\|_p = 1$ and $\left\|\sum_{t\in E} \lambda_p(t)\xi_i/|E|\right\| \to 1$. Since $l^p(G)$ is a uniformly convex Banach space for $p \in (1, \infty)$, it follows from [14] that $l^p(G)$ is a full k-convex Banach space for all positive integer $k \geq 2$. Then

$$\|\lambda_p(s)\xi_i - \lambda_p(t)\xi_i\|_p \to 0$$

for all $s, t \in E$. Since $e \in E$, it follows that

$$\|\lambda_p(s)\xi_i - \xi_i\|_p \to 0$$

for all $s \in E$. Then there exists a net (η_α) in $l^p(G)$ such that $\eta_\alpha \geq 0$, $\|\eta_\alpha\|_p = 1$ and

$$\|\lambda_p(s)\eta_\alpha - \eta_\alpha\|_p \to 0$$

for all $s \in G$. By Definition 2.2, we have that G is amenable.

(i) $\Rightarrow$ (iii): It follows from [1, Proposition 5.1].

(iii) $\Rightarrow$ (v): Let $\varphi_\alpha : PM_p(G) \to M^p_{n(\alpha)}$ and $\psi_\alpha : M^p_{n(\alpha)} \to PM_p(G)$ be the nets of weak* continuous p-completely contractive maps such that

$$\langle \psi_\alpha \circ \varphi_\alpha(a) - a, f\rangle \to 0$$

for all $a \in PM_p(G)$ and $f \in A_p(G)$.

Since $M^p_{n(\alpha)}$ has p-OAP, it follows from [1, Theorem 3.12] that there exists an isomorphism Ψ from $M^p_{n(\alpha)} \overset{\vee p}{\otimes} PM_p(G)$ to $M^p_{n(\alpha)} \overset{\wedge p}{\otimes} PM_p(G)$. By Lemmas 2.4 and 2.5, we have

$$\begin{tikzcd}
PM_p(G) \overset{\vee p}{\otimes} PM_p(G) \arrow[r, "\varphi_\alpha \overset{\vee p}{\otimes} \mathrm{Id}_{PM_p(G)}"] & M^p_{n(\alpha)} \overset{\vee p}{\otimes} PM_p(G) \arrow[d, "\Psi"] \\
PM_p(G) \overset{\wedge p}{\otimes} PM_p(G) & M^p_{n(\alpha)} \overset{\wedge p}{\otimes} PM_p(G) \arrow[l, "\psi_\alpha \overset{\wedge p}{\otimes} \mathrm{Id}_{PM_p(G)}"']
\end{tikzcd}$$

Since $\|\cdot\|_{\wedge_p}$ is the largest p-operator space norm [10, Proposition 4.8], it follows that the canonical linear map

$$h : PM_p(G) \otimes PM_p(G) \to B(l^p(G))$$

defined by

$$h(\lambda_p(s) \otimes \lambda_p(t)) = \lambda_p(s)\rho_p(t)$$

has a continuous extension on $PM_p(G) \overset{\wedge p}{\otimes} PM_p(G)$ and it is denoted by $(\lambda_p \cdot \rho_p)_\lambda$.

Then we can define a net of bounded linear maps

$$\Phi_\alpha : PM_p(G) \overset{\vee p}{\otimes} PM_p(G) \to \mathcal{B}(l^p(G))$$

by $\Phi_\alpha = ((\lambda_p \cdot \rho_p)_\lambda) \circ (\psi_\alpha \overset{\wedge p}{\otimes} \mathrm{Id}_{PM_p(G)}) \circ \Psi \circ (\varphi_\alpha \overset{\vee p}{\otimes} \mathrm{Id}_{PM_p(G)})$. Since $\mathcal{B}(l^p(G)) = \mathcal{N}(l^p(G))'$, it follows from [4, Theorem 1.3.7] that there exists a point-weak* cluster point Φ of the net (Φ_α). Then we get a bounded linear map

$$\Phi : PM_p(G) \overset{\vee}{\otimes}^{p} PM_p(G) \to \mathcal{B}\left(l^p(G)\right).$$

<u>Claim 2</u>: *The bounded linear map Φ extends the map $h : PM_p(G) \otimes PM_p(G) \to \mathcal{B}\,(l^p(G))$ given by $h\left(\lambda_p(s) \otimes \lambda_p(t)\right) = \lambda_p(s)\rho_p(t)$.*

Indeed, since $PM_p(G)$ is p-semidiscrete, we have

$$\langle \psi_\alpha \circ \varphi_\alpha(\lambda_p(s)) - \lambda_p(s), f\rangle = \sum_{n=1}^{\infty} \left\langle \xi_n, \left(\psi_\alpha \circ \varphi_\alpha(\lambda_p(s)) - \lambda_p(s)\right)(\eta_n)\right\rangle \to 0,$$

for all $f = \sum_{n=1}^{\infty} \xi_n * \check{\eta}_n \in A_p(G)$ and $s, t \in G$. Then, for any

$$g = \sum_{n=1}^{\infty} x_n \otimes y_n \in l^{p'}(G) \widehat{\otimes} l^p(G),$$

we have

$$\begin{aligned}
&\left\langle \Phi_\alpha(\lambda_p(s) \otimes \lambda_p(t)) - \lambda_p(s)\rho_p(t), g\right\rangle \\
&\qquad = \left\langle (\lambda_p \cdot \rho_p)_\lambda\left(\psi_\alpha \circ \varphi_\alpha(\lambda_p(s)) \otimes \lambda_p(t)\right) - \lambda_p(s)\rho_p(t), g\right\rangle \\
&\qquad = \left\langle \psi_\alpha \circ \varphi_\alpha(\lambda_p(s))\rho_p(t) - \lambda_p(s)\rho_p(t), g\right\rangle \\
&\qquad = \left\langle \left(\psi_\alpha \circ \varphi_\alpha(\lambda_p(s)) - \lambda_p(s)\right)\rho_p(t), g\right\rangle \\
&\qquad = \sum_{n=1}^{\infty} \langle x_n, \left(\psi_\alpha \circ \varphi_\alpha(\lambda_p(s)) - \lambda_p(s)\right)\rho_p(t) y_n\rangle \\
&\qquad \to 0.
\end{aligned}$$

It follows that

$$\begin{aligned}
&\left|\langle \Phi(\lambda_p(s) \otimes \lambda_p(t)) - \lambda_p(s)\rho_p(t), g\rangle\right| \\
&\qquad \le \left|\langle \Phi(\lambda_p(s) \otimes \lambda_p(t)) - \Phi_\alpha(\lambda_p(s) \otimes \lambda_p(t)), g\rangle\right| \\
&\qquad + \left|\langle \Phi_\alpha(\lambda_p(s) \otimes \lambda_p(t)) - \lambda_p(s)\rho_p(t), g\rangle\right| \\
&\qquad \to 0.
\end{aligned}$$

Hence $\Phi(\lambda_p(s) \otimes \lambda_p(t)) = h(\lambda_p(s) \otimes \lambda_p(t))$, proving Claim 2. Then (v) follows easily. ☐

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