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We previously explicitly described all homogeneous extensions ϕ_m, $m \in \mathbb{Z}$, of $C(S_0^{2\ell+1})$ by $C(\mathbb{T}) \otimes \mathcal{K}$ (*Pacific J. Math.* 288:2 (2017), 435–452). However, for $m = 0$, the given extension is not homogeneous. In this corrigendum, we correct this mistake by constructing a trivial homogeneous extension of $C(S_0^{2\ell+1})$ by $C(\mathbb{T}) \otimes \mathcal{K}$. The K_0-groups of the middle C^*-algebra turn out to be the same as those mentioned in the original article. Consequently, the proofs of all results in that paper remain valid.

1. Correction

For $m \in \mathbb{Z}$, define the $*$-homomorphisms ϕ_m as

$$\begin{aligned}
\phi_m : C(S_0^{2\ell+1}) &\to Q\big(\mathcal{K}(\ell^2(\mathbb{N})^{\otimes(\ell+1)} \otimes C(\mathbb{T}))\big),\\
S^* \otimes 1 \otimes \cdots \otimes 1 &\mapsto S^* \otimes 1 \otimes \cdots \otimes 1,\\
p \otimes S^* \otimes 1 \otimes \cdots \otimes 1 &\mapsto p \otimes S^* \otimes 1 \otimes \cdots \otimes 1,\\
&\vdots\\
p \otimes p \otimes \cdots \otimes p \otimes S^* \otimes 1 &\mapsto p \otimes p \otimes \cdots \otimes S^* \otimes 1 \otimes 1,\\
p \otimes p \otimes \cdots \otimes t &\mapsto p \otimes p \otimes \cdots \otimes p \otimes (S^*)^m \otimes 1.
\end{aligned}$$

In [Saurabh 2017], it is claimed that the ϕ_m's are essential unital extensions. Moreover, since the last component is 1, these extensions are also homogeneous. For $m \neq 0$, the claim is indeed true. However, for $m = 0$, the claim does not hold. To see this, note that

$$\phi_0(p^{\otimes\ell} \otimes t) = p^{\otimes\ell} \otimes 1 \otimes 1 = \phi_0(p^{\otimes\ell} \otimes 1).$$

This proves that ϕ_0 is not injective; hence, it is not an essential extension and, consequently, not a homogeneous extension.

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To rectify this, we need to construct a trivial homogeneous extension of $C(S_0^{2\ell+1})$ by $C(\mathbb{T})\otimes\mathcal{K}$. First, recall from [Saurabh 2017], the construction of a homogeneous extension $\Sigma^2\tau$ of $\Sigma^2 A$ induced from a homogeneous extension τ of A. Let $\mathcal{H}=\ell^2(\mathbb{N})$. Let τ be a unital homogeneous extension of A by $C(Y)\otimes\mathcal{K}(\mathcal{H})$. Define $\tilde{\tau}: A\to Q(C(Y)\otimes\mathcal{K}(\mathcal{H})\otimes\mathcal{K}(\mathcal{H}))$ by $\tilde{\tau}(a)=[\tau_a\otimes p]$, where $[\tau_a]=\tau(a)$. By the universal property of quantum double suspension (see [Hong and Szymański 2008, Proposition 2.2]), we have a homomorphism

$$\Sigma^2\tau : \Sigma^2 A \to Q(C(Y)\otimes\mathcal{K}(\mathcal{H})\otimes\mathcal{K}(\mathcal{H})) \tag{1-1}$$

such that

$$\Sigma^2\tau(a\otimes p)=\tilde{\tau}(a)=[\tau_a\otimes p] \quad \text{and} \quad \Sigma^2\tau(1\otimes S)=[1\otimes 1\otimes S].$$

One can show that $\Sigma^2\tau$ is a unital homogeneous extension. This induces a map

$$\beta : \mathrm{Ext}_{\mathrm{PPV}}(Y, A)\to \mathrm{Ext}_{\mathrm{PPV}}(Y, \Sigma^2 A), \quad [\tau]_u\mapsto[\Sigma^2\tau]_u.$$

From Proposition 2.4 and Lemma 2.8 in [Saurabh 2017], one concludes that if $Y=\mathbb{T}$, then β is an isomorphism between $\mathrm{Ext}_{\mathrm{PPV}}(\mathbb{T}, A)$ and $\mathrm{Ext}_{\mathrm{PPV}}(\mathbb{T}, \Sigma^2 A)$.

Let $\Upsilon := \{q\in\mathbb{C} : 0<|q|<1,\ \theta=\frac{1}{\pi}\arg(q) \text{ is irrational}\}$. Fix $q\in\Upsilon$. From Theorem 3.9 in [Guin and Saurabh 2022], it follows that the extension

$$\mathcal{E}_q : 0\to\mathcal{K}(\ell^2(\mathbb{N}))\otimes C(\mathbb{T})\to C(U_q(2)/_\psi\mathbb{T})\xrightarrow{\vartheta} C(\mathbb{T})\to 0$$

is a trivial homogeneous extension of $C(\mathbb{T})$ by $C(\mathbb{T})\otimes\mathcal{K}$. Therefore the equivalence class $[\phi_q]$ of the Busby invariant ϕ_q of $\mathcal{E}_q$ is a zero element in $\mathrm{Ext}_{\mathrm{PPV}}(\mathbb{T}, C(\mathbb{T}))$. Then $(\Sigma^2\Sigma^2\cdots\Sigma^2)^\ell\phi_q$ is a homogeneous extension of $(\Sigma^2\Sigma^2\cdots\Sigma^2)^\ell C(\mathbb{T})$ by $C(\mathbb{T})\otimes\mathcal{K}$. Hence its equivalence class $[(\Sigma^2\Sigma^2\cdots\Sigma^2)^\ell\phi_q]$ gives a zero element of $\mathrm{Ext}_{\mathrm{PPV}}(\mathbb{T}, C(S_0^{2\ell+1}))$. Further, note that the middle C^*-algebra of $(\Sigma^2\Sigma^2\cdots\Sigma^2)^\ell\mathcal{E}_q$ is $(\Sigma^2\Sigma^2\cdots\Sigma^2)^\ell C(U_q(2)/_\psi\mathbb{T})$. Using Theorem 3.4 of [Guin and Saurabh 2022] and the short term exact sequence

$$0\to A\otimes\mathcal{K}\to\Sigma^2 A\to C(\mathbb{T})\to 0$$

for any unital C^*-algebra A, it is easy to verify that

$$K_0((\Sigma^2\Sigma^2\cdots\Sigma^2)^\ell C(U_q(2)/_\psi\mathbb{T}))\cong\mathbb{Z}\oplus\mathbb{Z}.$$

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