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DESIGN CRITERIA FOR ADDED DAMPERS AND SUPPORTING BRACES

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The papers deals with the use of dissipative braces as retrofit solutions for existing moment resisting frame buildings. Braces are widely used in order to enhance performances of existing buildings under seismic loads, by adding stiffness and strength against inertial forces induced by earthquake ground motions. The braces can be equipped with supplemental dissipators in order to increase the overall energy dissipation capacity of the system and reduce stresses in the existing structures. In the present work, general design criteria for dampers and supporting braces are given and a simple design procedure based on the actual mechanical interaction between dampers and braces has been carried out. A number of design procedures have been proposed for dissipative bracing systems in frame structures. The procedures are often based on simplifying assumptions, due to the complexity of mechanical behavior of systems equipped with dissipative braces. Those assumptions make the procedures easier to use, but at the same time, less reliable in predicting the behavior of complex structures. In the present work, results are obtained without using two of the most common simplifying assumptions that neglect interaction between frame and braces: the use of the floor stiffness in order to characterize the frame behavior, and the use of equivalent systems with a single degree of freedom. The proposed design procedure has been tested on a moment resisting frame building and appears feasible for implementation on real structures.

1. Introduction

In recent years, many important changes in seismic codes are occurred. Most of the changes in the seismic design area derive from greater comprehension of actual poor buildings performances in recent earthquakes. Due to the renewed knowledge of the existing buildings behavior, retrofit of buildings is a paramount task in reducing seismic risk. New techniques for protecting buildings against earthquake have been developed with the aim of improving their capacity. Seismic isolation and energy dissipation are widely recognized as effective protection techniques for reaching the performance objectives of modern codes. However, many codes include design specifications for seismically isolated buildings, while there is still need of improved rules for energy dissipation protective systems. FEMA 356 [FEMA 2000] is one of the first prestandards that gives general criteria for the design of dissipative braces. According to this document, dissipative braces, added in a structure, should be able to ensure the necessary increment in stiffness for the protection in the Immediate Occupancy Performance and the necessary supplemental damping for the protection in the Life Safety Performance. However, design rules for specific devices (viscous, friction, steel devices) are still missing.

A large amount of research has been concerned with development of these innovative earthquake resistant systems. In many studies involving parametric analyses [Choi et al. 2003; Phocas and Pocanschi 2003; Whittaker et al. 2003a; 2003b; Wu and Ou 2003; Lin and Chopra 2002; Goel 2000; 2001; Singh

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and Moreschi 2001; Pekcan and Chen 1999; Shukla and Datta 1999; Fu and Kasai 1998] and design procedures [Park and Min 2004; Lee et al. 2004; Moreschi and Singh 2003; Singh and Moreschi 2002; Garcia 2001; Levy et al. 2000; Fu and Cherry 2000; Yamada 2000; Takewaki 1999; Gluck et al. 1997; Ciampi 1993; Filiatrault and Cherry 1990], the structure, equipped with braces and dampers, is modeled as a simple mechanical system. This approach leads to a significant reduction of the complexity of the problem that has to be solved for evaluating the response of the system, but at the same time it leads to approximations, in some cases not acceptable, in the assessment of the behavior of the actual system.

In this paper, two of the most common simplifications that neglect interaction between frame and braces have been analyzed and their effects on the evaluation of the actual behavior of structural systems have been quantified. The study has been conducted on systems that can be modeled as viscoelastic components according to FEMA 356. The proposed design procedure for dissipative braces uses the results of a parametric study considering the actual interaction between frame and different types of dissipative braces and has been validated on a seven-storey frame building through non linear numerical analyses.

2. Problem statement

The analyzed structural systems are the moment resisting frames that are typical systems for the modern building and well suited to be protected with braces equipped with dissipative braces. Only limited damages are tolerated on the frames so the structures are expected to remain in the elastic range. For this reason the frames are modeled in order to perform with a linear-viscous-elastic behavior.

2A. ***Frame without braces.*** The dynamic characteristic of the frames, subjected to external forces, is formulated for structures discretized with a finite number of degrees of freedom (DOFs) and defined in term of generalized displacements of the nodes. The equation of the motion for a generic elastic multiple degree of freedom (MDOF) frame structure is

$$m_s\ddot{u}(t)+c_s\dot{u}(t)+k_su(t)=p(t) \tag{2-1}$$

where m_s, c_s and k_s are the mass, damping and stiffness matrices of the frame structure without braces, respectively, u is the displacement vector, t is the time variable and p is the external force vector.

The following assumptions are generally accepted in design of frames subjected to seismic forces:

(i) Each floor is assumed to be rigid in its own plane.

(ii) A mass lumped matrix is used to describe inertial effects.

(iii) Structural damping is expressed as a function of the mass and stiffness matrices (*classical damping*).

Assumption (i) is generally appropriate for reinforced concrete buildings with floor slabs or in steel frame buildings with steel floor bracings; assumption (ii) is generally accepted for multi-storey buildings, in which the greater amount of mass is at the floor levels; assumption (iii) permits to neglect the terms related to the viscous forces and to consider only mass and stiffness proportional terms. Under those assumptions, the dynamic problem can be reduced to a smaller one by relating certain degrees of freedom to certain others by means of constraint equations and considering only the mass and stiffness terms. With this aim in mind, (2-1) can be written as

$$\begin{bmatrix} m_{s,tt} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{u}_t(t) \\ \ddot{u}_0(t) \end{bmatrix} + \begin{bmatrix} k_{s,tt} & k_{s,t0} \\ k_{s,0t} & k_{s,00} \end{bmatrix} \begin{bmatrix} u_t(t) \\ u_0(t) \end{bmatrix} = \begin{bmatrix} p_t(t) \\ 0 \end{bmatrix}, \tag{2-2}$$

where u_t and u_0 denote the displacements along DOFs with mass (dynamic DOFs) and with zero mass, respectively, and p_t are the external dynamic forces acting on the frame.

The displacement vector partition introduced in (2-2) leads to the equation

$$m_{s,tt}\ddot{u}_t(t) + \hat{k}_{s,tt}u_t(t) = p_t(t), \tag{2-3}$$

where $\hat{k}_{s,tt}$ is the condensed stiffness matrix of the frame defined as

$$\hat{k}_{s,tt} = k_{s,tt} - k^T_{s,0t}k^{-1}_{s,00}k_{s,0t}. \tag{2-4}$$

2B. *Frame with braces.* Following the same approach presented in the previous section, for the system composed by frame and braces the equation of motion could be expressed as

$$\begin{bmatrix} m_{s+b,tt} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{u}_t(t) \\ \ddot{u}_0(t) \end{bmatrix} + \begin{bmatrix} k_{s+b,tt} & k_{s+b,t0} \\ k_{s+b,0t} & k_{s+b,00} \end{bmatrix} \begin{bmatrix} u_t(t) \\ u_0(t) \end{bmatrix} = \begin{bmatrix} p_t(t) \\ 0 \end{bmatrix}, \tag{2-5}$$

where $m_{s+b} = m_s + m_b$ is the system mass matrix given by the sum of the frame mass matrix m_s and the braces mass matrix m_b, and $k_{s+b} = k_s + k_b$ is the system stiffness matrix given by the sum of the frame stiffness matrix k_s and the braces stiffness matrix k_b.

Considering (2-5), the static condensation of this system leads to the equation

$$m_{s+b,tt}\ddot{u}_t(t) + \hat{k}_{s+b,tt}u_t(t) = p_t(t), \tag{2-6}$$

where $\hat{k}_{s+b,tt}$ is the condensed stiffness matrix of the frame defined as

$$\hat{k}_{s+b,tt} = k_{s+b,tt} - k^T_{s+b,0t}k^{-1}_{s+b,00}k_{s+b,0t}. \tag{2-7}$$

From (2-5), the displacement vector associated to the DOFs with zero mass can be evaluated through the expression

$$u_0(t) = -k^{-1}_{s+b,00}k_{s+b,0t}u_t(t). \tag{2-8}$$

3. Interaction between frame and braces

Two of the most common assumptions used in design procedures and parametric studies of braces systems assume that (1) each floor of the frame is characterized by a floor stiffness, and (2) the frame can be reduced to an equivalent system with a single degree of freedom. According to the first assumption, the storey drift is function of the shear forces induced by horizontal seismic loads. Generally, this assumption leads to two possible model simplifications:

(1a) The shear-type floor stiffness is obtained imposing that flexural and shear deformations of the beams and axial deformations of the columns are null. This case will be referred as "shear-type" floor stiffness $k_{s,st}$.

(1b) The shear-type floor stiffness is obtained as the ratio between shear forces and storey drift displacements computed on the frame under known horizontal forces, without imposing null deformations. This case will be referred as "equivalent shear-type" floor stiffness, related to the condensed stiffness matrix of the frame without braces $\hat{k}_s$.

For the second assumption, the fundamental mode shape is used in order to reduce the multiple degree of freedom (MDOF) model to a single degree of freedom (SDOF) model. Both assumptions are not able to describe the actual interaction between the frame and braces but are commonly used to reduce the complexity of the problem to be solved and generally accepted for analyzing frames equipped with dissipative braces. Their validity is questionable when the actual system behavior becomes more complex, i.e., when the braces have a stiffness comparable to the frame elements stiffness and the building modifies the shape of its fundamental vibration modes after the insertion of the braces.

3A. ***Floor stiffness assumption.*** The use of the floor stiffness instead of the whole stiffness matrix is based on the assumption that the braces have no effects on the stiffness of the storey in which they are installed. In this case the floor stiffness of the frame can be added to the stiffness of the braces to evaluate the whole floor stiffness. It is clear, however, that the interaction between frame and braces modifies the frame behavior. The stiffness of each floor is, in fact, influenced by the braces as a function of k_b and k_s. The force vector $p_s(t)$, carried only by the frame for a given displacement vector $u(t)$, can be obtained as

$$\begin{bmatrix} k_{s,tt} & k_{s,t0} \\ k_{s,0t} & k_{s,00} \end{bmatrix} \begin{bmatrix} u_t(t) \\ u_0(t) \end{bmatrix} = \begin{bmatrix} p_{s,t}(t) \\ p_{s,t}(t) \end{bmatrix}. \tag{3-1}$$

Considering that $u_0(t)$ is given by (2-8), this equation can be replaced by

$$\hat{k}_{s*,tt} u_t(t) = p_{s,t}. \tag{3-2}$$

where the condensed stiffness matrix of the frame when the braces are installed is defined as

$$\hat{k}_{s*,tt} = k_{s,tt} - k^T_{s,0t} k^{-1}_{s+b,00} k_{s+b,0t}. \tag{3-3}$$

Similarly, the force vector $p_b(t)$, carried only by the braces for a given displacement vector $u(t)$, can be obtained as

$$\begin{bmatrix} k_{b,tt} & k_{b,t0} \\ k_{b,0t} & k_{b,00} \end{bmatrix} \begin{bmatrix} u_t(t) \\ u_0(t) \end{bmatrix} = \begin{bmatrix} p_{b,t}(t) \\ p_{b,t}(t) \end{bmatrix}, \tag{3-4}$$

where the condensed stiffness matrix of the braces is defined as

$$\hat{k}_{b,tt} = k_{b,tt} - k^T_{b,0t} k^{-1}_{s+b,00} k_{s+b,0t}. \tag{3-5}$$

The condensed stiffness matrices given by (2-4) and (3-3) represent the stiffness of the same frame when no braces are installed and when the braces are installed, respectively. A comparison between those equations indicates that the presence of braces provides a variation in the condensed stiffness matrix of the frame. The variation is expressed as

$$\Delta\hat{k}_{s,tt} = \hat{k}_{s*,tt} - \hat{k}_{s,tt} = k^T_{s,0t}\big(k^{-1}_{s,00} k_{s,0t} - k^{-1}_{s+b,00} k_{s+b,0t}\big). \tag{3-6}$$

In addition, the condensed stiffness matrix of the braces, given by (3-5), includes the terms $k_{s+b,00}$ and $k_{s+b,0t}$ function of the frame stiffness. The interaction between frame stiffness and brace performance is

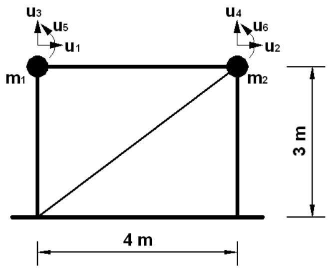

Figure 1. One-bay frame case study.

	EA	EI	GA_s
Beam	4.50×10^6 kN	1.13×10^6 kNm^2	1.44×10^6 kN
Columns	2.70×10^6 kN	2.43×10^5 kNm^2	8.64×10^5 kN
Brace	8.64×10^5 kN		

Table 1. Mechanical characteristics of the one-bay frame case study.

then to be expected. The floor and brace stiffnesses, computed separately, can be added only accepting an error in the final estimate of the whole floor-stiffness. In the following example, the interaction between the frame and brace stiffnesses has been studied for a simple one bay frame with a diagonal brace. A planar frame, with masses lumped at the column-beam joints and a steel brace with circular section pinned to the frame, was analyzed as first case study (Figure 1).

The mass of the brace is assumed to be negligible and inertial effects due to the ground motion are considered only in the horizontal direction. Seismic action on the frame have been described as equivalent horizontal static forces applied to the joints and the mechanical characteristics of the components are reported in Table 1, where A is the sectional area of the element, I is the moment of inertia, A_s is the shear area, E is the longitudinal elastic modulus and G is the shear elastic modulus. The variation of each stiffness contribution is reported in Figure 2 as a function of the cross sectional area of the brace A_b normalized to the reference area reported in Table 1. The reference area has been chosen in

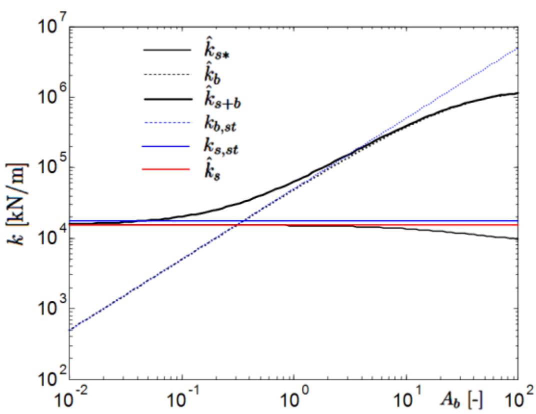

Figure 2. Stiffness components versus brace sectional area A_b.

order to represent a quite usual situation in steel frame protected by braces, corresponding to a brace stiffness approximatively two times higher than the frame stiffness versus horizontal forces. From the graph, it is evident that the stiffness $k_{s,st}$ overestimates of approximatively 30% $\hat{k}_s$. Moreover, the actual stiffness of the frame $\hat{k}_{s*}$ is affected by the presence of the brace and for $A_b = 1$ the stiffness reduction is approximatively 15% of the stiffness of the frame without brace $\hat{k}_s$. The frame stiffness reduction increases with the sectional area of the brace. For $A_b = 10$ the stiffness reduction of the frame is around 50%. Finally, the stiffness of the brace is also influenced by the interaction with the frame. For $A_b = 1$ the interaction produces a reduction of the brace stiffness $\hat{k}_b$ approximatively equal to 5% of its theoretical "shear-type" stiffness $k_{b,st}$, linearly increasing with A_b. For $A_b = 10$ the interaction produces a brace stiffness reduction of almost 90%.

To express the interaction between brace and frame, an index R was defined as

$$R = \frac{i^T p_{s,t}}{i^T p_t}, \tag{3-7}$$

where i is the dynamic coupling vector composed by unit-components in the earthquake direction and null-components in the other directions, $p_{s,t}$ is the portion of the force vector carried by the frame, as derived from (3-2), and p_t is the force vector acting on the whole system composed by frame and braces. Index R represents the ratio between the shear forces acting on the frame and the shear forces acting on the overall system, under a fixed displacement. It expresses the ratio between the floor stiffness of the frame and the floor stiffness of the whole system. For the case study of Figure 1, the index R is presented in Figure 3 as a function of the braces cross sectional area A_b. The graph shows how the stiffness of the overall braced frame is shared between the frame and the brace. As the area A_b of the brace increase, brace stiffness becomes the contribute more important to the whole stiffness while frame stiffness becomes less significant ($R \to 0$). Let us note that the R is strongly not linear, versus A_b, due to the interaction between frame and brace. By using the floor stiffness assumption a linear trend for R would instead be assumed. In the middle range of the horizontal axis, say $0.1 \le A_b \le 5$, corresponding to usual brace stiffness values varying between 0.1 and 10 times the frame stiffness, the curve trend is approximatively linear, meaning that the forces carried by the frame decrease with the logarithm of the sectional area A_b of the brace and not linearly with the area A_b, as expected according to the floor stiffness assumption.

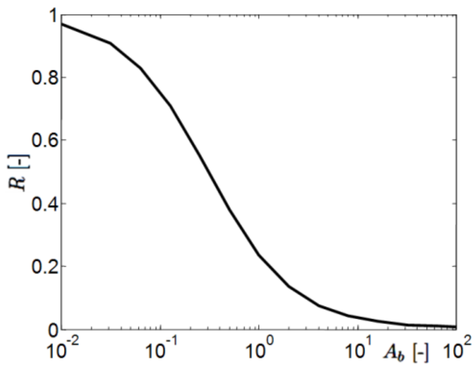

Figure 3. Index R versus brace sectional area A_b.

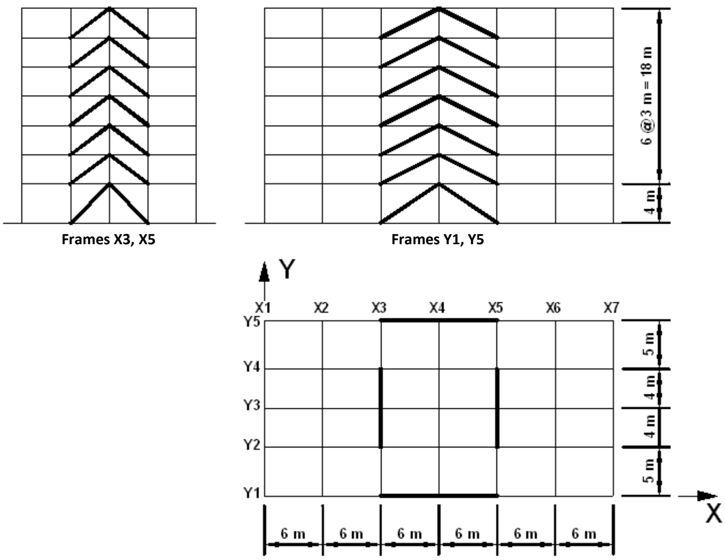

Figure 4. MDOF frame case study.

3B. ***Equivalent SDOF system assumption.*** Equivalent SDOF systems are used for reducing the complexity of the mechanical problem to be solved. A MDOF system can be reduced to an equivalent SDOF system assuming that its deformed shape is known under dynamic forces applied. In this case, mechanical parameters, describing the mechanical behavior of the system, are reduced to its fundamental vibration period T_s and damping ratio ξ_s. By using this simplification, effects of dissipative braces on the behavior of the SDOF system are easy to be quantified. They generally introduce a reduction of the vibration period, due to the increment of stiffness, from the value T_s to the value T_{s+b}, as well as the increment of the damping ratio, due to added energy dissipation capacity, from the value ξ_s to the value ξ_{s+b}. However the equivalent SDOF system can not describe the actual interaction between the frame structure and the bracing system, as shown as an example in the next case study. The 7-storey frame building, shown in Figure 4, was analyzed. The mechanical characteristics of the frame are listed in Table 2. Inertial forces are considered only in the horizontal directions and each floor is assumed to perform as a rigid diaphragm. Forces acting in the y direction were considered and the following linear path of seismic forces used was

$$p_t = F \frac{m_{s+b} h}{i^T m_{s+b} h} = \phi_p F, \tag{3-8}$$

	EA	EI	GA_s
Lateral and longitudinal beams	4.50×10^6 kN	1.13×10^6 kNm^2	1.44×10^6 kN
Internal transversal beams	4.32×10^6 kN	2.49×10^5 kNm^2	1.38×10^6 kN
Columns	6.30×10^6 kN	3.09×10^6 kNm^2	2.02×10^6 kN
Braces	2.36×10^8 kN		

Table 2. Mechanical characteristics of the MDOF frame case study.

where F is the total seismic force entity, h is a vector containing the heights of each floor, measured from the foundation level, and ϕ_p is the vector expressing the seismic force distribution. As described before, interaction effects are captured by R index. For the forces of (3-8) the following expression of R is obtained from (3-7) and considering (2-6) and (3-2):

$$R = \frac{i^T \hat{k}_{s*,tt} \hat{k}_{s+b,tt}^{-1} \phi_p}{i^T \phi_p}. \tag{3-9}$$

Note that R depends only on the shape of the seismic forces path and not on their amplitude. For the selected case study, index $R = 19.6\%$ indicates that the frame stiffness to the overall structural stiffness is 19.6%. Reduction of the MDOF system to an equivalent SDOF system overestimates this contribution. Since the total mass variation due to braces is neglected, for the equivalent SDOF system, the contribution of the frame stiffness to the whole system stiffness is given by

$$\left(\frac{T_s}{T_{s+b}} \right)^2 = \frac{k_s}{k_{s+b}} = 23.3\%. \tag{3-10}$$

with $T_{s+b} = 0.882\,\text{s}$ the fundamental vibration period of the whole system, $T_s = 1.826\,\text{s}$ the fundamental vibration period of the frame without braces and k_s, k_{s+b} the SDOF equivalent stiffnesses for the frame with no braces and with braces, respectively. For the selected case study, the frame stiffness contribution computed using the SDOF assumption in (3-10) appears about 1.19 times the effective one, computed with (3-9). By using the reduction to a SDOF system, the portion of seismic forces carried by the frame is hence overestimated by 19%.

4. Interaction between brace and damper

Dissipative braces are commonly applied to structure as integral devices that exhibit both functions of stiffnesses and energy dissipators in a single mechanical system as well as combination of different devices with different functional contribution. Both configurations will be referred here as dissipative braces, while the energy dissipation capabilities will be associated to elements generically defined as dampers to be intended as single units as well as components of integral devices. In dissipative bracing system, the interaction between the brace and the additional damping effects has to be considered. To evaluate this interaction, devices characterized by the following behaviors have been considered: viscous linear (VL), viscous nonlinear (VN), viscous elastic (VE), elastoplastic (EP), and friction (FR). The selected behaviors cover the most common typologies of dampers used for protection of frame buildings. It is clear that the installation of generic damper devices reduces the stiffness of the brace. Total strains in each dissipative brace are obtained as the sum of strains in the brace and strains in the damper. Accordingly, the R index evaluated for the structure with only the braces, is always not lower than the one computed for the dissipative bracing system R_d by means of (3-7). The portion of total seismic forces carried by the frame is, in fact, greater if there are dampers installed. This loss of efficiency, in terms of stiffness, for the braces alone can be quantified by the ratio

$$F_R = \frac{R_d}{R} \leq 1. \tag{4-1}$$

The larger the loss of efficiency of the braces due to the deformability of the dampers, the smaller is the parameter F_R.

The second effect of the addition of the dampers is the reduction of seismic forces acting on the whole system due to added energy dissipation capacity. The seismic force vector should be reduced due to higher dissipation capacity of the system. An important issue is the evaluation of the effective dissipation capacity of dissipative braces. It is evident also that the dissipated energy in dampers depends on the stiffness of the supporting braces. For a given damper, the maximum amount of dissipated energy is obtained if the supporting brace has null deformability. In this case, total strains in the dissipative brace affect the dampers performance and produce energy dissipation. The energy dissipation capacity is expressed in terms of equivalent damping ratio, evaluated for stationary oscillations as

$$\xi_{s+b} = \frac{1}{4\pi} \frac{E_d}{E_s} \frac{\omega_{s+b}}{\omega}, \tag{4-2}$$

where E_d is the dissipated energy in the deformation cycle, E_s is the maximum strain energy, ω is the circular frequency of the deformation cycle and ω_{s+b} is the equivalent circular frequency of the system composed by the structure and the braces, defined as

$$\omega_{s+b} = \sqrt{\frac{i^t p_{t,\max}}{i^t m_{s+b} u_{\max}}}, \tag{4-3}$$

where $p_{t,\max}$ is the force vector corresponding to the maximum displacement vector $u_{\max}$.

By using (4-2) and (4-3), the maximum equivalent damping ratio ξ_{s+d} can be computed assuming supporting braces of infinite stiffness. The ratio ξ_{s+d} results always greater than the effective damping ratio of the system ξ_{s+b} evaluated for braces with finite stiffness. Accordingly, the quantity

$$\xi_d = \xi_{s+d} - \xi_s \tag{4-4}$$

represents the maximum damping ratio that the given dampers can add to the original damping ratio ξ_s of the structure, while the actual damping ratio added by the dissipative braces is given by

$$\xi_b = \xi_{s+b} - \xi_s. \tag{4-5}$$

If the phenomenon of interaction between bracing effects and dissipation of energy is considered in terms of damping contribution, it is evident that braces make the dampers less efficient in supporting seismic forces because they reduce their energy dissipation capacity. This loss of efficiency, in terms of damping, can be quantified by the ratio

$$F_\xi = \frac{\xi_b}{\xi_d} \le 1. \tag{4-6}$$

The larger is the loss of efficiency of the dampers due to deformability of the braces, the smaller is the parameter F_ξ.

In Figures 5–9, the variation of R_d, F_R and ξ_{s+b}, F_ξ are represented versus R for different values of ξ_d with the aim of describing the loss of efficiency in terms of stiffness and in terms of damping, respectively. On the abscissa, lower values of R represent stiffer supporting braces, while $R = 1$ represents the frame without braces. The dampers mechanical characteristics have been chosen in order to obtain values of the maximum added damping ratio ξ_d in the range 0.05–0.25. For the damping exponent of the VN

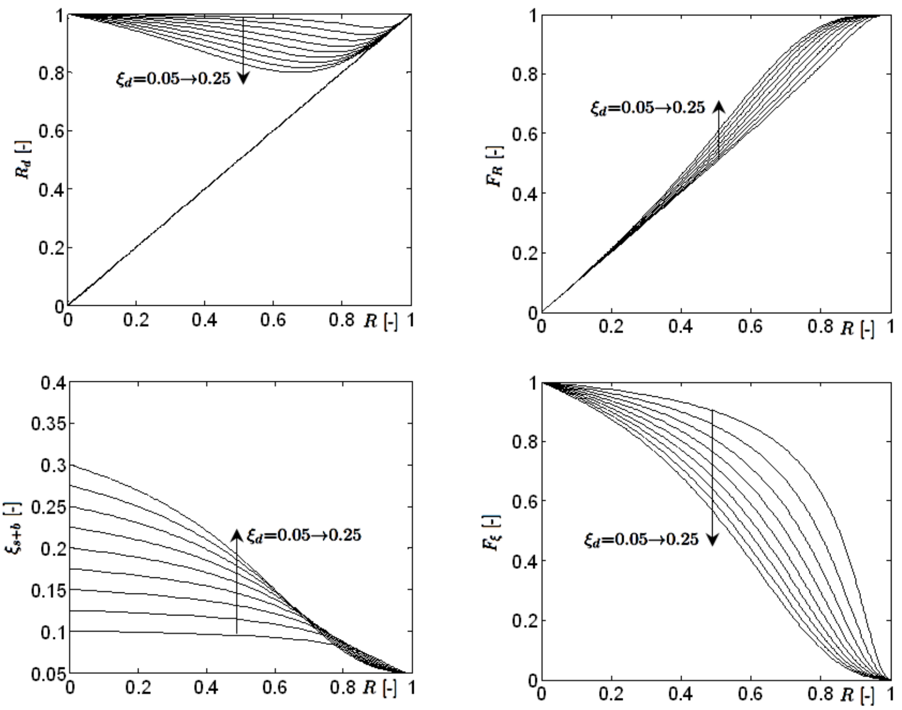

Figure 5. Variation of R_d, F_R. ξ_{s+b}, and F_ξ with R ($\xi_d = 0.05$–0.25) for VL dampers.

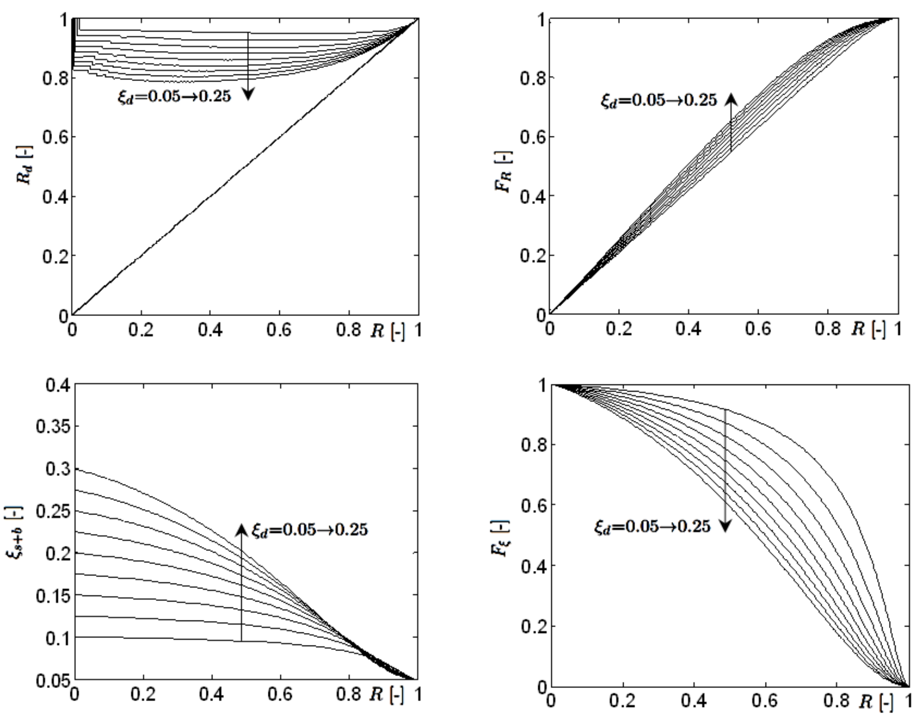

Figure 6. Variation of R_d, F_R. ξ_{s+b}, and F_ξ with R ($\xi_d = 0.05$–0.25) for VN dampers.

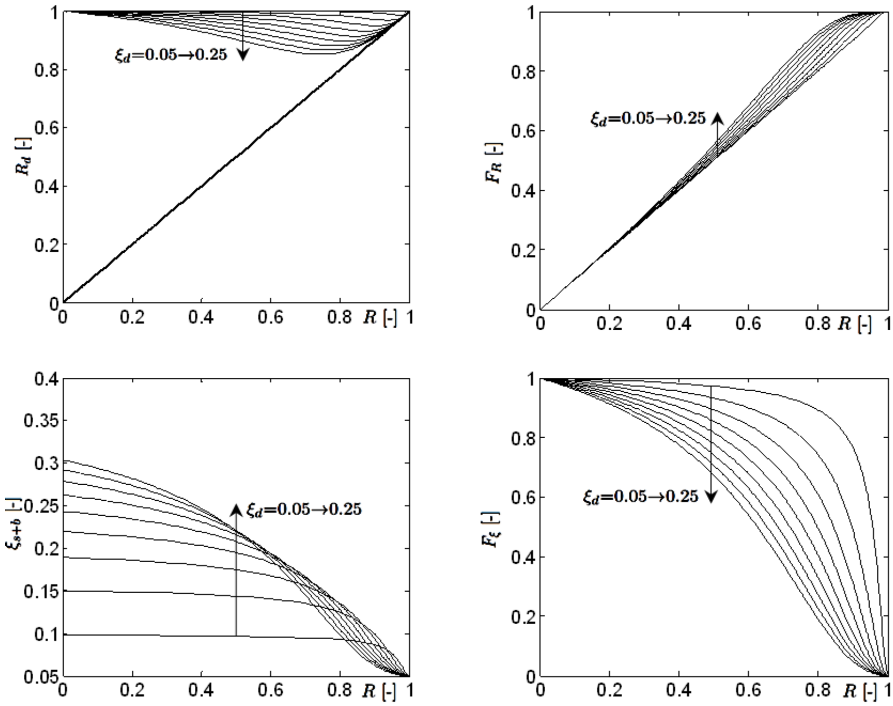

Figure 7. Variation of R_d, F_R. ξ_{s+b}, and F_ξ with R ($\xi_d = 0.05$–0.25) for VE dampers.

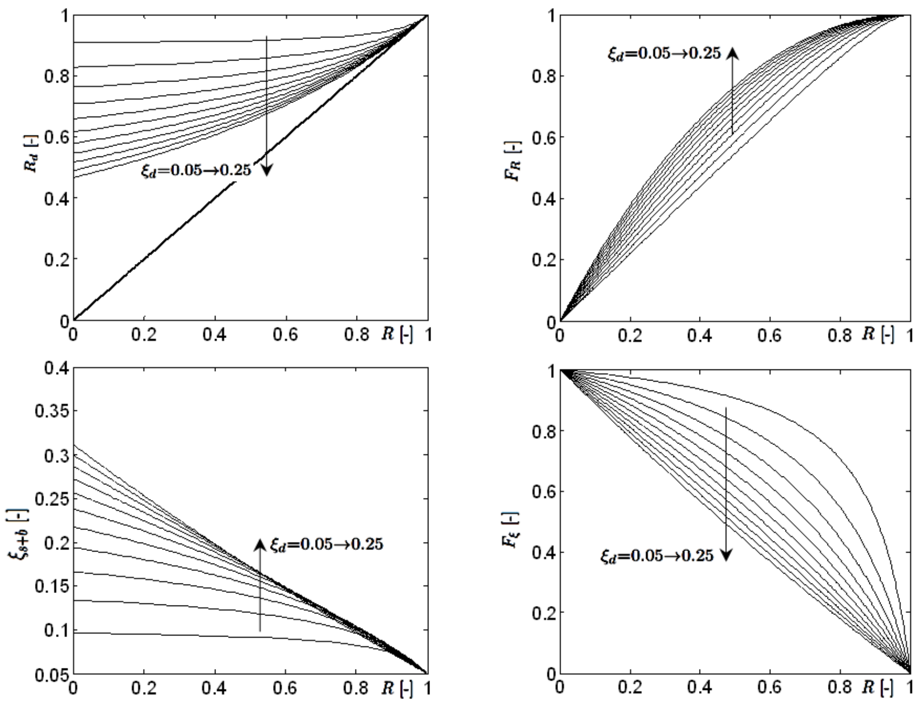

Figure 8. Variation of R_d, F_R. ξ_{s+b}, and F_ξ with R ($\xi_d = 0.05$–0.25) for EP dampers.

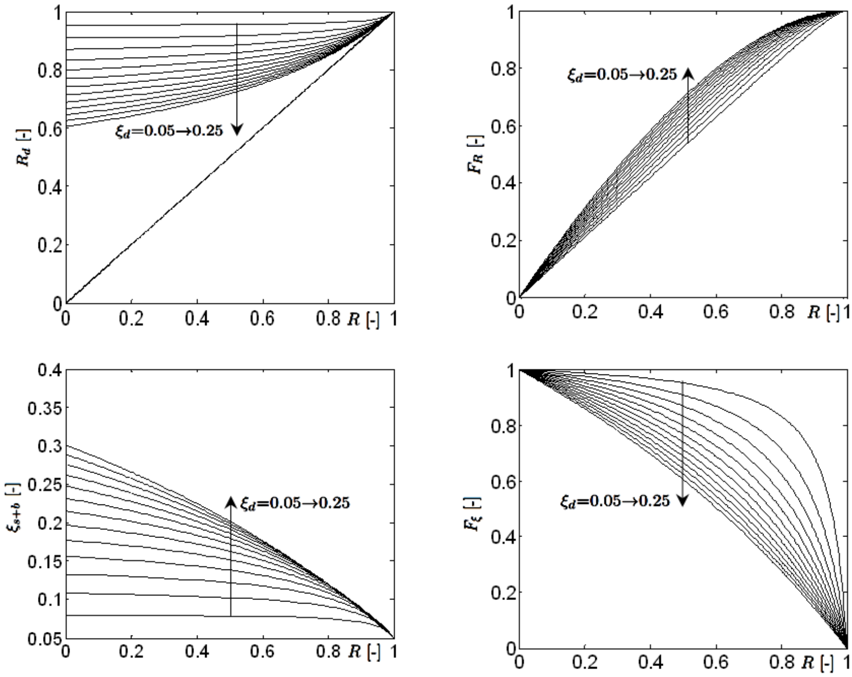

Figure 9. Variation of R_d, F_R, ξ_{s+b}, and F_ξ with R ($\xi_d = 0.05$–0.25) for FR dampers.

dampers the value $\alpha = 0.15$ was considered, for VE dampers a loss factor $\eta_L = 0.80$ was assumed and for the EP dampers the hardening ratio of $\beta = 0.01$ was used.

From these figures, it is evident that the losses of efficiency in terms of stiffness and damping act in a different way. Factor F_R shows that the greater losses of efficiency occurs at low values of R, (stiffer braces), and at low values of ξ_d (for less dissipative dampers). On the other side, factor F_ξ shows that the greater losses of efficiency occur for higher values of R (smaller braces), and for higher values of ξ_d (for more dissipative dampers). For supporting braces with the same stiffness of the frame ($R = 0.5$) and for a damping ratio $\xi_{s+b} = 0.30$, losses of stiffness of 64%, 58%, 78%, 24%, and 40% were experienced due to the insertion of damper types VL, VN, VE, EP, and FR, respectively. EP dampers are then preferable when higher stiffness increments are needed while VE dampers are most indicated for lower increments of stiffness. For supporting braces with $R = 0.5$ and with an additional damping $\xi_d = 0.25$, losses of damping of 33%, 30%, 23%, 40%, and 27% correspond to types VL, VN, VE, EP, and FR, respectively. VE dampers are then less affected by the brace stiffness while EP require very stiff supporting braces in order to ensure their dissipation capacity. The previously defined factors could be combined in a factor describing the global efficiency of the braces:

$$F(R, \xi_d) = F_R(R, \xi_d) \times F_\xi(R, \xi_d). \tag{4-7}$$

The factor F describes losses of efficiency in terms of both stiffness and damping: dissipative braces with greater values of F are the most efficient both for stiffness and damping. As an example, the left part of Figure 10 shows the F factor versus R index for braces equipped with VL dampers. Values of F factor are always lower than unity. In the presented case, maximum value of F is equal to 0.55 for a R

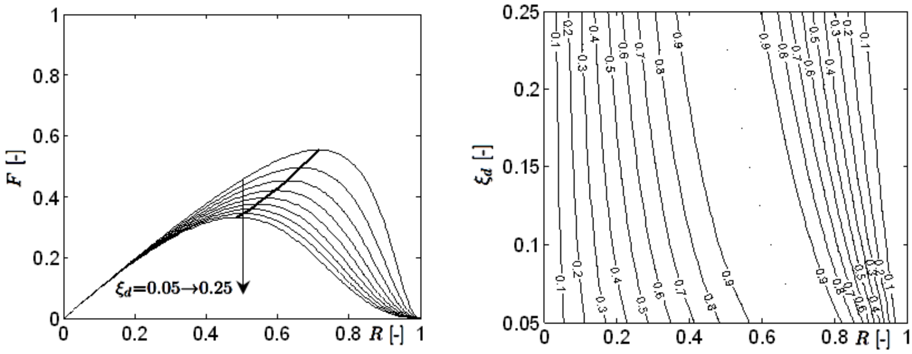

Figure 10. Variation of F with R (left) and contour levels of F^* in the plane R-ξ_d for VL dampers.

value of approximately 0.7. The solid line represents the maximum values of F factor for values of the damping ratio ranging from 0.05 to 0.25.

To assign the value of 1 to the most efficient case (maximum F) a normalization procedure was applied:

$$F^*(R, \xi_d) = \frac{F(R, \xi_d)}{\max\big(F_\xi(R, \xi_d)\big)} \leq 1. \tag{4-8}$$

Figure 10, right, shows the dependence of F^* on R and ξ_d using by contour levels. In the area between the two curves labeled 0.9, the maximum efficiency both in terms of stiffness and damping selection is achieved. Maximum efficiency is observed for the range of damping values 0.05–0.25 centered on different values of R, expressing the stiffness contribution to the total stiffness of the frame alone. It can be observed that, for instance, for a value of ξ_d equal to 0.05 (limited additional damping) the center of the efficiency band corresponds to R approximately equal to 0.7. This scenario indicates that a limited damping addition is particularly beneficial for frames quite stiff originally with limited contribution of the braces to the total stiffness. For larger introduced dissipation capability ($\xi_d = 0.25$) the efficiency is maximized for frames where braces contribute about 50% of the total stiffness. An increase or reduction of additional stiffness reduces the efficiency of the overall system.

Similar charts can be obtained for all the selected damper typologies and could indicate variations of the location of the maximum efficiency range. Figure 11 reports the maximum efficiency range

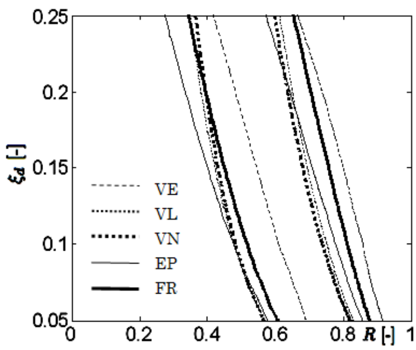

Figure 11. Contour levels corresponding to $F^* = 0.90$ in the plane R-ξ_d.

($F^* = 0.90$) for all the damper types considered. These charts provide useful information for the design and selection of dissipative braces.

5. Design procedure

This section shows how the presented charts can be used for the design of bracing systems. The proposed design procedure essentially follows five steps:

Step 1. Selection of the performance requirements (performance level and reference seismic action).

Step 2. Selection of the target damping level and reduction of the seismic action. The damping level is expressed by the damping ratio ξ_{s+b} for the whole system.

Step 3. Design of elastic braces. For the elastic bracing system, a vector containing R indices computed by (3-7) is evaluated for each level of the frame in order to quantify the interaction between the frame and the bracing system.

Step 4. Choice of characteristics of dissipative braces. The characteristics of dissipative braces depend on the R values selected at Step 3 and on the design charts presented in Figure 12.

Step 5. Validation of the design solution. The behavior of the system composed by the frame and the dissipative braces is studied through non linear analyses in order to verify that the performance requirements are satisfied.

As an example of the procedure application and validity, the 7-storey frame of Figure 4 is considered. The system has four diagonal bracings for each direction. The procedure is applied as described below.

Step A1. Performance requirements are statements of acceptable performance of the structure. The performance target can be specified as limits on any response parameter such as stresses, strains, displacements, accelerations. In the case study, target inter-storey drifts of 0.3% are considered, assuming that the frame should remain elastic under the design seismic action. The elastic spectrum Type 1 given by Eurocode 8 [CEN 2004] for ground type A, with ground acceleration equal to ag=0.35g, is assumed in the design. Seven ground motions were selected by means of specialized software [Gasparini and Vanmarcke 1976] in order to obtain an average acceleration spectrum matching the elastic design spectrum, in accordance with Eurocode 8.

Step A2. The damping ratio $\xi_{s+b} = 0.20$ is chosen as target damping level for the frame equipped with dissipative braces. According to Eurocode 8, the spectrum reduction factor which takes into account damping is $\eta = 0.63$.

Step A3. An elastic bracing system has been designed to ensure compliance with target inter-storey drifts. At each floor level all the braces have the same geometrical characteristics. The vector containing the R indices of the braces, at each floor level, from the bottom to the top of the building, is

$$R = [0.92\ \ 0.81\ \ 0.86\ \ 0.86\ \ 0.89\ \ 1.00\ \ 1.00]\,.$$

Note that for the two upper levels no braces are necessary in order to satisfy the performance requirements ($R = 1.00$).

Step A4. The dissipative braces at each level will have $R_d = R$ and they must provide an additional damping in order to obtain the target damping ratio ξ_{s+b} for the whole system. The supplemental damping

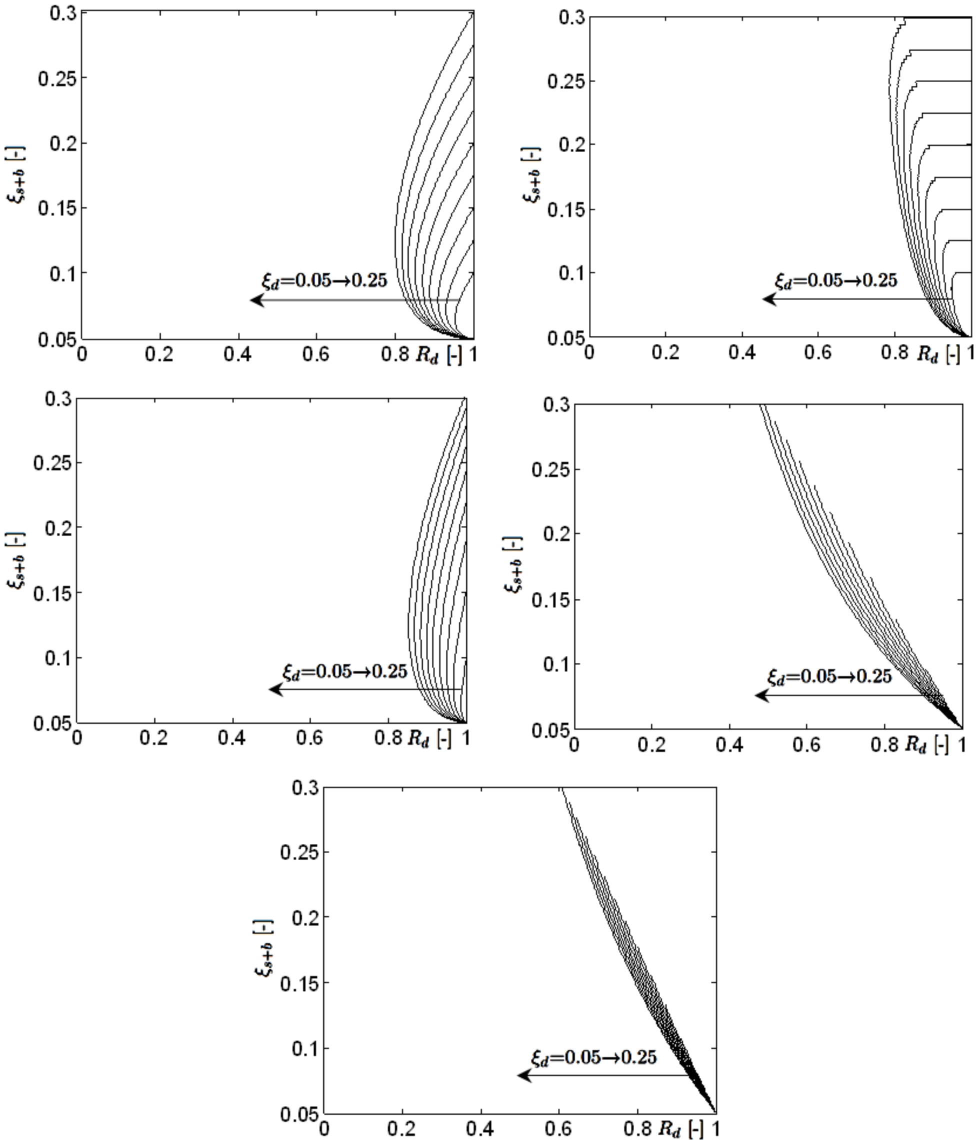

Figure 12. Design charts for different types of dampers: variation of ξ_{s+b} with R_d ($\xi_d = 0.05$–0.25). Clockwise from top left: VL, VN, EP, FR, VE.

value is obtained as the weighted average of the damping value of each storey, proportionally to the storey shear forces. In the analyzed case, the vector containing the damping ratios for each floor level is

$$\xi_{s+b} = [0.29 \;\; 0.27 \;\; 0.23 \;\; 0.18 \;\; 0.12 \;\; 0.05 \;\; 0.05]\,.$$

From R_d and ξ_{s+b} values, the additional damping ratios ξ_d provided by each damper can be estimated from the charts presented in Figure 12. The R values, characterizing the stiffness of the supporting braces, can be calculated from the charts presented in Figures 5–9. Different types of dampers can be chosen

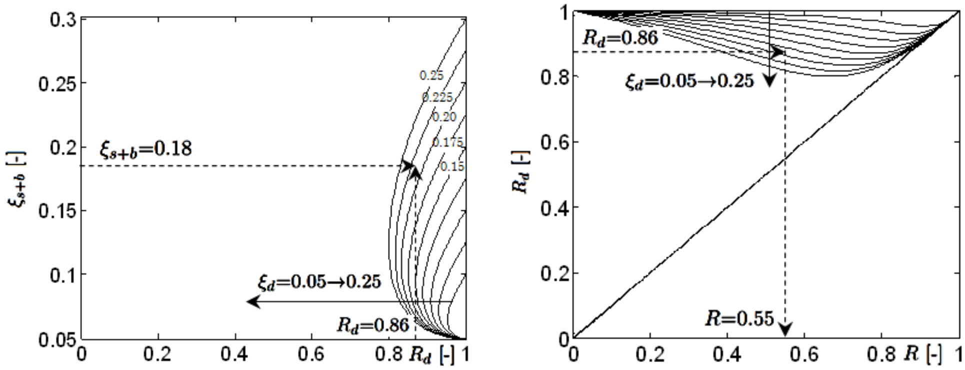

Figure 13. Left: estimation of ξ_d from ξ_{s+b} and R_d. Right: estimation of R from ξ_d and R_d values. In both cases, ξ_d ranges from 0.05 to 0.25.

in order to obtain the desired characteristics. The most efficient ones have been chosen according to the F^* index of (4-8). For example, selection for the fourth storey is presented in Figure 13, left. Given a value of $R_d = 0.86$ and a damping ratio $\xi_{s+b} = 0.18$, the chart indicates an optimum value for ξ_d equal to 0.21. The estimate for the R index is presented in Figure 13, right. Given $R_d = 0.86$ and $\xi_d = 0.21$, the chart indicates an optimum value of 0.55 for R.

The dissipative braces selected for the fourth level are VL dampers characterized by $\xi_d = 0.21$ and supported by elastic braces with $R = 0.55$. It is evident that the interaction between brace and damper implies that the additional damping ratio due to damper $\xi_d = 0.21$ must be higher than the value required at the storey level $\xi_b = \xi_{s+b} - \xi_s = 0.18 - 0.05 = 0.13$, where $\xi_s = 0.05$ is the damping ratio of the elastic structure, because of the loss of efficiency due to the deformability of the brace. At the same time, the elastic braces could support $(1 - R) = (1 - 0.55) = 0.45$ times the whole seismic forces acting at the storey level. They however suffer a reduction of stiffness for the presence of the damper and are able to carry only $(1 - R_d) = (1 - 0.86) = 0.14$ times the whole seismic forces. Results for all the levels of the frame are presented in Table 3.

Note that the required level of damping is obtained by the use of different dampers. VL dampers are most indicated for less stiff braces with lower dissipation capacity, while VN dampers are preferred where more stiffness and damping is required, i.e., at lower levels.

Level	Damper	R	ξ_d
1	Type VN	0.10	0.25
2	Type VN	0.45	0.22
3	Type VN	0.45	0.20
4	Type VL	0.55	0.21
5	Type VL	0.70	0.18
6	—	—	—
7	—	—	—

Table 3. Dissipative braces characteristics (R, ξ_d) derived from the design procedure.

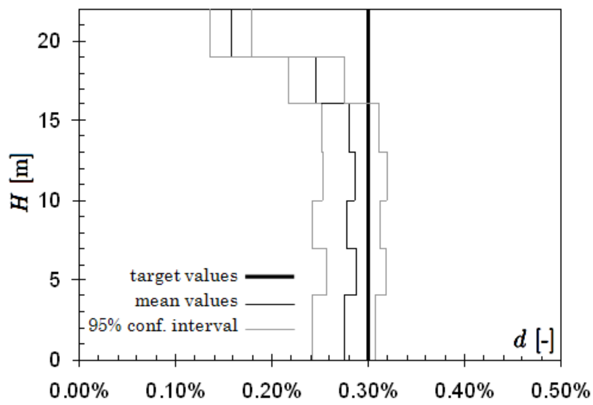

Figure 14. Inter-storey drifts d along the height H of the building.

Step A5. A numerical investigation has been carried out for the structure under consideration in order to evaluate the effects produced by the insertion of the dissipative braces. The braces were modeled as purely elastic springs and purely viscous dampers connected in series. The viscous dampers are characterized by a damping coefficient $c = \xi_d m_d \omega_{s+b}/2$, where m_d is the modal mass supported by the damper and ω_{s+b} is the circular frequency of the fundamental mode of the system. Values of the damping exponent α equal to 1 and 0.15 are assumed for VL and VN dampers, respectively. In addition to the damping provided by the dissipative braces, a global damping ratio of 0.05 for the structure was included using the Caughey damping model [Chopra 1995]. The interstorey drifts are presented as the average values over the seven time histories. The 95% confidence interval is also represented. Results reported in Figure 14 show a good agreement between the target inter-story drift values (bold line) and the average values obtained from the seven time histories. Sectional areas of the supporting braces A_d were estimated on the basis of the R values, assuming that the dissipative portion of the brace is one third of the total length:

$$A_d = [0 \;\; 0 \;\; 0.0076 \;\; 0.0131 \;\; 0.0184 \;\; 0.0146 \;\; 0.0225]\,\mathrm{m}^2.$$

The values A_d can be compared with the sectional areas A_{el} of the elastic bracing system that can ensure the same level of performance requirements without added dissipation:

$$A_{el} = [0 \;\; 0.0159 \;\; 0.0276 \;\; 0.0339 \;\; 0.0399 \;\; 0.0345 \;\; 0.0238]\,\mathrm{m}^2.$$

It is evident that the elastic braces that support the dampers have smaller sectional areas of the elastic bracing system. In the case study, sectional areas of the braces in the dissipative system are on average 0.43 times the sectional areas of the elastic system.

6. Conclusions

Effects associated with two of the most common assumptions adopted in design procedures for dissipative braces are studied. The assumption of constant floor stiffness and the reduction of MDOF systems to SDOF systems are shown to be limited in capturing the interaction between the frame structure and the installed braces. Index R is introduced in order to describe that interaction. Two additional interaction

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