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A generalization of a power-conjugacy problem in torsion-free negatively curved groups

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Let H and K be quasiconvex subgroups of a negatively curved torsion-free group G. We give an algorithm which decides whether an element of H is conjugate in G to an element of K.

1. Introduction

Max Dehn [1911] introduced three basic algorithmic problems in group theory: the word problem, the conjugacy problem, and the isomorphism problem. Let a group G be given by a presentation $G = \langle X | R \rangle$. The word problem asks if there exists an algorithm to decide if any word in the alphabet X represents the trivial element of G. The word problem was shown to be undecidable, in general, by Novikov [1955], and independently, by Boone [1958]. The conjugacy problem asks if there exists an algorithm which for any pair of words in the alphabet X decides whether they represent conjugate elements in G. A special case of the conjugacy problem, namely the existence of an algorithm deciding if a given word in the alphabet X represents an element of G conjugate to the identity of G, is the word problem. Hence the conjugacy problem is also undecidable, in general. The isomorphism problem asks if for any pair of presentations there exists an algorithm to decide if they define isomorphic groups. The isomorphism problem was shown to be undecidable, in general, by Adian [1957], and independently by Rabin [1958]. The membership problem for a subgroup H of a group G asks if there exists an algorithm deciding if any element of G belongs to H. As the word problem, in general, is undecidable, it follows that the membership problem is, in general, undecidable. The power-conjugacy problem for a group G asks if for any two elements of G there exists an algorithm to decide if one of them is conjugate to some power of the other. The generalized power-conjugacy problem for a group G asks if for any two elements of G there exists an algorithm to decide if some power of one of them is conjugate to some power of the other. A special case of the power-conjugacy problem, namely the existence of an algorithm deciding if

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any element of G is conjugate to some power of the identity element in G, is the word problem. Hence the power-conjugacy problem and the generalized power-conjugacy problem are undecidable, in general. For more detailed information about the aforementioned algorithmic problems see, for example, survey articles [Hurwitz 1984; Miller III 1992].

Even though the aforementioned algorithmic problems are undecidable in general, they are decidable in negatively curved groups. The solution of the word problem in negatively curved groups follows from the work of Greendlinger [1960]. The solution of the conjugacy problem for negatively curved groups was given by Gromov [1987, p. 199]. The solution of the isomorphism problem for negatively curved groups was given by Dahmani and Guirardel [2011]. The solution of the membership problem for quasiconvex subgroups of negatively curved groups was given by the author [Gitik 1996; 2017]. The power-conjugacy problem has been the subject of extensive research and was solved for several additional classes of groups, see for example, [Anshel and Stebe 1974; Bezverhnii 2016; Barker et al. 2016; Bezverkhniĭ and Kuznetsova 2008; Bogopolski et al. 2006; Comerford 1977; Lipschutz and Miller III 1971; Pride 2008]. In this paper we prove a generalized version of the power-conjugacy problem for torsion-free negatively curved groups.

Our solution of a generalized version of the power-conjugacy problem for torsion-free negatively curved groups implies that both the power-conjugacy problem and the generalized power-conjugacy problem are solvable in torsion-free negatively curved groups.

Theorem 1 (generalized power-conjugacy problem). *Let H and K be μ-quasiconvex subgroups of a δ-negatively curved torsion-free group G. There exists an algorithm to decide if an element of H is conjugate in G to an element of K.*

Corollary 2. *Let K be a quasiconvex subgroup of a torsion-free negatively curved group G, and let u be a nontrivial element of G. There exists an algorithm to decide whether some power of u is conjugate in G to an element of K.*

Proof. As a cyclic subgroup in a negatively curved group is quasiconvex [Gromov 1987, p. 210], we can apply Theorem 1 with H being the cyclic subgroup generated by u. □

Corollary 3. *Let K be a quasiconvex subgroup of a torsion-free negatively curved group G and let u be a nontrivial element of G. There exists an algorithm to decide whether u is conjugate in G to an element of K.*

Proof. Lemma 7, stated below, shows that if H is the cyclic subgroup generated by u and u is conjugate to an element of K, then there exists $g \in G$ with $|g| < C$ such that $gug^{-1} \in K$, (C is defined in the statement of Lemma 7). As G is finitely generated, there are only finitely many elements shorter than C in G. Hence we need to check if one of finitely many elements of the form gug^{-1} with $|g| < C$ is in K, which we can do because the membership problem for K in G is decidable. □

Corollary 4. *The power-conjugacy problem is decidable for torsion-free negatively curved groups.*

Proof. Let u be a nontrivial element of G and let v be any element of G. Corollary 3 implies that there is an algorithm to decide whether u is conjugate in G to an element of a cyclic group generated by v, which is the power-conjugacy problem. □

Corollary 5. *The generalized power-conjugacy problem is decidable for torsion-free negatively curved groups.*

Proof. Let u be a nontrivial element of G and let v be any element of G. Corollary 2 implies that there is an algorithm to decide whether some power of u is conjugate in G to an element of a cyclic group generated by v, which is the generalized power-conjugacy problem. □

Theorem 1 follows from three technical results stated below.

Remark 6. Note that if there exist $h \in H$ and $g \in G$ such that $ghg^{-1} \in K$, then for any $h_0 \in H$ and $k_0 \in K$, $(k_0gh_0)(h_0^{-1}hh_0)(h_0^{-1}g^{-1}k_0^{-1}) \in K$. So if $g \in G$ conjugates an element of H to an element of K, then any g_0 in the double coset KgH has the same property.

Lemma 7. *Let H and K be μ-quasiconvex subgroups of a δ-negatively curved torsion-free group G, and let $g \in G$ be a shortest representative of the double coset KgH such that ghg^{-1} is in K for some nontrivial element h of H. Then g is shorter than $C = 4\delta + 2\mu + (m^2 + 1) \cdot L$, where L is the number of words in G with length less than $8\delta + \mu$, and m is the number of elements in G with length not greater than $42\delta + 12\mu$.*

Lemma 8. *Let H and K be μ-quasiconvex subgroups of a δ-negatively curved torsion-free group G and let h be a shortest nontrivial element of H such that ghg^{-1} is in K for some $g \in G$ with $|g| < C$. Then h is shorter than*

$$C' = (L' + 2)2\mu + 8\delta,$$

where L' is the number of words in G shorter than $(2\delta + 2\mu)$.

Proof of Theorem 1. Assume that there exists a nontrivial element $h \in H$ and an element $g \in G$ such that $ghg^{-1} \in K$. Let g_1 be a shortest element in the double coset KgH. Lemma 7 states that $|g_1| < C$. Remark 6 implies that there exists an element $h' \in H$ such that $g_1h'g_1^{-1} \in K$. Let $h_1 \in H$ be a shortest nontrivial element such that $g_1h_1g_1^{-1} \in K$. Lemma 8 states that $|h_1| < C'$. As G is finitely generated, there are finitely many possible g_1 and h_1. Hence we need to form finitely many products $g_1 h_1 g_1^{-1}$ and to check if they belong to K, which we can verify because the generalized word problem is solvable for quasiconvex subgroups of negatively curved groups. □

2. Preliminaries

Let X be a set, let $X \cup X^{-1} = \{x, x^{-1} \mid x \in X\}$, and for $x \in X$ define $(x^{-1})^{-1} = x$. A word in $X \cup X^{-1}$ is any finite sequence of elements of $X \cup X^{-1}$. Denote the set of all words in $X \cup X^{-1}$ by $W(X)$, and denote the equality of two words by "$\equiv$".

Recall that the Cayley graph of $G = \langle X|R\rangle$, denoted Cayley(G), is an oriented graph whose set of vertices is G and the set of edges is $G \times (X \cup X^{-1})$, such that an edge (g, x) begins at the vertex g and ends at the vertex gx. Since the Cayley graph depends on the generating set of the group, we work with a fixed generating set.

A geodesic in the Cayley graph is a shortest path joining two vertices. A geodesic triangle in the Cayley graph is a closed path $p = p_1 p_2 p_3$, where each p_i is a geodesic. A group $G = \langle X|R\rangle$ is δ-negatively curved if any side of any geodesic triangle in the Cayley graph of $G = \langle X|R\rangle$ belongs to the δ-neighborhood of the union of the other two sides.

A subgroup H of a group $G = \langle X|R\rangle$ is μ-quasiconvex in $G = \langle X|R\rangle$ if any geodesic in the Cayley graph of $G = \langle X|R\rangle$ with endpoints in H belongs to the μ-neighborhood of H. A subgroup is quasiconvex in $G = \langle X|R\rangle$ if it is μ-quasiconvex in $G = \langle X|R\rangle$ for some μ. As usual, we assume that all negatively curved groups are finitely generated.

The label of a path $p = (g, x_1)(g \cdot x_1, x_2) \cdots (g \cdot x_1 \cdots x_{n-1}, x_n)$ in Cayley(G) is the function Lab$(p) \equiv x_1 x_2 \cdots x_n \in W(X)$. As usual, we identify the word Lab(p) with the corresponding element in G.

Theorem GMRS [Gitik et al. 1998]. *Let H be a μ-quasiconvex subgroup of a δ-negatively curved torsion-free group G, and let m be the number of elements in G with length not greater than $42\delta + 12\mu$. Let $S = \{g_i^{-1} H g_i \mid 1 \le i \le n\}$ be a collection of essentially distinct conjugates of H, where the conjugates $g_i^{-1} H g_i$ and $g_j^{-1} H g_j$ are called essentially distinct if $Hg_i \neq Hg_j$ for $i \neq j$. If $n > m^2$, then the intersection of some pair of elements of S is trivial.*

3. Proofs of the results

Let g be a shortest element in the double coset KgH such that $ghg^{-1} = k$ is in K for some nontrivial element h of H.

Let p, p_h and p' be geodesics in Cayley(G) such that Lab$(p) \equiv$ Lab$(p') \equiv g$, Lab$(p_h) = h$, p begins at 1 and ends at g, p' begins at ghg^{-1} and ends at gh, and p_h begins at g which is the endpoint of p and ends at gh which is the endpoint of p'.

Denote the vertices of p in their linear order by $1 = v_0, v_1, \ldots, v_n = g$ and denote the vertices of p' in their linear order by $ghg^{-1} = v'_0, v'_1, \ldots, v'_n = gh$. Note that $|g| = |p| = |p'| = n$.

Let p_k be a geodesic in Cayley(G) joining $v_0 = 1$ and $v'_0 = ghg^{-1}$. Then the paths p, p_k, p' and p_h form a geodesic 4-gon which is 2δ-thin in Cayley(G), because G is δ-negatively curved.

Lemma 9. *For any index i such that $2\delta + \mu \le i \le n - 2\delta - \mu$ the distance $d(v_i, v'_i)$ is less than $8\delta + \mu$.*

Proof. Let l be the biggest index such that the vertex v_l belongs to the 2δ-neighborhood of p_k, let w_l be a vertex in p_k closest to v_l, and let r be a geodesic joining w_l to v_l. By construction, Lab$(p_k) = k \in K$. As K is μ-quasiconvex, p_k belongs to the μ-neighborhood of K in Cayley(G), hence there exists a vertex $u_l \in K$ such that $d(w_l, u_l) < \mu$. Let r' be a geodesic joining u_l to w_l. Let s_l be the subpath of p joining v_0 to v_l, let $\bar{s_l}$ be the inverse of the path s_l, and let t_l be the subpath of p joining v_l to v_n. Note that $\mathrm{Lab}(r'rt_l) = \mathrm{Lab}(r'r\bar{s_l})(s_lt_l) = \mathrm{Lab}(r'r\bar{s_l})g \in Kg$. As g is a shortest representative of KgH, it follows that $|g| = |p| = |s_l| + |t_l| \le |r'rt_l| < 2\delta + \mu + |t_l|$, so $|s_l| = d(v_0, v_l) = l < 2\delta + \mu$. Hence if $i \ge \mu + 2\delta$, then $d(v_i, p_k) > 2\delta$.

Let i be the smallest index such that the vertex v_i belongs to the 2δ-neighborhood of p_h. An argument, similar to the above, shows that for any $j \le n - 2\delta - \mu$, $d(v_j, p_h) > 2\delta$.

Therefore, for any index i such that $2\delta + \mu \le i \le n - 2\delta - \mu$, the vertex v_i belongs to the 2δ-neighborhood of p'. Similarly, for any index i such that $2\delta + \mu \le i \le n - 2\delta - \mu$ the vertex v'_i belongs to the 2δ-neighborhood of p.

Let $b = n - 2\delta - \mu$. We claim that $d(v_b, v'_b) < 4\delta + \mu$. Indeed, let $j(b) \le b$ be an index such that $d(v_b, v'_{j(b)}) < 2\delta$. Let t_b be the subpath of p joining v_b and v_n, let $t'_{j(b)}$ be the subpath of p' joining $v'_{j(b)}$ to v'_n, and let γ be a geodesic joining v_b and $v'_{j(b)}$. Consider the geodesic 4-gon formed by t_b, p_h, $t'_{j(b)}$ and γ.

As $b \le n - 2\delta - \mu$, it follows that $d(v'_b, p_h) > 2\delta$. If $d(v'_b, \gamma) < 2\delta$, then $d(v_b, v'_b) \le |\gamma| + d(v'_b, \gamma) < 4\delta$. If $d(v'_b, t_b) < 2\delta$, then $d(v_b, v'_b) \le |t_b| + d(v'_b, t_b) < 4\delta + \mu$.

Now consider $2\delta + \mu \le i \le n - 2\delta - \mu$. Let $j(i)$ be an index such that $d(v_i, v'_{j(i)}) < 2\delta$. By interchanging v_i and $v'_{j(i)}$, if needed, we can assume that $j(i) \ge i$. As p is a geodesic, $d(v_i, v_b) = b - i \le d(v_i, v'_{j(i)}) + d(v'_{j(i)}, v'_b) + d(v_b, v'_b) < 2\delta + (b - j(i)) + 4\delta + \mu$, hence $0 \le j(i) - i < 6\delta + \mu$. But then $d(v_i, v'_i) \le d(v_i, v'_{j(i)}) + d(v'_{j(i)}, v'_i) < 2\delta + (j(i) - i) < 8\delta + \mu$, proving Lemma 9. □

Proof of Lemma 7. Assume that $|g| = n \ge C$, where C is defined in the statement of Lemma 7. It follows that $(n - 2\delta - \mu) - (2\delta + \mu) \ge C - 4\delta - 2\mu = L \cdot (m^2 + 1)$.

Therefore Lemma 9 implies there exists a set of distinct indexes $\{i_j \mid 1 \le j \le m^2+1\}$ such that

(1) $n - 2\delta - \mu \ge i_j \ge 2\delta + \mu$,

(2) the paths connecting v_{i_j} to v'_{i_j} have the same label, say a, for all i_j.

Recall that the paths p and p', defined at the beginning of the current section, have identical labels. Let s_{i_j} be the initial subpath of p connecting v_0 and v_{i_j} and let s'_{i_j} be the initial subpath of p' connecting v'_0 and v'_{i_j}. This definition implies that the paths s_{i_j} and s'_{i_j} have identical labels for all j.

Let k be the element of K and h be the element of H, defined at the beginning of the current section.

If $a = 1$, then $v_{i_1} = v'_{i_1}$. It follows that $\mathrm{Lab}(s_{i_1})^{-1} k\, \mathrm{Lab}(s'_{i_1}) = 1$, hence $k = h = 1$, contradicting the choice of h.

If $a \ne 1$, consider the set $S = \{\mathrm{Lab}(s_{i_j}^{-1}) k\, \mathrm{Lab}(s_{i_j}) \mid 1 \le i_j \le m^2+1\}$. As $\mathrm{Lab}(s_{i_j})^{-1} k\, \mathrm{Lab}(s_{i_j}) = a \ne 1$ for all $1 \le i_j \le m^2+1$, and as G is torsion-free, it follows that the intersection of any pair of elements of S is infinite.

However, the elements of S are essentially distinct. Indeed, assume that there exists $k_0 \in K$ such that $k_0\, \mathrm{Lab}(s_{i_j}) = \mathrm{Lab}(s_{i_l})$.

Without loss of generality, $i_l > i_j$. Let t_{i_l} be the subpath of p joining v_{i_l} to v_n. Then $g = \mathrm{Lab}(s_{i_l})\mathrm{Lab}(t_{i_l}) = k_0 \mathrm{Lab}(s_{i_j})\mathrm{Lab}(t_{i_l})$. Hence the element $\mathrm{Lab}(s_{i_j})\mathrm{Lab}(t_{i_l})$ belongs to Kg and $|\mathrm{Lab}(s_{i_j})\, \mathrm{Lab}(t_{i_l})| \le |s_{i_j}| + |t_{i_j}| < |s_{i_l}| + |t_{i_l}| = |g|$, contradicting the choice of g as a shortest representative of the double coset KgH. So S is a collection of m^2+1 distinct conjugates of K such that any two elements of S have infinite intersection, contradicting Theorem GMRS.

Hence $|g| < C$, proving Lemma 7. □

Remark 10. By increasing the quasiconvexity constant μ if needed, we can assume that μ is a positive integer.

Lemma 11. *Let g be an element shorter than $4\delta + 2\mu$ such that $ghg^{-1} \in K$ for a nontrivial $h \in H$. If h is longer than $(L'+2)2\mu + 8\delta$, where L' is the number of words in G shorter than $2\delta + 2\mu$, then there exist a nontrivial $h_0 \in H$ with $|h_0| \le 2\mu(L'+2)$ and $g_0 \in G$ with $|g_0| < 2\delta + 2\mu$ such that $g_0 h_0 g_0^{-1} \in K$.*

Proof. Let p, p_k, p' and p_h be a geodesic 4-gon, as in the proof of Lemma 9. Denote the vertices of p_h in their linear order by $g = v_0^h, v_1^h, \ldots, v_f^h = gh$.

Let q' be the maximal initial subpath of p_h which belongs to the 2δ-neighborhood of p. Note that the length of q' is at most $4\delta + \mu$. Indeed, let $v_{q'}^h$ be the terminal vertex of q'. Let $v_{q'}$ be a vertex of p such that $d(v_{q'}^h, v_{q'}) \le 2\delta$. Let α be a geodesic in $\mathrm{Cayley}(G)$ which begins at $v_{q'}$ and ends at $v_{q'}^h$. Let $s_{q'}$ be the initial subpath of p joining v_0 to $v_{q'}$ and let $t_{q'}$ be the terminal subpath of p joining $v_{q'}$ to $v_n = g$. As H is μ-quasiconvex in G, there exists a vertex $x_{q'}$ in $\mathrm{Cayley}(G)$ and a geodesic α'

joining $v^h_{q'}$ to $x_{q'}$ such that $\mathrm{Lab}(q'\alpha') \in H$ and $|\alpha'| < \mu$. As g is a shortest element in the double coset KgH, it follows that

$$|g| = |s_{q'}| + |t_{q'}| \le |s_{q'}| + |\alpha| + |\alpha'| \le |s_{q'}| + 2\delta + \mu.$$

Hence $|t_{q'}| \le 2\delta + \mu$. It follows that $|q'| \le |t_{q'}| + |\alpha| \le 4\delta + \mu$.

Similarly, the length of the maximal subpath of p_h which belongs to the 2δ-neighborhood of p' is at most $4\delta + \mu$.

Assume that h is longer than $(L'+2)2\mu + 8\delta$. Then there exists a subpath q of p_h of length at least $(L'+1)2\mu$ which belongs to the 2δ-neighborhood of p_k. By construction, q begins at the vertex $v^h_{q'}$. By definition of the path q, for any vertex v^h_i of q there exists a vertex $w(v^h_i)$ in p_k such that $d(v^h_i, w(v^h_i)) < 2\delta$. As H is μ-quasiconvex in G, for any vertex v^h_i of q there exists a vertex x_i such that $d(v^h_i, x_i) < \mu$, and the element x_i belongs to the coset gH. Similarly, there exists a vertex $k(v^h_i)$ such that $d(w(v^h_i), k(v^h_i)) < \mu$ and the element $k(v^h_i)$ belongs to K. Let β_i be a geodesic joining $k(v^h_i)$ and x_i. Then $|\beta_i| < 2\mu + 2\delta$.

Consider the subset of vertices of p_h with indexes

$$v^h_{q'}, v^h_{q'+2\mu}, \ldots, v^h_{q'+j\cdot 2\mu}, \ldots, v^h_{q'+L'\cdot 2\mu}.$$

The distance between two consecutive vertices in this subset is 2μ, hence $x_{q'+i\cdot 2\mu} \neq x_{q'+j\cdot 2\mu}$ for $i \neq j$.

By definition of the constant L', there exist indexes $i \neq j$ such that $\mathrm{Lab}(\beta_{q'+i\cdot 2\mu}) = \mathrm{Lab}(\beta_{q'+j\cdot 2\mu})$. By construction, $d(v_{q'+i\cdot 2\mu}, v_{q'+j\cdot 2\mu}) \le 2\mu(L'+1)$, so

$$d(x_{q'+i\cdot 2\mu}, x_{q'+j\cdot 2\mu}) < 2\mu + 2\mu(L'+1) = 2\mu(L'+2).$$

By construction, if ν is a geodesic joining $x_{q'+i\cdot 2\mu}$ and $x_{q'+j\cdot 2\mu}$, then $\mathrm{Lab}(\nu) \in H$. Similarly, if ν' is a geodesic joining $k(v_{q'+i\cdot 2\mu})$ and $k(v_{q'+j\cdot 2\mu})$, then $\mathrm{Lab}(\nu') \in K$. So take $g_0 = \mathrm{Lab}(\beta(v_{q'+i\cdot 2\mu})$ and $h_0 = \mathrm{Lab}(\nu)$, proving Lemma 11. □

Proof of Lemma 8. Let h be a nontrivial element of H such that $ghg^{-1} \in K$ for some $g \in G$ with $|g| < C$, where C is defined in the statement of Lemma 7. We want to find $h_0 \in H$ with $|h_0| < C'$, where C' is defined in the statement of Lemma 8, and $g_0 \in G$, which might be different from g, with $|g_0| < C$ such that $g_0 h_0 g_0^{-1} \in K$.

Consider three cases.

(1) If $|g| < 4\delta + 2\mu$ and $|h| \le (L'+2)2\mu + 8\delta$, take $g_0 = g$ and $h_0 = h$.

(2) If $|g| < 4\delta + 2\mu$ and $|h| > (L'+2)2\mu + 8\delta$, then Lemma 11 states that there exist a nontrivial $h_0 \in H$ with $|h_0| \le 2\mu(L'+2)$ and $g_0 \in G$ with $|g_0| < 2\delta + 2\mu$ such that $g_0 h_0 g_0^{-1} \in K$.

(3) If $C > |g| \geq 4\delta + 2\mu$, let p, p', v_b, v_b' and p_h be as in the proof of Lemma 9. It is shown in Lemma 9 that $d(v_b, v_b') < 4\delta + \mu$. Then

$$|h| = |p_h| \leq d(v_b, v_n) + d(v_b, v_b') + d(v_b', v_n') < (\mu + 2\delta) + (\mu + 4\delta) + (\mu + 2\delta) < (3\mu + 8\delta).$$

Hence we can take $g_0 = g$ and $h_0 = h$, proving Lemma 8. □

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RITA GITIK:

ritagtk@umich.edu
Department of Mathematics, University of Michigan, Ann Arbor, MI, United States

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