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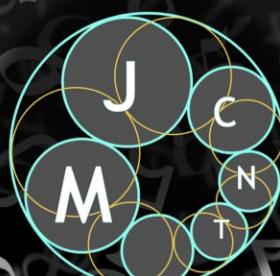
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Naum Illyitch Feldman

Nikolay Moshchevitin

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## Naum Ilyitch Feldman

by Nikolay Moshchevitin



This volume gathers papers dedicated to the memory of Professor Naum Ilyitch Feldman (1918–1994). For many years he worked at Moscow Lomonosov State University, in the Department of Number Theory. I remember him as one of my favorite teachers at the University.

Naum Ilyitch Feldman was born in a Jewish family on November 26, 1918, in the city of Melitopol in the south of Russia. Those were difficult times for Russia. Toward the end of World War I a revolution befell the Russian Empire, followed by a civil war.

After graduating from secondary school in 1936, N. I. Feldman studied at Saint Petersburg University (called at that time Leningrad University) under the supervision of Professor Rodion Osievich Kuzmin.

From 1941 till the end of the World War II he served in the Soviet Army as an artillery officer. After the war he entered a PhD program at Moscow University. His scientific adviser was Alexandre Osipovich Gelfond. In 1949 he published his first papers, devoted to approximations to logarithms of algebraic numbers and got a doctoral degree. From 1961 till his death on April 20, 1994, he worked at Moscow Lomonosov State University. I remember him as an excellent lecturer, a widely educated mathematician and an extremely friendly person.

His research was mostly devoted to transcendence theory and its applications. I can say that he was a great admirer of numbers. He truly loved them. I will mention three of his best known results.

- Following earlier papers by Alexander Gelfond, and developing a method invented by Alan Baker, Naum Ilyitch Feldman was the first to obtain in 1967–68 effective polynomial lower bounds for linear forms in logarithms of algebraic numbers, that is, bounds of the form

$$|h_1 \log \alpha_1 + \dots + h_d \log \alpha_d| \geq C_1 \left( \max_{1 \leq j \leq d} |h_j| \right)^{-C_2}, \quad \forall (h_1, \dots, h_d) \in \mathbb{Z}^d \setminus \{(0, \dots, 0)\},$$

where  $\alpha_1, \dots, \alpha_d$  are algebraic numbers such that their logarithms are linearly independent over  $\mathbb{Q}$ . In fact, he obtained a more general result, namely that the restriction on the linear form coefficients can be relaxed: they can be assumed to be arbitrary algebraic numbers. A result of this type with more appropriate constants  $C_1, C_2$  was later obtained by Eugenii Matveev, one of Feldman's students. Matveev's result has many applications, especially in Diophantine equations.

- N. I. Feldman was the first to obtain (in 1971) an effective improvement of Liouville's theorem, i.e., an inequality of the form

$$\left| \alpha - \frac{p}{q} \right| \geq \frac{C_1}{q^{n-C_2}}, \quad \forall \frac{p}{q} \in \mathbb{Q},$$

where  $\alpha$  is an algebraic number of degree  $n$  and  $C_1, C_2$  are effective positive constants.

- For many years Naum Ilyitch worked on bounds for the transcendence measure of  $\pi$ . His first result in this direction was published in 1951. His last result was announced in 1993 at a conference at Oberwolfach Mathematical Institute. It gives a lower bound

$$|P(\pi)| \geq e^{-C(d \log d + \log H)d(1+\log d)}$$

for an integer polynomial  $P \in \mathbb{Z}[z]$  with  $d = \deg P \geq d_0$ ,  $C = 151$ , and  $H$  denoting the height of  $P$ . In 1999, Yuri Aleksentsev reduced the constant 151 to 21.4708.

In June 2019 Moscow Institute of Physics and Technology in collaboration with Moscow Lomonosov State University organised a conference in the memory of Professor Feldman. The current issue of the Moscow Journal of Combinatorics and Number Theory comprises papers of the participants, as well as some other papers dedicated to Naum Ilyitch.

# Moscow Journal of Combinatorics and Number Theory

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