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S. Orlik and M. Strauch have studied locally analytic principal series representation for general p-adic reductive groups generalizing an earlier work of P. Schneider for GL(2) and related the condition of irreducibility of such locally analytic representation with that of a suitable Verma module. We take the case of GL(n) and study the globally analytic principal series representation under the action of the pro-p Iwahori subgroup of $GL(n, \mathbb{Z}_p)$, following the notion of globally analytic representations introduced by M. Emerton. Furthermore, we relate the condition of irreducibility of our globally analytic principal series to that of a Verma module. Finally, using the Steinberg tensor product theorem, we construct the Langlands base change of our globally analytic principal series to a finite unramified extension of $\mathbb{Q}_p$.

1. Introduction

In this paper we construct a globally analytic (also called rigid analytic) principal series representation of the pro-p Iwahori subgroups of $\mathrm{GL}_n(\mathbb{Z}_p)$ and determine when it is irreducible. Furthermore, we construct base change of our rigid analytic representation to a finite unramified extension of $\mathbb{Q}_p$. This extends earlier works of Robert [1984; 1985] for SL_2 and Clozel [2018] for GL_2.

Denote by G the pro-p Iwahori subgroup of $\mathrm{GL}_n(\mathbb{Z}_p)$ (the group of matrices in $\mathrm{GL}_n(\mathbb{Z}_p)$ that are lower unipotent modulo $p\mathbb{Z}_p$) and by B the subgroup of matrices in $\mathrm{GL}_n(\mathbb{Z}_p)$ which are lower triangular modulo $p\mathbb{Z}_p$. Let P_0 and T_0 be the set of upper triangular and diagonal matrices in B, respectively. Let $Q_0 = P_0 \cap G$. Let P^+ be the Borel subgroup of upper triangular matrices in $\mathrm{GL}_n(\mathbb{Z}_p)$, and let W be the Weyl group (isomorphic to the permutation group S_n) of $\mathrm{GL}_n(\mathbb{Q}_p)$ with respect to its maximal torus. Define

$$P_w^+ := B \cap w P^+ w^{-1}, \quad w \in W.$$

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Let K be a finite extension of $\mathbb{Q}_p$, let $\chi : T_0 \to K^\times$ be a locally analytic character with $\chi(t_1, \dots, t_n) = \chi_1(t_1) \cdots \chi_n(t_n)$, and $\chi_i(t) = t^{c_i} = e^{c_i \log(t)}$, where $c_i \in K$, for t sufficiently close to 1 in $\mathbb{Z}_p$ and where e is the exponential function.

Throughout this article, we use the following definitions.

(1) Let $\mathcal{A}_{\mathrm{loc}}(B, K)$ be the space of locally analytic functions on B with values in K. These are functions from B to K such that for any $x \in B$ we can find a ball $B_{r_x}(x)$ of radius r_x (depending on x) around x such that f can be written as a power series with coefficients in K which converges on $B_{r_x}(x)$ [Schneider 2011].

(2) Let $\mathrm{ind}_{P_0}^B(\chi)_{\mathrm{loc}} = \{f \in \mathcal{A}_{\mathrm{loc}}(B, K) : f(gb) = \chi(b^{-1}) f(g),\ b \in P_0,\ g \in B\}$.

(3) Let $\mathrm{ind}_{Q_0}^G(\chi)_{\mathrm{loc}} = \{f \in \mathcal{A}_{\mathrm{loc}}(G, K) : f(gb) = \chi(b^{-1}) f(g),\ b \in Q_0,\ g \in G\}$. Note that $B = UP_0$ and $G = UQ_0$, where U is the lower unipotent subgroup of B (or G). Therefore, we can obviously see that, as a vector space,

$$\mathrm{ind}_{P_0}^B(\chi)_{\mathrm{loc}} \cong \mathcal{A}_{\mathrm{loc}}(U, K) \cong \mathrm{ind}_{Q_0}^G(\chi)_{\mathrm{loc}}.$$

(4) Let d be the dimension of the p-adic Lie group U. Let $\mathcal{A}(U, K)$ be the space of globally analytic functions inside $\mathcal{A}_{\mathrm{loc}}(B, K)$. Any element $f \in \mathcal{A}(U, K)$ is of the form $f = \sum_{\nu \in \mathbb{N}^d} c_\nu a^\nu$ with $\lim_{|\nu| \to \infty} |c_\nu| = 0$. The space $\mathcal{A}(U, K)$ is a K-Banach space with the sup norm on f defined by

$$|f| = \sup |c_\nu|$$

(see [Bosch 2014, Chapter 2]). This is also known as the Tate algebra of globally analytic (sometimes called "rigid analytic") functions on U (see [Bosch 2014]). By (2) above, the vector space of globally analytic functions inside $\mathrm{ind}_{P_0}^B(\chi)_{\mathrm{loc}}$ (or $\mathrm{ind}_{Q_0}^G(\chi)_{\mathrm{loc}}$) is isomorphic to $\mathcal{A}(U, K)$.

(5) The action of G on the globally analytic vectors $\mathcal{A}(U, K)$ of $\mathrm{ind}_{P_0}^B(\chi)_{\mathrm{loc}}$ is given by the left translation $h \cdot f(g) \mapsto f(h^{-1}g),\ h \in G$.

(6) Recall that for any K-Banach space V with norm $|\cdot|$, a representation π of G on V is called a globally analytic representation if the map

$$g \mapsto g \cdot v = \pi(g) v$$

is globally analytic on G for all $v \in V$. Therefore, in coordinates $(x_1, \dots, x_l)$ with $l = \dim(G)$,

$$g \cdot v = \sum_k x^k v_k,$$

where $v_k \in V$ and $|v_k| \to 0$. Here $k = (k_1, \dots, k_l)$ and $x^k = x_1^{k_1} \cdots x_l^{k_l},\ k_i \in \mathbb{N}$ (see [Emerton 2017; Clozel 2018, Section 2]). For a detailed discussion on globally analytic representation, see [Emerton 2017].

(7) Write $\chi = (\chi_1, \dots, \chi_n)$, $\chi_i(1 + pu_i) = e^{c_i \log(1+pu_i)}$ for $c_i \in K$, u_i close to 0, $i \in [1, n]$. The exponential is analytic (in K) in the domain $v_p(z) > e/(p-1)$ where $e = e(K)$ is the ramification index of K and v_p is the normalized valuation, $v_p(p) = 1$. Now,

$$v_p(c_i \log(1 + pu_i)) = v_p(c_i) + 1 + v_p(u_i).$$

So we say that χ is analytic if and only if $v_p(c_i) > e/(p-1) - 1$ (see (3-10)).

Then, we show in Lemmas 3.2, 3.3 and 3.7 that the action of G on the *globally analytic vectors* $\mathcal{A}(U, K)$ of $\mathrm{ind}_{P_0}^{B}(\chi)_{\mathrm{loc}}$ is a globally analytic action of G, that is, it gives a globally analytic representation of G, in the sense of Emerton [2017].

Let μ be the linear form from the Lie algebra of the torus T_0 to K given by

$$\mu = (-c_1, \dots, c_n) : \mathrm{Diag}(t_1, \dots, t_n) \mapsto \sum_{i=1}^{n} -c_i t_i,$$

where $t = (t_i) \in \mathrm{Lie}(T_0)$ and $c_i \in K$. For negative root $\alpha = (i, j)$, $i > j$, let $H_{(i,j)} = E_{i,i} - E_{j,j}$ where $E_{i,i}$ is the standard elementary matrix.

Using Theorems 3.8 and 3.9, we will show the following:

Theorem. *Assume $p > n+1$ and χ is analytic. Then the space of globally analytic vectors of* $\mathrm{ind}_{P_0}^{B}(\chi)_{\mathrm{loc}}$ *is an admissible and globally analytic representation of G. Furthermore, the space of globally analytic vectors of* $\mathrm{ind}_{P_0}^{B}(\chi)_{\mathrm{loc}}$ *is irreducible if and only if $-\mu(H_\alpha) + i - j \notin \{1, 2, 3, \dots\}$ for all $\alpha = (i, j) \in \Phi^-$.*

Here, the admissibility is in the sense of [Emerton 2017] (see also [Clozel 2018, Section 2.3]).

For global analyticity, we compute explicitly the action of G on the Tate algebra of globally analytic functions of $\mathrm{ind}_{P_0}^{B}(\chi)_{\mathrm{loc}}$ and show that the action map is a globally analytic function on G seen as a rigid analytic space. For this, we have to do a lot of new technical computations, which were not necessary for the GL(2) case by Clozel [2018]. In particular, see Lemmas 3.4–3.7. For the irreducibility we first use the action of the Lie algebra of G to show that any nonzero closed G-invariant subspace of the globally analytic vectors of $\mathrm{ind}_{P_0}^{B}(\chi)_{\mathrm{loc}}$ contains the constant function 1. Unlike the GL(2) case, the remaining part of the argument for the proof of irreducibility uses the notion of Verma modules and its condition of irreducibility, a result of Bernstein, Gelfand and Gelfand.

Strikingly, one can check easily that the condition that we obtain for irreducibility of the globally analytic principal series is the same condition as the irreducibility of the locally analytic principal series $\mathrm{ind}_{P_0}^{B}(\chi)_{\mathrm{loc}}$, deduced by Schneider and Teitelbaum [2002, Theorems 5.4 and 5.6, Corollary 5.7] for GL(2) and Orlik and Strauch for GL(n) [Orlik and Strauch 2010, Theorems 3.4.12 and 4.1.1] although our style of proof is completely different from their proof. Note that in Theorem 4.1.1 of [Orlik and Strauch 2010], there is a typo where the authors quote the result in [Dixmier 1977] of the irreducibility of a Verma module due to Bernstein, Gelfand and Gelfand. For the correct result of irreducibility one should look at [Dixmier 1977, Theorem 7.6.24].

The preliminaries are included in Section 2. The main result is discussed in Section 3A. In Section 3B, we extend these results to the pro-p Iwahori group of $\mathrm{GL}_n(L)$ where L is an unramified finite extension of $\mathbb{Q}_p$. Then, in Theorem 3.19, we use the Steinberg tensor product [1963] to construct base change in the context of Langlands functoriality.

In Section 4, we deal with the globally analytic vectors induced from the Weyl orbit of the upper triangular Borel subgroup of the Iwahori subgroup B, i.e., the globally analytic vectors of $\mathrm{ind}_{P_w^+}^B(\chi^w)_{\mathrm{loc}}$, where $\chi_w(h) = \chi(w^{-1}hw)$.

Our work is just the tip of an iceberg in the domain of globally analytic representations and it leads to a plethora of future questions; some of them are discussed at the end of Section 4.

2. Base change maps for analytic functions

We introduce the basic notions of rigid analytic geometry, including a brief discussion on the restriction of scalars. Then we briefly recall (following [Clozel 2018]) the notions of holomorphic and Langlands base change functors from a globally analytic representation over $\mathbb{Q}_p$ to a representation over L. The Langlands base change is related to the "Steinberg tensor product" described at the end of Section 1.1 of [Clozel 2017] for GL(2).

2A. Let L be a finite unramified extension of $\mathbb{Q}_p$ of degree N, with ring of integers $\mathcal{O}_L$. Given a formal scheme $\mathbb{X}_{\mathcal{O}_L}$ over $\mathcal{O}_L$, Bertapelle [2000] constructs a Weil restriction functor which associates to $\mathbb{X}_{\mathcal{O}_L}$ another formal scheme over $\mathbb{Z}_p$. Let $\mathbb{X}_L$ be the rigid analytic space associated to the formal scheme $\mathbb{X}_{\mathcal{O}_L}$. Bertapelle's construction gives a Weil restriction functor (we will call it as restriction of scalars) which associates to $\mathbb{X}_L$ another rigid analytic space $\mathrm{Res}_{L/\mathbb{Q}_p}(\mathbb{X}_L)$ over $\mathbb{Q}_p$. Although we will not recall the construction of this functor for general rigid analytic spaces and formal schemes, we do recall how this functor behaves with respect to affinoid rigid analytic spaces which is what we will need in this article. An interested reader should consult [Bertapelle 2000] for the most general construction of this restriction functor. Let (B^1/L) be the (rigid analytic affinoid) closed unit ball over L with its Tate algebra of analytic functions $\mathcal{T}_L = L\langle x\rangle$ and G_L be a rigid analytic group isomorphic as a rigid analytic space to $(B^1/L)^d$ which is a rigid analytic space with affinoid algebra $\mathcal{A}(G_L) := \widehat{\otimes}^d \mathcal{T}_L := \mathcal{T}_d(L) = L\langle x_1, \dots, x_d\rangle$, the Tate algebra of analytic functions in d variables with coefficients in L. The restriction of scalars functor [Bertapelle 2000] associates to G_L a rigid analytic space $\mathrm{Res}_{L/\mathbb{Q}_p} G_L$ over $\mathbb{Q}_p$. In general, this functor does not behave trivially, but L being unramified, we obtain

$$\mathrm{Res}_{L/\mathbb{Q}_p}(B^1/L) \cong (B^1/\mathbb{Q}_p)^N,$$

[Clozel 2018, Lemma 1.1] which is canonically obtained by the choice of a basis (e_i) of $\mathcal{O}_L$ over $\mathbb{Z}_p$. This is defined in the following way. For an affinoid $\mathbb{Q}_p$-algebra $\mathcal{B}$ and for $f \in \mathrm{Hom}_L(L\langle x\rangle, \mathcal{B} \otimes_{\mathbb{Q}_p} L)$ with $f(x) = \sum b_i e_i \ \ (b_i \in \mathcal{B})$, we canonically define a function $g \in \mathrm{Hom}_{\mathbb{Q}_p}(\mathbb{Q}_p\langle x_1, \dots, x_N\rangle, \mathcal{B})$ with $g(x_i) = b_i$ which is given by

$$g(x_1, \dots, x_N) = f\left(\sum e_i x_i\right) \tag{2-1}$$

([Clozel 2018, Section 1.1]; see also [Bertapelle 2000, Proposition 1.8]). Since e_i is integral and $|x_i| \leq 1$ it is easy to see that the series on the right converges. As the restriction of scalars is compatible with direct products [Bertapelle 2000, Proposition 1.8], $\mathrm{Res}_{L/\mathbb{Q}_p} G_L = (B^1/\mathbb{Q}_p)^{dN}$. Henceforth, we write $\mathrm{Res}\, G_L$ to denote $\mathrm{Res}_{L/\mathbb{Q}_p} G_L$.

2B. Assume now that $G_L \cong (B^1/L)^d$ is obtained by *extension* of scalars from $\mathbb{Q}_p$. Then, the Tate algebra $\mathcal{A}(G_L)$ is equal to $\mathcal{A}(G_{\mathbb{Q}_p}) \otimes L$. The comultiplication map m^*, defined by a morphism

$$m^* : \mathcal{A}(G_L) \to \mathcal{A}(G_L) \widehat{\otimes} \mathcal{A}(G_L)$$

with image inside the completed tensor product, is obtained by extension of scalars from

$$m_0^* : \mathcal{A}(G_{\mathbb{Q}_p}) \to \mathcal{A}(G_{\mathbb{Q}_p}) \widehat{\otimes} \mathcal{A}(G_{\mathbb{Q}_p}).$$

Note that (2-1) associates to $f \in \mathcal{A}(G_L)$ (with L-coefficients, i.e., in $\mathcal{T}_d(L)$) a function $g \in \mathcal{A}(\mathrm{Res}\, G_L) \otimes L$ (the function g given in (2-1) will have coefficients in L). In particular, we get a map $\mathcal{A}(G_{\mathbb{Q}_p}) \to \mathcal{A}(\mathrm{Res}\, G_L) \otimes L$ in composing with the "tautological map"

$$\mathcal{A}(G_{\mathbb{Q}_p}) \to \mathcal{A}(G_L).$$

This gives us the map

$$b_1 : \mathcal{A}(G_{\mathbb{Q}_p}) \to \mathcal{A}(\mathrm{Res}\, G_L) \otimes L, \tag{2-2}$$

and we call it as a "holomorphic base change" map. The Galois group

$$\Sigma = \mathrm{Gal}(L/\mathbb{Q}_p)$$

naturally acts on L which induces a natural action of Σ on the pro-p Iwahori with coefficients in L. This action is $\mathbb{Q}_p$-linear. Therefore Σ acts naturally on G_L with a $\mathbb{Q}_p$-linear action. Note also that the action of Σ on $\mathrm{Res}\, G_L$ is $\mathbb{Q}_p$-linear. Recall from (2-2) that b_1 sends $\mathcal{A}(G_{\mathbb{Q}_p})$ to the functions that are L-holomorphic (given by power series $\sum a_m \underline{x}^m,\ a_m \in L$, with $\underline{x} = (x_1, \dots, x_d)$ being the variable).

Let $\sigma \in \Sigma$. Then the action associated to σ sends a power series in $\mathcal{A}(G_{\mathbb{Q}_p})$ to $\left(\sum a_m \underline{x}^m\right)^\sigma := \sum \sigma(a_m)\underline{x}^m$. This gives rise to a map

$$b : \mathcal{A}(G_{\mathbb{Q}_p}) \to \mathcal{A}(\operatorname{Res} G_L) \otimes L, \tag{2-3}$$

$$b(f) = \prod_{\sigma\in\Sigma} b_1(f)^\sigma. \tag{2-4}$$

We now consider all Tate algebras to have coefficients in L and we denote them by $\mathcal{A}_L$. That is, $\mathcal{A}_L(\operatorname{Res} G_L) = \mathcal{A}(\operatorname{Res} G_L) \otimes L$ and $\mathcal{A}_L(G_L) := \mathcal{A}(G_L)$. Clozel [2018, Proposition 1.5] has shown that the map b constructed in (2-3) is actually a tensor product and gives rise to an isomorphism $\mathcal{A}_L(\operatorname{Res} G_L) \cong \widehat{\otimes}_\sigma \mathcal{A}_L(G_L)$.

Fix a finite extension K of $\mathbb{Q}_p$ and an injection $i : L \subset K$. If $\sigma \in \operatorname{Gal}(L/\mathbb{Q}_p)$, we then have the injection

$$i \circ \sigma : L \to K.$$

Denote by V a (globally) analytic representation of $G_{\mathbb{Q}_p}$ on a K- Banach space. Then V naturally extends to an analytic representation of G_L; this is called the *holomorphic base change* of V in [Clozel 2018]. For $\sigma \in \operatorname{Gal}(L/\mathbb{Q}_p)$, write V^σ the representation of G_L associated to $i \circ \sigma$. Then, the *full (Langlands) base change* of V is defined to be the globally analytic representation of $\operatorname{Res}_{L/\mathbb{Q}_p}(G_L)$ on $\widehat{\otimes}_\sigma V^\sigma$ (see [Clozel 2018, Definition 3.2]).

3. Globally analytic principal series for GL(n)

We first recall the notion of locally analytic principal series representation induced from the Borel to the Iwahori subgroup of $\mathrm{GL}(n, \mathbb{Z}_p)$. Then we treat the action of the pro-p Iwahori on the subspace of rigid analytic functions within the locally analytic principal series and show that this action is a globally analytic action (Theorem 3.8). This gives us the globally analytic induced principal series representation under the pro-p Iwahori subgroup G. Furthermore, we treat the condition of irreducibility of the globally analytic principal series by translating an irreducibility condition of a suitable Verma module (Theorem 3.9). Finally in Section 3B we base change our globally analytic representation to a finite unramified extension L of $\mathbb{Q}_p$.

3A. We consider the case of principal series for $\mathrm{GL}_n(\mathbb{Z}_p)$. Denote by G the pro-p Iwahori subgroup of $\mathrm{GL}_n(\mathbb{Z}_p)$, i.e., the group of matrices in $\mathrm{GL}_n(\mathbb{Z}_p)$ that are lower unipotent modulo $p\mathbb{Z}_p$ and by B the subgroup of matrices in $\mathrm{GL}_n(\mathbb{Z}_p)$ which are lower triangular modulo $p\mathbb{Z}_p$. Let $P_0 \supset T_0$ be the set of upper triangular and diagonal matrices in B, and let $\chi : T_0 \to K^\times$ be a locally analytic character with

$$\chi(t_1, \ldots, t_n) = \chi_1(t_1) \cdots \chi_n(t_n),$$

and $\chi_i(t) = t^{c_i}$. Here t^{c_i} is the exponential $e^{c_i \log(t)}$ where $c_i \in K$ for t sufficiently close to 1 in $\mathbb{Z}_p$.

We first consider the locally analytic induced representation of B,

$$J_{\text{loc}} = \text{ind}_{P_0}^{B}(\chi)_{\text{loc}} = \{f \in \mathcal{A}_{\text{loc}}(B, K) : f(gb) = \chi(b^{-1}) f(g),\ b \in P_0,\ g \in B\},$$

where χ is naturally extended to P_0 and $\mathcal{A}_{\text{loc}}(B, K)$ is the space of locally analytic functions on B. With U the lower unipotent subgroup of B with entries in $\mathbb{Z}_p$ in the lower triangular part, 1 in the diagonal entries and 0 elsewhere, we have the natural decomposition

$$B = U P_0.$$

Since χ is fixed and G is an open normal subgroup of B, the restriction of the functions of J_{loc} to $G \subset B$ is injective [Clozel 2018, Section 3.3]. With $Q_0 = P_0 \cap G$, we deduce that the vector space of J_{loc} is

$$I_{\text{loc}} = \{f \in \mathcal{A}_{\text{loc}}(G, K) : f(gb) = \chi(b^{-1}) f(g),\ b \in Q_0,\ g \in G\}. \tag{3-1}$$

With the decomposition $G = U Q_0$, we see that $I_{\text{loc}} \cong \mathcal{A}_{\text{loc}}(\mathbb{Z}_p^{(n(n-1))/2}, K) = \mathcal{A}_{\text{loc}}(U, K)$. Here, $\mathbb{Z}_p$ is seen as the rigid analytic (additive) group $B^1(\mathbb{Z}_p)$. The group G acts on I_{loc} by the left translation

$$h \cdot f(g) \mapsto f(h^{-1} g). \tag{3-2}$$

Let $E_{i,j}$ be the elementary matrices with 1 in the (i, j)-th place and 0 elsewhere. From now on, we assume

$$p > n + 1;$$

then G is p-saturated in the sense of [Lazard 1965, III, 3.2.7.5] and thus, it is the ordered product (as a rigid analytic group) of the following one-parameter subgroups:

(1) First, for $y \in \mathbb{Z}_p$, take the one-parameter lower unipotent matrices by the following lexicographic order: the one-parameter group of matrices $(1 + yE_{i,j})$ comes before the one-parameter group of matrices$(1 + yE_{k,l})$ if and only if $i < k$ or $i = k$ and $j < l$. Note that $(1 + yE_{i,j})$ and $(1 + yE_{k,l})$ are lower unipotent, and hence $i > j$ and $k > l$.

(2) Then, for $t_k \equiv 1[p]$ and $k \in [1, n]$, take the one-parameter diagonal subgroups $(t_k E_{k,k} + \sum_{i=1, i\neq k}^{n} E_{i,i})$ starting from the top left extreme to the low right extreme.

(3) Finally, for $y \in p\mathbb{Z}_p$, take the upper unipotent matrices in the following order: the one-parameter group of matrices $(1 + yE_{i,j})$ comes before the one-parameter group of matrices $(1 + yE_{k,l})$ if and only if $i \geq k$ or $i = k$ and $j > l$. Note that $(1 + yE_{i,j})$ and $(1 + yE_{k,l})$ are upper unipotent, and hence $i < j$ and $k < l$.

That is, for the lower unipotent matrices, we start with the top and left extreme and then fill the lines from the left, going down and for the upper unipotent matrices we start with the low and right extreme and then fill the lines from the right, going up. (See [Lazard 1965, III, 3.3.2] for the rigid analyticity and see Theorem 2.2.1 and Remark 2.2.2 of [Ray 2020] for the order of the product, i.e., an ordered Lazard basis of G, although in [Ray 2020] we have taken G to be upper unipotent matrices modulo p but this does not matter).

Let now

$$\mathcal{A} = \mathcal{A}(U, K) = \mathcal{A}(\mathbb{Z}_p^{(n(n-1))/2}, K)$$

be the subspace of globally analytic functions of

$$I_{\text{loc}} = \mathcal{A}_{\text{loc}}(U, K).$$

Thus $f \in \mathcal{A}$ is a globally analytic function in the variables $a_{i,j}$ on U, that is,

$$f(A) = \sum_{\nu \in \mathbb{N}^d} c_\nu a^\nu$$

such that $c_\nu \in K$ and $|c_\nu| \to 0$ as $|\nu| \to \infty$. Here $d = (n(n-1))/2$, $a = (a_{2,1}, a_{3,1}, a_{3,2}, \ldots, a_{n,n-1}) \in \mathbb{Z}_p^d$ with the lexicographic ordering of $a_{i,j}$ as in (1), $\nu = (\nu_{2,1}, \nu_{3,1}, \ldots, nu_{n,n-1}) \in \mathbb{N}^d$, $a^\nu = a_{2,1}^{\nu_{2,1}} \cdots a_{n,n-1}^{\nu_{n,n-1}}$ and $|\nu| = \nu_{2,1} + \cdots + \nu_{n,n-1}$.

We now seek conditions such that if f is a globally analytic function on G and the action of G is defined as above, then the map

$$h \mapsto h \cdot f(g) = f(h^{-1}g)$$

is globally analytic.

Lemma 3.1. *With the above notation, for $p > n+1$, the action of G on $f \in \mathcal{A}(U, K)$, i.e., the map $h \mapsto h \cdot f$, is a globally analytic function on G if and only if it is so for all one-parameter (rigid analytic) subgroups and the diagonal subgroup of which G is the product.*

Proof. This follows from the same argument as in the discussion after Lemma 3.4 of [Clozel 2018]. □

Thus, our goal is to verify the analyticity of the action of the diagonal subgroup, the one-parameter lower unipotent subgroups and the one-parameter upper unipotent subgroups of G which are treated in Lemmas 3.2, 3.3 and 3.7, respectively.

Let $A = (a_{i,j})_{i,j}$ be any matrix in U (i.e., $a_{i,i} = 1$ and $a_{i,j} = 0$ for $i < j$) and

$$T = \text{Diag}(t_1, \ldots, t_n) = \sum_{k=1}^{n} t_k E_{k,k}$$

be any element in the diagonal $T_0 \cap G$, where $t_k \in 1 + p\mathbb{Z}_p$. Assume $f \in I_{\text{loc}}$, then the action of T on f, given by (3-2), is

$$\begin{aligned} T\cdot f(A) &= f(\mathrm{Diag}(t_1^{-1},\dots,t_n^{-1})A)\\ &= f\Bigl(\Bigl(\sum_{k=1}^{n} t_k^{-1}E_{k,k}\Bigr)\Bigl(\sum_{i,j=1}^{n} a_{i,j}E_{i,j}\Bigr)\Bigr)\\ &= f\Bigl(\sum_{j,k=1}^{n} t_k^{-1}a_{k,j}E_{k,j}\Bigr)\\ &= f\Bigl(\Bigl(\sum_{k,j=1}^{n} t_k^{-1}t_j a_{k,j}E_{k,j}\Bigr)\Bigl(\sum_{j=1}^{n} t_j^{-1}E_{j,j}\Bigr)\Bigr)\\ &= f\Bigl(\sum_{k,j=1}^{n} t_k^{-1}t_j a_{k,j}E_{k,j}\Bigr)\chi(t_1,\dots,t_n) \qquad \text{(from (3-1))}. \end{aligned}$$

Interchanging indices $k\to i$, we obtain

$$\Bigl(\sum_{i=1}^{n} t_iE_{i,i}\Bigr)\cdot f\Bigl(\sum_{i,j=1}^{n} a_{i,j}E_{i,j}\Bigr) = f\Bigl(\sum_{i,j=1}^{n} t_i^{-1}t_j a_{i,j}E_{i,j}\Bigr)\chi(t_1,\dots,t_n) \tag{3-3}$$

with $a_{i,i}=1,\ a_{i,j}=0$ for $i<j$ and $t_i\equiv 1 \pmod p$.

Taking $f=1$ we see that $\chi(t_1,\dots,t_n)$ must be an analytic function. By (3-3), for fixed $k\in[1,n]$ considering the action of the matrix $\bigl(t_kE_{k,k}+\sum_{i=1,i\neq k}^{n}E_{i,i}\bigr)$ on f we obtain

$$\begin{aligned} &\Bigl(t_kE_{k,k}+\sum_{i=1,i\neq k}^{n}E_{i,i}\Bigr)f(A)\\ &\quad= f\Bigl(\sum_{\substack{u,v\neq k\\ u>v}} a_{u,v}E_{u,v}+a_{k,k}E_{k,k}+\sum_{j=1}^{k-1}t_k^{-1}a_{k,j}E_{k,j}+\sum_{i=k+1}^{n}t_ka_{i,k}E_{i,k}\Bigr)\\ &\qquad\times\chi(1,\dots,t_k,\dots,1)\\ &\quad:= f(\mathcal{C})\chi(1,\dots,t_k,\dots,1) \end{aligned} \tag{3-4}$$

where $\mathcal{C}$ is the matrix

$$\Bigl(\sum_{\substack{u,v\neq k\\ u>v}} a_{u,v}E_{u,v}+a_{k,k}E_{k,k}+\sum_{j=1}^{k-1}t_k^{-1}a_{k,j}E_{k,j}+\sum_{i=k+1}^{n}t_ka_{i,k}E_{i,k}\Bigr).$$

Assume now that f is globally analytic in the variables $a_{i,j}$ on U, that is,

$$f(A)=\sum_{\nu\in\mathbb{N}^d} c_\nu a^\nu, \tag{3-5}$$

such that $c_\nu \in K$ and $|c_\nu| \to 0$. Then with $t_k = 1 + p\xi_k,\ \xi_k \in \mathbb{Z}_p$,

$$f(\mathcal{C}) = \sum_\nu c_\nu \Bigg(a_{k,k}^{\nu_{k,k}} \prod_{\substack{u,v\neq k\\ u>v}} a_{u,v}^{\nu_{u,v}}\Bigg)\Bigg(\prod_{j=1}^{k-1}(t_k^{-1}a_{k,j})^{\nu_{k,j}}\Bigg)\Bigg(\prod_{i=k+1}^{n}(t_k a_{i,k})^{\nu_{i,k}}\Bigg) \tag{3-6}$$

$$= \sum_\nu c_\nu \Bigg(a_{k,k}^{\nu_{k,k}} \prod_{\substack{u,v\neq k\\ u>v}} a_{u,v}^{\nu_{u,v}}\Bigg)\Bigg(\prod_{j=1}^{k-1}(1+p\xi_k)^{-\nu_{k,j}}a_{k,j}^{\nu_{k,j}}\Bigg)\Bigg(\prod_{i=k+1}^{n}(1+p\xi_k)^{\nu_{i,k}}a_{i,k}^{\nu_{i,k}}\Bigg). \tag{3-7}$$

Recall that for $|v| < 1,\ m \in \mathbb{N}$, we have $(1-v)^{-m} = \sum_{q=0}^{\infty}\binom{m+q-1}{q}v^q$. Now, inserting the expressions

$$(1+p\xi_k)^{-\nu_{k,j}} = \sum_{q_{k,j}=0}^{\infty}\binom{\nu_{k,j}+q_{k,j}-1}{q_{k,j}}(-p\xi_k)^{q_{k,j}}$$

and

$$(1+p\xi_k)^{\nu_{i,k}} = \sum_{u_{i,k}=0}^{\nu_{i,k}}\binom{\nu_{i,k}}{u_{i,k}}p^{u_{i,k}}\xi_k^{u_{i,k}}$$

into (3-7) we obtain, with $|q| := q_{k,1} + \cdots + q_{k,k-1},\ |u| = u_{k+1,k} + \cdots + u_{n,k}$ and $v_{\max} = \prod_{i=k+1}^{n} \nu_{i,k}$,

$$\begin{aligned}
f(\mathcal{C}) &= \sum_\nu c_\nu \Bigg(a_{k,k}^{\nu_{k,k}} \prod_{\substack{u,v\neq k\\ u>v}} a_{u,v}^{\nu_{u,v}}\Bigg)\Bigg(\prod_{j=1}^{k-1}\Bigg(\sum_{q_{k,j}=0}^{\infty}\binom{\nu_{k,j}+q_{k,j}-1}{q_{k,j}}(-p\xi_k)^{q_{k,j}}a_{k,j}^{\nu_{k,j}}\Bigg)\Bigg)\\
&\qquad\times\Bigg(\prod_{i=k+1}^{n}\sum_{u_{i,k}=0}^{\nu_{i,k}}\binom{\nu_{i,k}}{u_{i,k}}p^{u_{i,k}}\xi_k^{u_{i,k}}a_{i,k}^{\nu_{i,k}}\Bigg)\\
&= \sum_\nu c_\nu \Bigg(a_{k,k}^{\nu_{k,k}} \prod_{\substack{u,v\neq k\\ u>v}} a_{u,v}^{\nu_{u,v}}\Bigg)\Bigg(\sum_{N\geq 0}^{\infty}\xi_k^N\Bigg(\sum_{|q|=N}\prod_{j=1}^{k-1}\binom{\nu_{k,j}+q_{k,j}-1}{q_{k,j}}(-p)^{q_{k,j}}a_{k,j}^{\nu_{k,j}}\Bigg)\Bigg)\\
&\qquad\times\Bigg(\sum_{M=0}^{v_{\max}}\xi_k^M\Bigg(\sum_{|u|=M}\prod_{i=k+1}^{n}\binom{\nu_{i,k}}{u_{i,k}}p^{u_{i,k}}a_{i,k}^{\nu_{i,k}}\Bigg)\Bigg).
\end{aligned}$$

Let f_N and g_M be defined by

$$f_N = \Bigg(\sum_{|q|=N}\prod_{j=1}^{k-1}\binom{\nu_{k,j}+q_{k,j}-1}{q_{k,j}}(-p)^{q_{k,j}}a_{k,j}^{\nu_{k,j}}\Bigg), \tag{3-8}$$

$$g_M = \Bigg(\sum_{|u|=M}\prod_{i=k+1}^{n}\binom{\nu_{i,k}}{u_{i,k}}p^{u_{i,k}}a_{i,k}^{\nu_{i,k}}\Bigg). \tag{3-9}$$

Then,

$$f(\mathcal{C}) = \sum_{\nu} c_\nu (a_{k,k}^{\nu_{k,k}} \prod_{\substack{u,v\neq k\\ u>v}} a_{u,v}^{\nu_{u,v}}) \Big(\sum_{N\geq 0}^{\infty} \xi_k^N f_N\Big)\Big(\sum_{M=0}^{\nu_{\max}} \xi_k^M g_M\Big)$$

$$= \sum_{m\geq 0}^{\infty} \xi_k^m \Big(\sum_{\nu} c_\nu (a_{k,k}^{\nu_{k,k}} \prod_{\substack{u,v\neq k\\ u>v}} a_{u,v}^{\nu_{u,v}}) \sum_{N+M=m} f_N g_M\Big).$$

Recall from the introduction that any element $f \in \mathcal{A}(U, K)$ is of the form

$$f = \sum_{\nu\in\mathbb{N}^d} c_\nu a^\nu$$

with $\lim_{|\nu|\to\infty} |c_\nu| = 0$. The space $\mathcal{A}(U, K)$ is a K-Banach space with the sup norm on f defined by

$$|f| = \sup|c_\nu|$$

(see [Bosch 2014, Chapter 2]). Recall that for any K-Banach space V with norm $|\cdot|$, a representation π of G on V is called a globally analytic representation if the map

$$g \mapsto g\cdot v = \pi(g)v$$

is globally analytic on G for all $v \in V$. Thus, in coordinates $(x_1, \ldots, x_l)$ with $l = \dim(G)$,

$$g\cdot v = \sum_k x^k v_k$$

where $v_k \in V$ and $|v_k| \to 0$. Here $k = (k_1, \ldots, k_l)$ and $x^k = x_1^{k_1} \cdots x_l^{k_l}$, $k_i \in \mathbb{N}$ (see [Emerton 2017; Clozel 2018, Section 2]).

Now, with $t_k = 1 + p\xi_k$, $\xi_k \in \mathbb{Z}_p$, in order to show that the action of the one-parameter diagonal subgroup $t_k(E_{k,k}) + \sum_{i\neq k,i=1}^{n} E_{i,i}$ on $f \in \mathcal{A}(U, K)$ is analytic we have to show that the map

$$\mathbb{Z}_p \to \mathcal{A}(U, K),$$
$$\xi_k \mapsto \Big((1 + p\xi_k)E_{k,k} + \sum_{i=1,i\neq k}^{n} E_{i,i}\Big) f = f(\mathcal{C})\chi(1, \ldots, 1 + p\xi_k, \ldots, 1)$$

is a globally analytic map on $\mathbb{Z}_p$. The norm of the coefficient of ξ_k^m, in (3-10), is

$$\bigg|\Big(\sum_{\nu} c_\nu \Big(a_{k,k}^{\nu_{k,k}} \prod_{\substack{u,v\neq k\\ u>v}} a_{u,v}^{\nu_{u,v}}\Big) \sum_{N+M=m} f_N g_M\Big)\bigg|.$$

Notice that, since $N, M \leq m$ and $f_N, g_M \in \mathbb{Z}_p$ from (3-8) and (3-9), the quantity $\big(a_{k,k}^{\nu_{k,k}} \prod_{u,v\neq k,\, u>v} a_{u,v}^{\nu_{u,v}} \sum_{N+M=m} f_N g_M\big)$ has finite sum and product and hence lies in $\mathbb{Z}_p$. Hence,

$$\left|\left(\sum_{\nu} c_\nu \left(a_{k,k}^{\nu_{k,k}} \prod_{\substack{u,v\neq k \\ u>v}} a_{u,v}^{\nu_{u,v}}\right) \sum_{N+M=m} f_N g_M\right)\right| \to 0$$

as $|c_\nu| \to 0$ with $\nu \to \infty$. This gives the analyticity of the action $f \to f(\mathcal{C})$. We treat the analyticity of the character χ in general. Write $\chi = (\chi_1, \dots, \chi_n)$, $\chi_i(1 + pu_i) = e^{c_i \log(1+pu_i)}$ for $c_i \in K$, u_i close to 0, $i \in [1, n]$. The exponential is analytic (in K) in the domain $v_p(z) > e/(p-1)$ where $e = e(K)$ is the verification index and v_p is the normalized valuation, $v_p(p) = 1$. Now,

$$v_p(c_i \log(1 + pu_i)) = v_p(c_i) + 1 + v_p(u_i).$$

So we must have

$$v_p(c_i) + 1 > \frac{e}{p-1};$$

that is,

$$v_p(c_i) > \frac{e}{p-1} - 1. \tag{3-10}$$

We say that χ is "analytic" if and only if the c_i's verify the condition (3-10) and in the rest of this text we assume that our character χ is analytic. It is easy to see that if χ is analytic, then $\chi(1, \dots, 1 + p\xi_k, \dots, 1)$ is an analytic function on ξ_k. The character

$$\begin{aligned}\chi(1, \dots, 1+p\xi_k, \dots, 1) = \chi_k(1+p\xi_k) &= \sum_{n=0}^{\infty} c_n (1+p\xi_k)^n \quad \text{(since } \chi_k \text{ is analytic)} \\ &= \sum_{n=0}^{\infty} c_n \sum_{u=0}^{n} \binom{n}{u} p^u \xi_k^u \\ &= \sum_{u=0}^{\infty} \xi_k^u \left(p^u \sum_{n\geq u}^{\infty} c_n \binom{n}{u}\right).\end{aligned}$$

The norm of the coefficient of ξ_k^u is

$$\left|p^u \sum_{n\geq u}^{\infty} c_n \binom{n}{u}\right|$$

which goes to 0 as $|c_n| \to 0$ with $n \to \infty$. Thus, we have shown:

Lemma 3.2. *Under the hypothesis (3-10), for each $k \in [1, n]$, the action of the one-parameter diagonal subgroup $\left(t_k E_{k,k} + \sum_{i=1, i\neq k}^{n} E_{i,i}\right)$ of G on $\mathcal{A}(\mathbb{Z}_p^{(n(n-1))/2}, K)$ given by (3-4) is an analytic action.*

For $y \in \mathbb{Z}_p$ and $i > j$, with i, j fixed between $1, \ldots, n$, the action of the one-parameter (rigid analytic) subgroup $(1 + yE_{i,j})$ on $f(A)$, given by (3-2) is

$$(1 + yE_{i,j})f(A) = f((1 + yE_{i,j})^{-1}A) = f((1 - yE_{i,j})A) \tag{3-11}$$
$$= f\Bigg((1 - yE_{i,j})\Bigg(\sum_{\substack{k \geq l \\ k,l \in [1,n]}} a_{k,l}E_{k,l}\Bigg)\Bigg)$$
$$= f\Bigg(\sum_{\substack{k \geq l \\ k,l \in [1,n]}} a_{k,l}E_{k,l} - \sum_{l=1,\ldots,j} ya_{j,l}E_{i,l}\Bigg) := f(\mathcal{B}),$$

where $\mathcal{B}$ is the matrix

$$\sum_{k \geq l,\ k,l \in [1,n]} a_{k,l}E_{k,l} - \sum_{l=1}^{j} ya_{j,l}E_{i,l}.$$

One can easily see that the matrix $\mathcal{B} = (b_{u,v})$ is lower unipotent and differs from matrix A only in the first j entries of its i-th row. In particular,

$$b_{i,v} = a_{i,v} - ya_{j,v}$$

for all $v \in [1, j]$, $a_{j,j} = 1$, and all other $b_{u,v}$ are the same as $a_{u,v}$ (recall that A is lower unipotent).

Now, let f be a globally analytic function on U as in (3-5). That is

$$f(a) = \sum_{\nu \in \mathbb{N}^d} c_\nu a^\nu$$

with $a^\nu = a_{2,1}^{\nu_{2,1}} \cdots a_{n,n-1}^{\nu_{n,n-1}}$ and $|c_\nu| \to 0$. Then, we have to show that

$$(1 + yE_{i,j})f = f(\mathcal{B})$$

gives an analytic map

$$\mathbb{Z}_p \to \mathcal{A}(U, K),$$
$$y \to (1 + yE_{i,j})f = f(\mathcal{B}).$$

The power series $f(\mathcal{B})$ is equal to

$$\sum_\nu c_\nu \Bigg(\Bigg(\prod_{\substack{u > v \\ u = i \Rightarrow v > j}} a_{u,v}^{\nu_{u,v}}\Bigg)\Bigg(\prod_{k=1}^{j}(a_{i,k} - ya_{j,k})^{\nu_{i,k}}\Bigg)\Bigg).$$

For each $k \in [1, j]$, inserting the expansion

$$(a_{i,k} - ya_{j,k})^{\nu_{i,k}} = \sum_{m_{i,k}=0}^{\nu_{i,k}} \binom{\nu_{i,k}}{m_{i,k}} y^{m_{i,k}}(-a_{j,k})^{m_{i,k}} a_{i,k}^{\nu_{i,k} - m_{i,k}}$$

into $f(\mathcal{B})$ we obtain, for $M=\prod_{k=1}^{j}\nu_{i,k},\ |m|=m_{i,1}+\cdots+m_{i,j}$,

$$f(\mathcal{B})=\sum_{\nu}c_\nu\Biggl(\Biggl(\prod_{\substack{u>v\\ u=i\Rightarrow v>j}}a_{u,v}^{\nu_{u,v}}\Biggr)\Biggl(\prod_{k=1}^{j}\Biggl(\sum_{m_{i,k}=0}^{\nu_{i,k}}\binom{\nu_{i,k}}{m_{i,k}}y^{m_{i,k}}(-a_{j,k})^{m_{i,k}}a_{i,k}^{\nu_{i,k}-m_{i,k}}\Biggr)\Biggr)\Biggr)$$

$$=\sum_{\nu}c_\nu\Biggl(\Biggl(\prod_{\substack{u>v\\ u=i\Rightarrow v>j}}a_{u,v}^{\nu_{u,v}}\Biggr)\Biggl(\sum_{N=0}^{M}y^N\Biggl(\sum_{\substack{|m|=N\\ m_{i,\star}\in[0,\nu_{i,\star}]}}\prod_{k=1}^{j}\binom{\nu_{i,k}}{m_{i,k}}(-a_{j,k})^{m_{i,k}}a_{i,k}^{\nu_{i,k}-m_{i,k}}\Biggr)\Biggr)\Biggr)$$

$$=\sum_{\nu}c_\nu\Biggl(\sum_{N=0}^{M}y^N\Biggl(\Biggl(\prod_{\substack{u>v\\ u=i\Rightarrow v>j}}a_{u,v}^{\nu_{u,v}}\Biggr)\sum_{\substack{|m|=N\\ m_{i,\star}\in[0,\nu_{i,\star}]}}\prod_{k=1}^{j}\binom{\nu_{i,k}}{m_{i,k}}(-a_{j,k})^{m_{i,k}}a_{i,k}^{\nu_{i,k}-m_{i,k}}\Biggr)\Biggr)$$

$$=\sum_{N=0}^{M}y^N\Biggl(\sum_{\nu}c_\nu\Biggl(\Biggl(\prod_{\substack{u>v\\ u=i\Rightarrow v>j}}a_{u,v}^{\nu_{u,v}}\Biggr)\sum_{\substack{|m|=N\\ m_{i,\star}\in[0,\nu_{i,\star}]}}\prod_{k=1}^{j}\binom{\nu_{i,k}}{m_{i,k}}(-a_{j,k})^{m_{i,k}}a_{i,k}^{\nu_{i,k}-m_{i,k}}\Biggr)\Biggr)$$

$$=\sum_{N=0}^{M}y^N f_N,$$

where

$$f_N:=\sum_{\nu}c_\nu\Biggl(\Biggl(\prod_{\substack{u>v\\ u=i\Rightarrow v>j}}a_{u,v}^{\nu_{u,v}}\Biggr)\sum_{\substack{|m|=N\\ m_{i,\star}\in[0,\nu_{i,\star}]}}\prod_{k=1}^{j}\binom{\nu_{i,k}}{m_{i,k}}(-a_{j,k})^{m_{i,k}}a_{i,k}^{\nu_{i,k}-m_{i,k}}\Biggr).$$

Define

$$s(N,\nu):=\Biggl(\Biggl(\prod_{\substack{u>v\\ u=i\Rightarrow v>j}}a_{u,v}^{\nu_{u,v}}\Biggr)\sum_{\substack{|m|=N\\ m_{i,\star}\in[0,\nu_{i,\star}]}}\prod_{k=1}^{j}\binom{\nu_{i,k}}{m_{i,k}}(-a_{j,k})^{m_{i,k}}a_{i,k}^{\nu_{i,k}-m_{i,k}}\Biggr)$$

such that $f_N=\sum_\nu c_\nu s(N,\nu)$. Notice that since $m_{i,k}\le\nu_{i,k}$ for all $k\in[1,j]$, the sum in $s(N,\nu)$ is a finite sum and thus $s(N,\nu)$ lies in $\mathbb{Z}_p$. Therefore, the norm of the coefficient of y^N is $|f_N|=\bigl|\sum_\nu c_\nu s(N,\nu)\bigr|$ which goes to 0 as $|c_\nu|\to 0$ with $|\nu|\to\infty$. This gives the analyticity of the map $y\to(1+E_{i,j})f=f(\mathcal{B})$.

Therefore, we have shown:

Lemma 3.3. *For $y\in\mathbb{Z}_p$ and $i>j$, the action of the lower unipotent* (*rigid analytic*) *one-parameter subgroup $(1+yE_{i,j})$ of G on $f\in\mathcal{A}(\mathbb{Z}_p^{(n(n-1))/2},K)$ given by (3-11) is an analytic action.*

It remains to check the analyticity of the action (3-2) by triangular superior matrices of the form $(1+yE_{i,j})$ for $i<j,\ i,j\in[1,n],\ y\in p\mathbb{Z}_p$. Recall that the action of $(1+yE_{i,j})$ on $f\in I_{\mathrm{loc}}$ given by (3-2) is

$$(1+yE_{i,j})f(A)=f((1+yE_{i,j})^{-1}A)=f((1-yE_{i,j})A).$$

Recall the action of Q_0 given by (3-1), that is, $f(gb) = \chi(b^{-1}) f(g)$ with $b \in Q_0$. Hence, our objective is to write the matrix $(1 - yE_{i,j})A$ as the product of two matrices X and Z with $X \in U$ and $Z \in Q_0$, that is,

$$(1 - yE_{i,j})A = XZ,$$

where X is a lower unipotent matrix with entries in $\mathbb{Z}_p$ and Z is an upper triangular matrix with diagonal elements in $1 + p\mathbb{Z}_p$ and such that the elements above the diagonal have entries in $p\mathbb{Z}_p$.

Lemma 3.4. *For $i < j$ and $y \in p\mathbb{Z}_p$, there exists a unique matrix decomposition $(1 - yE_{i,j})A = XZ$ with $X = (x_{k,l})_{k,l} \in U$ and $Z = (z_{r,s})_{r,s} \in Q_0$. Also,*

(i) *all the diagonal elements $z_{r,r}$ of Z are of the form*

$$\frac{1 - yh_{r,r}(y, a)}{1 - yg_{r,r}(y, a)},$$

(ii) *all the elements $z_{r,s}$, for $r < s$, of Z are of the form*

$$\frac{yh_{r,s}(y, a)}{1 - yg_{r,s}(y, a)},$$

(iii) *all the elements $x_{k,l}$ with $k > l$ of the lower triangular unipotent matrix X are of the form*

$$\frac{h_{k,l}(y, a)}{1 - yg_{k,l}(y, a)},$$

where $h_{\star,\star}(y, a)$ and $g_{\star,\star}(y, a)$ are polynomial functions with integral coefficients in y and $a_{2,1}, a_{3,1}, a_{3,2}, \dots, a_{n,n-1}$ (entries of the lower unipotent matrix A).

Proof. We prove the lemma by an easy inductive argument. The base case $n = 2$ is clear from the matrix equation

$$\begin{pmatrix} 1 & -y \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ a_{2,1} & 1 \end{pmatrix} = \begin{pmatrix} 1 - ya_{2,1} & -y \\ a_{2,1} & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ x_{2,1} & 1 \end{pmatrix} \begin{pmatrix} z_{1,1} & z_{1,2} \\ 0 & z_{2,2} \end{pmatrix}$$
$$= \begin{pmatrix} z_{1,1} & z_{1,2} \\ z_{1,1}x_{2,1} & z_{2,2} + z_{1,2}x_{2,1} \end{pmatrix}$$

with $x_{2,1} = a_{2,1}/(1 - ya_{2,1})$, $z_{1,1} = 1 - ya_{2,1}$, $z_{1,2} = -y$, $z_{2,2} = 1/(1 - ya_{2,1})$. Assume, by induction hypothesis that our lemma is true for GL$(n-1)$. We show it for GL(n). Let us first suppose that $i > 1$, that is, with some elementary matrix E', where $1 - yE' \in \mathrm{GL}(n-1)$, we have

$$(1 - yE_{i,j}) = \left(\begin{array}{c|c} 1 & 0 \cdots 0 \\ \hline 0 & \\ \vdots & 1 - yE' \\ 0 & \end{array}\right).$$

The matrix A, being lower unipotent, can be written in the following block form:

$$A = \left(\begin{array}{c|c} 1 & 0\cdots 0 \\ \hline a_{2,1} & \\ \vdots & A' \\ a_{n,1} & \end{array}\right),$$

with $A' \in \mathrm{GL}(n-1)$. Setting $\underline{a}$ to be the column vector

$$\begin{pmatrix} a_{2,1} \\ a_{3,1} \\ \vdots \\ a_{n,1} \end{pmatrix},$$

we have

$$(1 - yE_{i,j})A = \left(\begin{array}{c|c} 1 & 0\cdots 0 \\ \hline 0 & \\ \vdots & 1 - yE' \\ 0 & \end{array}\right) \left(\begin{array}{c|c} 1 & 0\cdots 0 \\ \hline a_{2,1} & \\ \vdots & A' \\ a_{n,1} & \end{array}\right) = \left(\begin{array}{c|c} 1 & 0 \\ \hline (1-yE')\underline{a} & (1-yE')A' \end{array}\right).$$

We want to decompose the above matrix in the form

$$\left(\begin{array}{c|c} 1 & 0 \\ \hline (1-yE')\underline{a} & (1-yE')A' \end{array}\right) = \left(\begin{array}{c|c} 1 & 0\cdots 0 \\ \hline x_{2,1} & \\ \vdots & X' \\ x_{n,1} & \end{array}\right) \left(\begin{array}{c|c} z_{1,1} & z_{1,2}\cdots z_{1,n} \\ \hline 0 & \\ \vdots & Z' \\ 0 & \end{array}\right)$$

with $x_{2,1}, \ldots, x_{n,1} \in \mathbb{Z}_p$, $z_{1,1} \in 1 + p\mathbb{Z}_p$ and $z_{1,2}, \ldots, z_{1,n} \in p\mathbb{Z}_p$. Denote $\underline{z}$ to be the row vector $[z_{1,2}, \ldots, z_{1,n}]$, $\underline{x}$ to be the column vector

$$\begin{pmatrix} x_{2,1} \\ x_{3,1} \\ \vdots \\ x_{n,1} \end{pmatrix}.$$

Hence, we want to solve

$$\left(\begin{array}{c|c} 1 & 0 \\ \hline (1-yE')\underline{a} & (1-yE')A' \end{array}\right) = \left(\begin{array}{c|c} z_{1,1} & \underline{z} \\ \hline z_{1,1}\underline{x} & \underline{x}\cdot\underline{z} + X'Z' \end{array}\right).$$

So we must have

(1) $z_{1,1} = 1$,

(2) $\underline{z} = 0$,

(3) $z_{1,1}\underline{x} = \underline{x} = (1 - yE')\underline{a}$ (using $z_{1,1} = 1$ from (1)),

(4) $\underline{x}\cdot\underline{z} + X'Z' = X'Z' = (1 - yE')A'$ (as $\underline{z} = 0$ from (2)).

By the induction hypothesis, we can find X' and Z' satisfying (4) with entries as in Lemma 3.4. Also, (3) is of the form

$$\begin{pmatrix} 1 & & \\ & -y & \\ & \ddots & \\ & & 1 \end{pmatrix} \begin{pmatrix} a_{2,1} \\ a_{3,1} \\ \vdots \\ a_{n,1} \end{pmatrix} = \begin{pmatrix} x_{2,1} \\ x_{3,1} \\ \vdots \\ x_{n,1} \end{pmatrix}$$

Clearly, we can solve $x_{2,1}, \ldots, x_{n,1}$ from the above matrix equation satisfying Lemma 3.4 and in fact the solutions do not have any denominators.

So by induction we are reduced to the case $i = 1$, that is, when

$$(1 - yE_{i,j}) = \left(\begin{array}{c|c} 1 & 0 \cdots -y \cdots 0 \\ \hline 0 & \\ \vdots & 1 \\ 0 & \end{array}\right).$$

Our goal is to solve, for X and Z, the matrix equation

$$(1 - yE_{1,j})A = XZ. \tag{3-12}$$

Expanding right-hand side of (3-12), we obtain

$$\begin{aligned} \mathcal{B} = (b_{u,v})_{u,v} = XZ &= \Bigg(1 + \sum_{\substack{k\in[1,n] \\ l\in[1,k-1]}} x_{k,l} E_{k,l}\Bigg)\Bigg(\sum_{\substack{r\in[1,n] \\ s\in[r,n]}} z_{r,s} E_{r,s}\Bigg) \\ &= \sum_{\substack{r\in[1,n] \\ s\in[r,n]}} z_{r,s} E_{r,s} + \sum_{\substack{k\in[1,n] \\ r\in[1,k-1] \\ s\in[r,n]}} x_{k,r} z_{r,s} E_{k,s}. \end{aligned}$$

Therefore,

$$b_{u,v} = \begin{cases} \displaystyle\sum_{r=1}^{v} x_{u,r} z_{r,v} & \text{if } u > v, \\ \displaystyle z_{u,v} + \sum_{r=1}^{u-1} x_{u,r} z_{r,v} & \text{if } u \leq v. \end{cases} \tag{3-13}$$

Recall that our matrix

$$A = \sum_{k\geq l} a_{k,l} E_{k,l}$$

is lower unipotent, that is, $a_{k,k} = 1$ for all $k \in [1, n]$ and $a_{k,l} = 0$ for $k < l$. Expanding the left-hand side of (3-12), we obtain

$$(1-yE_{1,j})A=(1-yE_{1,j})\Big(\sum_{k\geq l}a_{k,l}E_{k,l}\Big)=\sum_{k\geq l}a_{k,l}E_{k,l}-\sum_{l=1}^{j}ya_{j,l}E_{1,l}$$
$$=\sum_{\substack{k\in[1,n]\\ l\in[1,k]\\ k\neq 1}}a_{k,l}E_{k,l}+\sum_{l=2}^{j}(-ya_{j,l})E_{1,l}+(1-ya_{j,1})E_{1,1}.$$

Note that the first row of the matrix $(1-yE_{1,j})A$ is

$$\sum_{l=2}^{j}(-ya_{j,l})E_{1,l}+(1-ya_{j,1})E_{1,1}.$$

From (3-12), the matrices $(1-yE_{1,j})A$ and $\mathcal{B}=(b_{u,v})_{u,v}$ are equal. Thus, equating $b_{u,v}$ from (3-13) with the above expression of the matrix $(1-yE_{1,j})A$, we obtain the following equations (with the convention that $x_{k,l}=0$ for $k\leq l$ and $z_{r,s}=0$ for $r>s$):

(1) For $u\neq 1$ and $u>v$, $b_{u,v}=\sum_{r=1}^{v}x_{u,r}z_{r,v}=a_{u,v}$.

(2) For $u\neq 1$ and $u=v$, $b_{u,v}=z_{u,u}+\sum_{r=1}^{u-1}x_{u,r}z_{r,v}=a_{u,u}=1$.

(3) For $u\neq 1$ and $u<v$, $b_{u,v}=z_{u,v}+\sum_{r=1}^{u-1}x_{u,r}z_{r,v}=a_{u,v}=0$.

(4) For $u=v=1$, $b_{1,1}=z_{1,1}=1-ya_{j,1}$.

(5) For $u=1$ and $u<v$, $b_{1,v}=z_{1,v}=-ya_{j,v}$.

Note that in (5), for $v>j$, $b_{1,v}=-ya_{j,v}=0$ (as A is lower unipotent). Setting $v=1$ in (1), for $u\in[2,n]$, we obtain

$$x_{u,1}=\frac{a_{u,1}}{z_{1,1}}=\frac{a_{u,1}}{1-ya_{j,1}}\quad\text{(as } z_{1,1}=1-ya_{j,1}\text{ from (4))}.\tag{3-14}$$

Now, let $C=(c_{k,l})_{k,l}=(1-yE_{1,j})A$ and $\mathcal{B}=(b_{u,v})_{u,v}$ as above. We proceed by equating, in stage 1, the first row of the matrix $\mathcal{B}$ with the first row of the matrix C, starting from the leftmost entry (i.e., given by (4) and (5) above) and solve for $z_{\star,\star}$. Then in the next stage (say, stage $1+\frac{1}{2}$) we equate the first column of the matrix $\mathcal{B}$ with the first column of the matrix C starting from the uppermost entry ($b_{2,1}=c_{2,1}$) and solve for $x_{\star,\star}$ (i.e., those given by (3-14)). In stage 2, we do the same with the second row and in the stage $2+\frac{1}{2}$ we equate the second column of the matrix $\mathcal{B}$ with C (given by (1), (2) and (3)) and proceed like this until the last (n-th) stage. Our objective is to solve $x_{\star,\star}$ and $z_{\star,\star}$ while equating the matrix $\mathcal{B}$ with C and to show (i), (ii) and (iii) of Lemma 3.4. We prove this by induction.

Assume, by induction hypothesis, at stages m and $m+\frac{1}{2}$, $1\leq m<n$, that we have found $x_{k,l}$ for $k\in[2,n]$, $l\in[1,m]$, $k>l$ and $z_{r,s}$ for $r\in[1,m]$, $s\in[1,n]$, $r\leq s$, having the forms (i), (ii) and (iii), respectively. Then, at stage $(m+1)$, we have to equate $b_{m+1,v} = c_{m+1,v}$ for $v \in [m+1, n]$. Equating $b_{m+1,m+1} = c_{m+1,m+1}$, we deduce, by (2), that

$$\begin{aligned} z_{m+1,m+1} &= 1 - \sum_{r=1}^{m} x_{m+1,r} z_{r,m+1} \\ &= 1 - \frac{y h_1(y,a)}{1 - y g_1(y,a)} = \frac{1 - y(h_1 + g_1)}{1 - y g_1}, \end{aligned}$$

where the second equality is by induction hypothesis, for some polynomial functions $h_1(y,a)$ and $g_1(y,a)$ with integral coefficients in y and $a_{2,1}, a_{3,1}, a_{3,2}, \dots, a_{n,n-1}$.

Similarly, equating $b_{m+1,v} = c_{m+1,v}$ for $v \in [m+2, n]$, we obtain, by (3), that

$$\begin{aligned} z_{m+1,v} &= -\sum_{r=1}^{m} x_{m+1,r} z_{r,v} \\ &= \frac{-y h_2(y,a)}{1 - y g_2(y,a)} \qquad \text{(again by induction hypothesis).} \end{aligned}$$

At stage $(m+1) + \frac{1}{2}$ we have to equate $b_{u,m+1} = c_{u,m+1}$ for all $u \in [m+2, n]$. So, by (1), we get

$$\begin{aligned} x_{u,m+1} z_{m+1,m+1} &= a_{u,m+1} - \sum_{r=1}^{m} x_{u,r} z_{r,m+1} \\ &= a_{u,m+1} - \frac{y h_3(y,a)}{1 - y g_3(y,a)} \qquad \text{(again by induction hypothesis)} \\ &= \frac{h_4(y,a)}{1 - y g_3(y,a)} \end{aligned}$$

for some h_4 and g_3 with integral coefficients. Therefore,

$$x_{u,m+1} = \frac{h_4(y,a)}{1 - y g_3(y,a)} \cdot \frac{1}{z_{m+1,m+1}} = \frac{h_4(y,a)}{1 - y g_3(y,a)} \frac{1 - y g_1}{1 - y(h_1 + g_1)} = \frac{h_5(y,a)}{1 - y g_5(y,a)},$$

with polynomials h_5 and g_5 having integral coefficients. This completes our induction argument and finishes the proof of Lemma 3.4. □

Now, let $f \in I_{\text{loc}}$. Then, by Lemma 3.4, and with X and $z_{\star,\star}$ as contained therein, the action of $(1 + y E_{i,j})$ on f is given by

$$(1 + y E_{i,j}) f(A) = f(X) \chi(z_{1,1}^{-1}, \dots, z_{n,n}^{-1}). \tag{3-15}$$

Recall that for $|v| < 1$, we have

$$(1 - v)^{-m} = \sum_{q=0}^{\infty} \binom{m+q-1}{q} v^q.$$

Assume now that $f \in \mathcal{A}(\mathbb{Z}^{(n(n-1))/2}, K)$ is a globally analytic function. Thus, f is an element in the Tate algebra of U with $\frac{1}{2}n(n-1)$ variables. In order to show that the action of $(1 + yE_{i,j})$ on $f \in \mathcal{A}(U, K)$, given by (3-15), is globally analytic we have to show that

$$\prod_{r=1}^{n} \chi_r(z_{r,r}^{-1}) f(X)$$

is a globally analytic function in y.

Lemma 3.5. *If the action $f \to g,\ g(A) = f(X)$, where $A = XZ$, is globally analytic, then $f \to \prod_{r=1}^{n} \chi_r(z_{r,r}^{-1}) g$ is globally analytic.*

Proof. With (3-10), our character χ is analytic. Hence,

$$\begin{aligned}\chi_r(z_{r,r}^{-1}) &= \chi_r\left(\frac{1 - yg_{r,r}(y, a)}{1 - yh_{r,r}(y, a)}\right) && \text{(from Lemma 3.4)}\\ &= \sum_{n=0}^{\infty} c_n \left(\frac{1 - yg_{r,r}(y, a)}{1 - yh_{r,r}(y, a)}\right)^n && (\text{for } |c_n| \to 0).\end{aligned}$$

We are reduced to showing that $(y, a) \to (1 - yg_{r,r}(y, a))/(1 - yh_{r,r}(y, a))$ is analytic in y and this is true because

$$\frac{1 - yg_{r,r}(y, a)}{1 - yh_{r,r}(y, a)} = (1 - yg_{r,r}(y, a))\left(\sum_{n=0}^{\infty}(yh_{r,r}(y, a))^n\right). \qquad \square$$

Therefore, with Lemma 3.5, to prove that the action of $(1 + yE_{i,j})$ on $f \in \mathcal{A}(U, K)$, given by (3-15), is globally analytic, we only need to show that the action $f \to g,\ g(A) = f(X)$, where $A = XZ$, is globally analytic.

Lemma 3.6. *The action $f \to g,\ g(A) = f(X)$, where $A = XZ$, is globally analytic.*

Proof. Recall that the lower unipotent matrix X is $((x_{k,l})_{k,l})$ with

$$x_{k,l} = \frac{h_{k,l}(y, a)}{1 - yg_{k,l}(y, a)}$$

given by Lemma 3.4. Write

$$x_{k,l} = h_{k,l}(y, a) \sum_{n=0}^{\infty} y^n g_{k,l}(y, a)^n = \sum_{n=0}^{\infty} y^n g_{n,k,l}(y, a).$$

Since f is analytic, $f = \sum_N f_N x^N$ with $N = (N_{k,l}) \in \mathbb{Z}_p^{n(n-1)/2},\ x^N = \prod_{k>l} x_{k,l}^{N_{k,l}}$. The norm $|f_N| \to 0$ as $N \to \infty$. Then,

$$f(X) = f((x_{k,l})_{k,l}) = \sum_N f_N \prod_{\substack{k,l\\k>l}} \left(\sum_{n=0}^{\infty} y^n g_{n,k,l}(y, a)\right)^{N_{k,l}}.$$

As

$$\left(\sum_{n=0}^{\infty} y^n g_n\right)^M = \sum_{v\geq 0} y^v \sum_{v_1+\cdots+v_M=v} g_{v_1}\cdots g_{v_M},$$

we obtain that

$$f(X) = \sum_{N=(N_{k,l})} f_N \prod_{k,l} \sum_{v\geq 0} y^v \Bigg(\sum_{v_1+\cdots+v_{N_{k,l}}=v} g_{v_1,k,l}\cdots g_{v_{N_{k,l}},k,l} \Bigg).$$

Define $a_{k,l}(v) = \left(\sum_{v_1+\cdots+v_{N_{k,l}}=v} g_{v_1,k,l}\cdots g_{v_{N_{k,l}},k,l}\right)$; then,

$$\begin{aligned} f(X) &= \sum_{N=(N_{k,l})} f_N \prod_{k,l} \sum_{v\geq 0} y^v a_{k,l}(v) \\ &= \sum_{N=(N_{k,l})} f_N \sum_{v\geq 0}^{\infty} y^v \sum_{\sum v_{k,l=v}} \prod_{k,l} a_{k,l}(v_{k,l}) \\ &= \sum_{N=(N_{k,l})} f_N \sum_{v\geq 0}^{\infty} y^v \sum_{\sum v_{k,l=v}} \prod_{k,l} \sum_{v_1+\cdots+v_{N_{k,l}}=v_{k,l}} g_{v_1,k,l}\cdots g_{v_{N_{k,l}},k,l}. \end{aligned}$$

The coefficient of y^v is

$$\sum_{N=(N_{k,l})} f_N \sum_{\sum v_{k,l=v}} \prod_{k,l} \sum_{v_1+\cdots+v_{N_{k,l}}=v_{k,l}} g_{v_1,k,l}\cdots g_{v_{N_{k,l}},k,l} := \sum_{N=(N_{k,l})} f_N s_N,$$

where

$$s_N := \sum_{\sum v_{k,l=v}} \prod_{k,l} \sum_{v_1+\cdots+v_{N_{k,l}}=v_{k,l}} g_{v_1,k,l}\cdots g_{v_{N_{k,l}},k,l}.$$

Here $N_{k,l}$ is finite and $v_{k,l} \leq v$ and hence the sum s_N is a finite sum in $\mathbb{Z}_p$. As, with $N \to \infty$, $|f_N| \to 0$, we obtain that $\left|\sum_N f_N s_N\right| \to 0$ and this completes the proof. □

This shows the analyticity of the action given by (3-15). So we have shown:

Lemma 3.7. *For $y \in p\mathbb{Z}_p$ and $i < j$, the action of the upper unipotent (rigid analytic) one-parameter subgroup $(1 + yE_{i,j})$ of G on $f \in \mathcal{A}(\mathbb{Z}_p^{n(n-1)/2}, K)$, given by (3-15) is an analytic action.*

Note that, by Section 3A, the vector space of locally analytic functions of principal series

$$\operatorname{ind}_{P_0}^B(\chi)_{\mathrm{loc}} = \{f \in \mathcal{A}_{\mathrm{loc}}(B, K) : f(gb) = \chi(b^{-1})f(g),\ b \in P_0,\ g \in B\}$$

is isomorphic to the vector space of the locally analytic functions

$$I_{\mathrm{loc}} \cong \mathcal{A}_{\mathrm{loc}}(\mathbb{Z}_p^{n(n-1)/2}, K).$$

Denote by $\mathrm{ind}_{P_0}^B(\chi)$ the space of globally analytic vectors of $\mathrm{ind}_{P_0}^B(\chi)_{\mathrm{loc}}$, which is

$$\mathcal{A} := \mathcal{A}(\mathbb{Z}_p^{n(n-1)/2}, K).$$

Also, the representation on $\mathcal{A}$ is admissible: indeed, $\mathcal{A}$ is a subspace of $\mathcal{A}(G)$ defined by the conditions $f(gb) = \chi(b^{-1}) f(g)$ (f is then analytic on G since χ is) and this is a closed subspace. Thus by Lemmas 3.1–3.3 and Lemma 3.7 we have shown the following theorem.

Theorem 3.8. *Assume $p > n+1$. Let χ be an analytic character of T_0 (see (3-10)). The action of G on the induced principal series $\mathrm{ind}_{P_0}^B(\chi)$ is a globally analytic action. Moreover, the globally analytic representation of G on $\mathrm{ind}_{P_0}^B(\chi)$ is admissible in the sense of [Emerton 2017].*

Recall that $\chi = (\chi_1, \dots, \chi_n)$ where $\chi_i(1 + pu_i) = e^{c_i \log(1+pu_i)}$ for $c_i \in K$, with u_i close to 0, $i \in [1, n]$. Also, recall from (3-5) that $f \in \mathcal{A}$ implies that $f(A) = \sum_{\nu \in \mathbb{N}^d} c_\nu a^\nu$ with $|c_\nu| \to 0$ as $|\nu| = \nu_{2,1} + \nu_{3,1} + \cdots + \nu_{n,n-1} \to \infty$.

In the following, we will have conditions on the character χ such that the globally analytic representation of G on $\mathcal{A}$ is irreducible.

Let μ be the linear form from the Lie algebra of the torus T_0 to K given by

$$\mu = (-c_1, \dots, c_n) : \mathrm{Diag}(t_1, \dots, t_n) \mapsto \sum_{i=1}^{n} -c_i t_i,$$

where $t = (t_i) \in \mathrm{Lie}(T_0)$. For negative root $\alpha = (i, j)$, $i > j$, let $H_{(i,j)}$ be the matrix $E_{i,i} - E_{j,j}$ where $E_{i,i}$ is the standard elementary matrix.

Theorem 3.9. *Let the c_i's satisfy (3-10) and $p > n+1$, then the globally analytic representation $\mathcal{A} \cong \mathrm{ind}_{P_0}^B(\chi)$ of G is topologically irreducible if and only if $-\mu(H_{\alpha=(i,j)}) + i - j \notin \{1, 2, 3, \dots\}$ for all $\alpha = (i, j) \in \Phi^-$.*

Assume $\mathcal{X} \subset \mathcal{A}$ is a closed nontrivial G-invariant subspace. Let Φ, Φ^-, Φ^+, Π be the roots, negative roots, positive roots and simple roots, respectively, associated to G. Consider $f \in \mathcal{A}$. Then, from (3-5),

$$f = \sum_{\nu \in \mathbb{N}^d} c_\nu a^\nu$$

where $d = n(n-1)/2$, $c_\nu \in K$, $|c_\nu| \to 0$ as $|\nu| := \sum_{\alpha \in \Phi^-} \nu_\alpha \to \infty$.
Here, $\nu = (\nu_\alpha, \alpha \in \Phi^-) \in \mathbb{N}^d$ and $a_\nu = \prod_{\alpha \in \Phi^-} a_\alpha^{\nu_\alpha}$. In some arguments we will have to order the exponents ν_α. We use the following lexicographic order. Let $\alpha = (i, j)$ and $\alpha' = (k, l)$. Then ν_α comes before $\nu_{\alpha'}$ if and only if $i < k$ or $i = k$ and $j < l$, i.e., $\nu = (\nu_{2,1}, \nu_{3,1}, \nu_{3,2}, \dots, \nu_{n,n-1})$ (see also the discussion before (3-5)). For $N \geq 0$, let τ_N be the natural truncation

$$\mathcal{A} \to K[a]_N := \oplus_{|\nu| \leq N} K a^\nu.$$

The latter space is the space of polynomials in several variables with total degree $\leq N$. As τ_N is equivariant under the action of the diagonal subgroup of G given by formulas (3-3) and (3-4) and the associated characters of the diagonal torus of G are linearly independent, $\tau_N(\mathcal{X})$ is a direct sum of monomials given by

$$\mathcal{X}_N := \tau_N(\mathcal{X}) = \Big\{ \sum_{\nu \in M_N} c_\nu a^\nu \Big\},$$

where M_N is the set of exponents of all the elements in $\mathcal{X}_N$. If $N \leq N'$ and $\nu \in M_N$, then as $\mathcal{X}_N$ is the image of the degree N truncation operator τ_N on $\mathcal{X}$, the surjectivity

$$K[a]_{N'} \twoheadrightarrow K[a]_N$$

implies that $M_N \subset M_{N'}$. This is because supposing

$$a^\nu \in \mathcal{X}_N \subset K[a]_N$$

(i.e., $\nu \in M_N$), then there exists a pullback $a^\nu + g \in \mathcal{X}_{N'}$ of a under the surjection $\mathcal{X}_{N'} \twoheadrightarrow \mathcal{X}_N$ (this map is a surjection by definition of $\mathcal{X}_N$ and $\mathcal{X}_{N'}$) such that g is a power series with the total degree of all its monomials strictly higher than N but less than or equal to N'. As $a^\nu + g \in \mathcal{X}_{N'}$ we see that $\nu \in M_{N'}$.

This shows that if $N \leq N'$, then $M_N \subset M_{N'}$. Conversely, $\nu \in M_{N'}$ and $|\nu| \leq N$ implies $\nu \in M_N$. Therefore, the multisets M_N and $M_{N'}$ are compatible and by letting $N \to \infty$ we see that, as $\mathcal{X}$ is closed, there exists M (the exponents of elements of $\mathcal{X}$) such that

(1) $f \in \mathcal{X} \implies c_\nu = 0$ for all $\nu \notin M$ and

(2) if $\nu \in M$, $a^\nu \in \tau_N(\mathcal{X})$ for all $N \geq |\nu|$; thus there exists

$$f := a^\nu + \sum_{|r|>N} c_r a^r \in \mathcal{X},$$

where $r = (r_\alpha, \alpha \in \Phi^-) \in \mathbb{N}^d$, $|c_r| \to 0$.

For $\alpha \in \Phi^-$, let $Y_\alpha \in \mathfrak{g} = \mathrm{Lie}(G)$ be the infinitesimal generator associated to the unipotent subgroup $1 + yE_\alpha$, $y \in \mathbb{Z}_p$, E_α being the standard elementary matrix at α.

Lemma 3.10. *The multi-index* 0 *is in M.*

Proof. $M \neq \varnothing$, because if so, then $\mathcal{X} = 0$, which is not true as, by assumption, $\mathcal{X}$ is nontrivial. Now if $\nu = (\nu_\alpha, \alpha \in \Phi^-) \in M$, then by (2) above,

$$f = a^\nu + \sum_{|r|>N} c_r a^r \in \mathcal{X},$$

where $N \geq |\nu|$ and $r \in \mathbb{N}^d$. By (3-11), the action of $Y_\beta = Y_{(i,j)}$ on f (where

$\beta = (i, j) \in \Phi^-$ is fixed) is given by

$$Y_\beta(f) = \frac{d}{dy}\bigg|_{y=0} \Bigg(\Bigg(\prod_{\substack{\alpha=(u,v) \\ u=i \Rightarrow v>j}} a_\alpha^{\nu_\alpha} \Bigg) \Bigg(\prod_{k=1}^{j} (a_{i,k} + y a_{j,k})^{\nu_{i,k}} \Bigg) + \sum_{|r|>N} c_r \Bigg(\prod_{\substack{\alpha=(u,v) \\ u=i \Rightarrow v>j}} a_\alpha^{r_\alpha} \Bigg) \Bigg(\prod_{k=1}^{j} (a_{i,k} + y a_{j,k})^{r_{i,k}} \Bigg) \Bigg)$$

$$= \prod_{\substack{\alpha=(u,v) \\ u=i \Rightarrow v>j}} a_\alpha^{\nu_\alpha} \Bigg(\sum_{l=1}^{j} \nu_{i,l} a_{j,l} a_{i,l}^{\nu_{i,l}-1} \prod_{\substack{k\in[1,j] \\ k\neq l}} a_{i,k}^{\nu_{i,k}} \Bigg) \times \sum_{|r|>N} c_r \prod_{\substack{\alpha=(u,v) \\ u=i \Rightarrow v>j}} a_\alpha^{r_\alpha} \Bigg(\sum_{l=1}^{j} r_{i,l} a_{j,l} a_{i,l}^{r_{i,l}-1} \prod_{\substack{k\in[1,j] \\ k\neq l}} a_{i,k}^{r_{i,k}} \Bigg)$$

$$= A + \sum_{|r|>N} c_r B.$$

The first term in the right-hand side of the above equation is

$$A := \prod_{\substack{\alpha=(u,v) \\ u=i \Rightarrow v>j}} a_\alpha^{\nu_\alpha} \Bigg(\sum_{l=1}^{j} \nu_{i,l} a_{j,l} a_{i,l}^{\nu_{i,l}-1} \prod_{\substack{k\in[1,j] \\ k\neq l}} a_{i,k}^{\nu_{i,k}} \Bigg) = \sum_{l=1}^{j} \nu_{i,l} a_{j,l}^{\nu_{j,l}+1} a_{i,l}^{\nu_{i,l}-1} \prod_{\substack{\alpha\neq(i,l) \\ \alpha\neq(j,l)}} a_\alpha^{\nu_\alpha}$$

and

$$B = \prod_{\substack{\alpha=(u,v) \\ u=i \Rightarrow v>j}} a_\alpha^{r_\alpha} \Bigg(\sum_{l=1}^{j} r_{i,l} a_{j,l} a_{i,l}^{r_{i,l}-1} \prod_{\substack{k\in[1,j] \\ k\neq l}} a_{i,k}^{r_{i,k}} \Bigg) = \sum_{l=1}^{j} r_{i,l} a_{j,l}^{r_{j,l}+1} a_{i,l}^{r_{i,l}-1} \prod_{\substack{\alpha\neq(i,l) \\ \alpha\neq(j,l)}} a_\alpha^{r_\alpha}.$$

Notice that the monomials in B have total degree $|r|$ except, when $l = j$, the term $r_{i,j} a_{i,j}^{r_{i,j}-1} \prod_{\alpha\neq(i,j)} a_\alpha^{r_\alpha}$ (note that $a_{j,j} = 1$ by convention) which has total degree $|r| - 1$.

As $Y_{(i,j)}(f) \in \mathcal{X}$, we see that $(\nu_\alpha, \nu_{i,j} - 1, \alpha \in \Phi^-, \alpha \neq (i, j)) \in M$; these are the exponents when we take $l = j$ in A. This shows that if $M \neq \varnothing$, then $0 \in M$ because we can descend the $v_{i,j}$'s successively for every negative root (i, j) and this completes the proof of Lemma 3.10. □

Lemma 3.11. *The constants a^0 are in $\mathcal{X}$.*

Proof. Let $T_k \in \mathfrak{g}$ be the infinitesimal generator associated to the diagonal subgroup $\mathrm{Diag}(1, \dots, t_k, \dots, 1)$, where $t_k \in 1 + p\mathbb{Z}_p$ and t_k is at the (k, k)-th place. By

Lemma 3.10, $0 \in M$. This implies there exists c_r for $|r| > 0$ such that

$$f = c_0 + \sum_{|r|>0} c_r a^r \in \mathcal{X}$$

(where $c_0 \neq 0$). By (3-6), from the action of $\mathrm{Diag}(1, \dots, t_k, \dots, 1)$ on f, the function obtained from $T_k(f)$ gives that

$$\sum_{|r|>0} c_r \Big(\sum r_\delta - \sum r_\beta\Big) a^r \in \mathcal{X}, \tag{3-16}$$

where $\sum r_\delta$ is $\sum_{i\in[k+1,n],\, \delta=(i,k)} r_\delta$ and $\sum r_\beta$ is $\sum_{j\in[1,k-1],\, \beta=(k,j)} r_\beta$.

The function obtained from $T_k^{p-1}(f)$ gives that

$$\sum_{|r|>0} c_r \Big(\sum r_\delta - \sum r_\beta\Big)^{p-1} a^r \in \mathcal{X}.$$

This implies that

$$E_k f := c_0 + \sum_{|r|>0} c_r \bigg(1 - \Big(\sum r_\delta - \sum r_\beta\Big)^{p-1}\bigg) a^r \in \mathcal{X}.$$

If $p \mid \sum r_\delta - \sum r_\beta$, then

$$\bigg(1 - \Big(\sum r_\delta - \sum r_\beta\Big)^{p-1}\bigg)^l \to 1 \quad \text{as } l \to \infty.$$

If $p \nmid \big(\sum r_\delta - \sum r_\beta\big)$, then

$$\bigg(1 - \Big(\sum r_\delta - \sum r_\beta\Big)^{p-1}\bigg)^l \to 0 \quad \text{as } l \to \infty.$$

Then

$$A_{k,1} f := c_0 + \sum_{\substack{|r|>0 \\ p \mid (\sum r_\delta - \sum r_\beta)}} c_r a^r \in \mathcal{X}.$$

Similar to (3-16), applying now the transformation T_k on $A_{k,1} f$, dividing by p, and iterating all the above steps, we see that

$$A_{k,2}(f) := c_0 + \sum_{\substack{|r|>0 \\ p^2 \mid (\sum r_\delta - \sum r_\beta)}} c_r a^r \in \mathcal{X}.$$

Repeating this s times, for $s \in \mathbb{N}$, we obtain

$$A_{k,s}(f) := c_0 + \sum_{\substack{|r|>0 \\ p^s \mid (\sum r_\delta - \sum r_\beta)}} c_r a^r \in \mathcal{X}.$$

This implies, for $s \in \mathbb{N}$,

$$\left(\prod_{k=1}^{n} A_{k,s}\right)(f) = c_0 + Q_s(f) \in \mathcal{X}, \tag{3-17}$$

where $Q_s(f) = \sum c_r a^r$ where the sum runs over all $r = (r_\alpha, \alpha \in \Phi^-)$ with $|r| > 0$ such that for all $k \in [1, n]$,

$$p^s \mid \left(\sum_{\substack{\delta=(i,k)\\ i\in[k+1,n]}} r_\delta - \sum_{\substack{\beta=(k,j)\\ j\in[1,k-1]}} r_\beta \right).$$

We need to show that $Q_s(f) \to 0$ as $s \to \infty$, i.e., we have to show that

$$\forall N_\epsilon, \exists S, \text{ such that } \forall s > S, v_p(c_r) > N_\epsilon, \forall r \in \mathbb{N}^d \text{ such that } |r| > 0, \tag{3-18}$$

whenever

$$p^s \mid \left(\sum_{\substack{\delta=(i,k)\\ i\in[k+1,n]}} r_\delta - \sum_{\substack{\beta=(k,j)\\ j\in[1,k-1]}} r_\beta \right) \qquad \text{for all } k \in [1, n]. \tag{3-19}$$

But as f is globally analytic, $|c_r| \to 0$ as $|r| = \sum_{\alpha\in\Phi^-} r_\alpha \to \infty$, which means that

$$\forall N_\epsilon, \exists S' \text{ such that } v_p(c_r) > N_\epsilon \text{ whenever } |r| > S'. \tag{3-20}$$

Choose an S such that $p^S > S'$.

For $k = 1$, (3-19) implies $p^s \mid r_{2,1} + r_{3,1} + r_{4,1} + \cdots + r_{n,1}$ which means that $r_{2,1}+r_{3,1}+r_{4,1}+\cdots+r_{n,1} \geq p^s > S'$ except when $r_{2,1} = r_{3,1} = r_{4,1} = \cdots = r_{n,1} = 0$. If this happens, then consider (3-19) with $k = 2$, i.e., $p^s \mid r_{3,2}+r_{4,2}+\cdots+r_{n,2}-r_{2,1}$ (where $r_{2,1} = 0$), i.e., $r_{3,2} + r_{4,2} + \cdots + r_{n,2} \geq p^s > S'$ except when $r_{3,2} = r_{4,2} = \cdots = r_{n,2} = 0$. Repeating this process, since we have started with an r such that $|r| > 0$, we see that any r as in (3-19), with $|r| > 0$, satisfies $|r| > S'$ for all $s > S$ and by (3-20) this implies that $v_p(c_r) > N_\epsilon$, which was the desired condition in (3-18). This shows that $Q_s(f) \to 0$ as $s \to \infty$ which gives that c_0 is in $\mathcal{X}$ (see (3-17)). This completes the proof of Lemma 3.11. □

In the following, we complete the proof of Theorem 3.9 which was to find conditions such that the globally analytic representation $\mathcal{A}$ of G is topologically irreducible. It uses an argument concerning Verma modules and the condition of irreducibility of $\mathcal{A}$ comes from a result of Bernstein, Gelfand and Gelfand determining the condition of irreducibility of that Verma module.

Let $\mathfrak{g} = \mathrm{Lie}(G)$, let $\mathfrak{h} = \mathrm{Lie}(T_0)$, and let $\mathfrak{b}$ (resp. $\mathfrak{b}^-$) be the upper (resp. lower) triangular Borel subalgebra containing $\mathfrak{h}$. Let $\mathfrak{u}^- = \mathrm{Lie}(U)$. Therefore $\mathfrak{g} \cong \mathfrak{gl}_n$, the set of all $n \times n$ matrices with coefficients in $\mathbb{Z}_p$. So $\mathfrak{h}$ and $\mathfrak{b}$ are the subalgebras of $\mathfrak{gl}_n$ consisting of diagonal and upper triangular matrices, respectively.

Recall that here c_i's $\in K$ are such that $\chi_i(t) = t^{c_i}$, for $t \to 1$. Let

$$V_{-\mu} := U(\mathfrak{g}) \otimes_{U(\mathfrak{b}^-)} K$$

be the Verma module where $U(\mathfrak{b}^-)$ acts on K via the action of $\mathfrak{b}^- = \mathfrak{u}^- \oplus \mathfrak{h}$, $\mathfrak{u}^-$ acting trivially, and $\mathfrak{h}$ via $-\mu \in \mathfrak{h}^\star = \mathrm{Hom}(\mathfrak{h}, K)$ given by

$$\mu = (-c_1, \dots, -c_n) : \mathrm{Diag}(t_1, \dots, t_n) \mapsto \sum_{i=1}^{n} -c_i t_i, \tag{3-21}$$

where $t = (t_i) \in \mathfrak{h}$, $U(\mathfrak{g})$ is the universal enveloping algebra of $\mathfrak{g}$. (Note that Dixmier [1977, Section 7.1.14] has a different normalization for the Verma module).

Let $\mathcal{A}_{\mathrm{fin}}$ be the set of polynomials within the rigid analytic functions $\mathcal{A}$. For $k \in [1, n]$, let $T_k \in \mathfrak{h}$ be the infinitesimal generator associated to the one parameter diagonal subgroup $\mathrm{Diag}(1, \dots, t_k, \dots, 1)$, $t_k \in 1 + p\mathbb{Z}_p$, where t_k is at the (k, k)-th place and $f = a^r \in \mathcal{A}_{\mathrm{fin}}$. The elements T_k form a basis of $\mathfrak{h}$. By (3-4) and (3-6), the action of $\mathrm{Diag}(1, \dots, t_k, \dots, 1)$ on f is given by

$$\begin{aligned}\mathrm{Diag}(1, \dots, t_k, \dots, 1)(f)&\\ &= \Bigg(\Bigg(\prod_{\substack{\alpha=(u,v)\\ u,v\neq k}} a_\alpha^{r_\alpha}\Bigg)\Bigg(\prod_{\substack{\delta=(i,k)\\ i\in[k+1,n]}} a_\delta^{r_\delta} t_k^{r_\delta}\Bigg)\Bigg(\prod_{\substack{\beta=(k,j)\\ j\in[1,k-1]}} a_\beta^{r_\beta} t_k^{-r_\beta}\Bigg)\Bigg)(\chi_k(t_k)).\end{aligned}$$

As $\chi_k(t_k) = t_k^{c_k}$, so the action of T_k on f is

$$\begin{aligned}T_k \cdot f &= c_k a^r + \Bigg(\sum_{\substack{\delta=(i,k)\\ i\in[k+1,n]}} r_\delta - \sum_{\substack{\beta=(k,j)\\ j\in[1,k-1]}} r_\beta\Bigg) a^r\\ &= \Bigg(c_k + \sum_{\substack{\delta=(i,k)\\ i\in[k+1,n]}} r_\delta - \sum_{\substack{\beta=(k,j)\\ j\in[1,k-1]}} r_\beta\Bigg) a^r = \Bigg(-\mu - \sum_{i=1}^{d} \alpha_i r_{\alpha_i}\Bigg)(T_k) a^r.\end{aligned}$$

Here the α_i's are the negative roots.

Thus if $H \in \mathfrak{h}$, then

$$H \cdot a^r = \Bigg(-\mu - \sum_{i=1}^{d} \alpha_i r_{\alpha_i}\Bigg)(H) a^r. \tag{3-22}$$

Decomposing

$$\mathcal{A}_{\mathrm{fin}} = \oplus_{\xi\in\mathfrak{h}^\star} \mathcal{A}_{\mathrm{fin}}(\xi)$$

in the form of $\mathfrak{h}$-eigenspaces, we see from (3-22) that the monomials a^r are $\mathfrak{h}$-finite and the dimensions of eigenspaces of $\mathcal{A}_{\mathrm{fin}}$ under $\mathfrak{h}$ are finite: The eigenvectors are of the form $\xi \in -\mu - \sum_{i=1}^{d} \mathbb{N}\alpha_i \in \mathfrak{h}^\star$ and the multiplicity $\mathrm{mult}(\xi) = \dim \mathcal{A}(\xi)$ of ξ is equal to

$$\dim \mathcal{A}(\xi) = \mathrm{mult}(\xi) = \left\{ \text{number of families } (r_{\alpha_i}) \in \mathbb{N}^d \;\middle|\; \xi = -\mu - \sum_{i=1}^{d} r_{\alpha_i}\alpha_i \right\}, \tag{3-23}$$

which is finite.

With $f_0 = 1 \in \mathcal{A}_{\text{fin}}$, $H \cdot f_0 = -\mu(H) f_0$ and the action $\mathfrak{u}^- \cdot f_0$ is equal to 0 because the action of any element of $\mathfrak{u}^-$ on f_0 is given by derivation (see proof of Lemma 3.10). So, the map $u \to u \cdot f_0$ for $u \in \mathfrak{g}$ induces a $\mathfrak{g}$-homomorphism

$$\phi : V_{-\mu} \to \mathcal{A}, \tag{3-24}$$

where $V_{-\mu} := U(\mathfrak{g}) \otimes_{U(\mathfrak{b}^-)} K$.

Moreover, $v \in V_{-\mu}$ implies v is $\mathfrak{h}$-finite (see [Dixmier 1977, Chapter 7]). This gives $\phi(v) \in \mathcal{A}$ is $\mathfrak{h}$-finite which means that $\phi(v) \in \mathcal{A}_{\text{fin}}$. This is because (3-22) gives, by continuity, that $f \in \mathcal{A}$; hence $f = \sum_{r=(r_{\alpha_i})} c_r a^r$ implies

$$H \cdot f = \sum_{r=(r_{\alpha_i})} \left(-\mu - \sum_{i=1}^{d} r_{\alpha_i}\alpha_i \right)(H) c_r a^r .$$

Then $H \cdot f = \lambda f$ implies $\lambda = \left(-\mu - \sum_{i=1}^{d} r_{\alpha_i}\alpha_i\right)(H)$ if $c_r \neq 0$. Therefore the cardinality of the set $\{c_r \neq 0\}$ is finite and the $\mathfrak{h}$-finite vectors of $\mathcal{A}$ are just $\mathcal{A}_{\text{fin}}$.

The map $\phi : V_{-\mu} \to \mathcal{A}_{\text{fin}}$ in (3-24) is clearly nonzero because the vector $1 \in V_{-\mu}$ goes to f_0.

Lemma 3.12. *If the Verma module $V_{-\mu}$ is irreducible then the globally analytic G-representation $\mathcal{A}$ is irreducible.*

Proof. Suppose the Verma module $V_{-\mu}$ is irreducible. Then the map $\phi : V_{-\mu} \to \mathcal{A}_{\text{fin}}$ is injective. Also by (3-23), under the action of $\mathfrak{h}$, since the eigenvectors of $V_{-\mu}$ and $\mathcal{A}_{\text{fin}}$ and their multiplicities match, that is $\dim \mathcal{A}(\xi) = \dim \mathcal{A}_{\text{fin}}(\xi) = \dim V_{-\mu}(\xi)$, we deduce that ϕ is an isomorphism.

The dimension of $\dim \mathcal{A}(\xi)$ is given by (3-23). On the other hand, using that our Verma module $V_{-\mu}$ is defined by $\mathfrak{b}^-$ and $-\mu$ (rather than $\lambda - \rho^-$ as in Dixmier's parametrization [1977, 7.1.4]), Dixmier's formula [1977, 7.1.6] yields

$$\dim V_{-\mu}(\xi) = \mathrm{mult}(\xi) = \left\{ \text{number of families } (r_{\alpha_i}) \in \mathbb{N}^d \;\middle|\; \xi = \lambda - \rho^- - \sum_{i=1}^{d} r_{\alpha_i}\alpha_i \right\},$$

where $\rho^- = \frac{1}{2} \sum_{\alpha \in \Phi^-} \alpha$ is half the sum of negative roots (since we have used $\mathfrak{b}^-$ to define the Verma module instead of Dixmier's $\mathfrak{b}^+$). We easily see that the above dimension $\dim V_{-\mu}(\xi)$ is equivalent to $\dim \mathcal{A}(\xi)$ (3-23) with $\lambda - \rho^- = -\mu$.

So $V_{-\mu} \cong \mathcal{A}_{\text{fin}}$. Suppose $\mathcal{X}$ is a nonzero closed subspace of $\mathcal{A}$. Then by Lemma 3.11, we have $1 \in \mathcal{X}$. Thus $\mathcal{A}_{\text{fin}} = U(\mathfrak{g}) \cdot 1 \subset \mathcal{X}$. Since $\mathcal{X}$ is closed, $\mathcal{X} = \mathcal{A}$. □

Now we prove the converse of Lemma 3.12.

Recall that a closed subspace of $\mathcal{A}$ is G-invariant if and only if it is invariant by $\mathfrak{g}$ [Clozel 2018, Proposition 2.4]. Moreover it follows from the definition of globally analytic representations (compare [Clozel 2018, Section 2.2]) that the action of $\mathfrak{g}$ on $\mathcal{A}$ is continuous. If $V \subset \mathcal{A}_{\text{fin}}$ is invariant by $\mathfrak{g}$, it follows that its closure $\overline{V}$ is G-invariant.

Recall that $\mathcal{A}_{\text{fin}}$ is the set of $\mathfrak{h}$-finite vectors in $\mathcal{A}$. In particular, if $\mathcal{X} \subset \mathcal{A}$ is closed, the space $\mathcal{X}_{\mathfrak{h}\text{-fin}}$ of $\mathfrak{h}$-finite vectors in $\mathcal{X}$ is $\mathcal{X} \cap \mathcal{A}_{\text{fin}}$.

Lemma 3.13. *Assume $V \subset \mathcal{A}_{\text{fin}}$ is invariant by $\mathfrak{g}$. Then $V = \overline{V} \cap \mathcal{A}_{\text{fin}} = \overline{V}_{\mathfrak{h}\text{-fin}}$.*

Proof. By (3-23), $\mathcal{A}(\xi)$ is the subspace of the Tate algebra spanned by a finite number of monomials a^r. In particular, the obvious projection $p_\xi : \mathcal{A} \to \mathcal{A}(\xi)$ is continuous. Assume $v \in \overline{V} \cap \mathcal{A}_{\text{fin}}$. Thus $v \in \bigoplus_\xi \mathcal{A}(\xi)$ (finite sum of finite-dimensional subspaces) and $v = \lim v_m,\ v_m \in V$. If P is the projection on $\bigoplus_\xi \mathcal{A}(\xi),\ v = Pv = \lim Pv_m$. But $Pv' \in V \cap \bigoplus_\xi \mathcal{A}(\xi)$ for any $v' \in V$. Thus $v \in V$, as a limit in a finite-dimensional space. □

Lemma 3.13 obviously gives the following Corollary.

Corollary 3.14. *Suppose V is a nonzero proper subspace of $\mathcal{A}_{\text{fin}}$ stable by $\mathfrak{g}$. Then $\overline{V}$ is a nonzero proper closed G-invariant subspace of $\mathcal{A}$.*

Lemma 3.15. *If the globally analytic G- representation $\mathcal{A}$ is irreducible then the Verma module $V_{-\mu}$ is irreducible.*

Proof. Let $W \subset \mathcal{A}_{\text{fin}}$ be the image of $V_{-\mu}$ by ϕ. Then $W \neq 0$. If $\mathcal{A}$ is an irreducible G-module, $W = \mathcal{A}_{\text{fin}}$ by Corollary 3.14. Thus we have a surjective map $\phi : V_{-\mu} \to \mathcal{A}_{\text{fin}}$. But, as we noticed, the dimensions of $V_{-\mu}(\xi)$ and of $\mathcal{A}_{\text{fin}}(\xi)$ coincide. This implies that ϕ is an isomorphism. On the other hand (again by the Corollary 3.14), W is irreducible. Thus $V_{-\mu}$ is irreducible. □

Now we determine the condition when the Verma module $V_{-\mu}$ is irreducible. Recall that

$$\mu = (-c_1, \dots, -c_n) : \text{Diag}(t_1, \dots, t_n) \mapsto \sum_{i=1}^{n} -c_i t_i,$$

where $t = (t_i) \in \mathfrak{h}$. For negative root $\alpha = (i, j),\ i > j$, let $H_{\alpha=(i,j)}$ be the matrix $E_{i,i} - E_{j,j}$ where $E_{i,i}$ is the standard elementary matrix.

Lemma 3.16. *The Verma module $V_{-\mu}$ is irreducible if and only if*

$$(-\mu)(H_{\alpha=(i,j)}) + i - j \notin \{1, 2, 3, \dots\}$$

for all $\alpha = (i, j) \in \Phi^-$.

Proof. Let $\rho^- = \frac{1}{2}\sum_{\alpha\in\Phi^-}\alpha$. For $\alpha = (i, j) \in \Phi^-$, $H_\alpha = H_{i+1,i} + \cdots + H_{j,j-1}$ and $\rho^-(H_{k+1,k}) = 1$. This gives that $\rho^-(H_{\alpha=(i,j)}) = i - j$. By Theorem 7.6.24 of [Dixmier 1977], the condition of irreducibility of our $V_{-\mu}$ is $(-\mu+\rho^-)(H_\alpha) \notin \{1, 2, 3, \ldots\}$ for all negative roots $\alpha \in \Phi^-$. (This is because Dixmier's $\mathfrak{b}^+$ is our $\mathfrak{b}^-$ and so we have to work with negative roots.) This gives the condition

$$(-\mu)(H_\alpha) + i - j \notin \{1, 2, 3, \ldots\}. \qquad \square$$

Lemmas 3.16, 3.15, and 3.12 together prove Theorem 3.9.

3B. With L an unramified finite extension of $\mathbb{Q}_p$, suppose V is a globally analytic representation of $G(\mathbb{Q}_p)$ on a K-Banach space where $L \subset K$. Then Clozel showed the following proposition for holomorphic base change.

Proposition 3.17 (Clozel). *V extends naturally to a globally analytic representation of $G(L)$.*

Proof. See [Clozel 2018, Proposition 3.1]. $\square$

All the arguments of Section 3A extend automatically to the group $G(L)$. As L is unramified, the conditions for the character χ to be analytic, that is, those given by (3-10), remain unchanged. Moreover, note that the representation $\mathcal{A}(B_1^{n(n-1)/2}, K)$ (where now $B_1^{n(n-1)/2}$ is seen as a product of $\frac{1}{2}n(n-1)$ closed rigid balls of radius 1 as an L-analytic space) given by Lemmas 3.2, 3.3 and 3.7 is L-analytic. The restriction of $\mathcal{A}(B_1^{n(n-1)/2}, K)$ to $G(\mathbb{Q}_p)$ is simply the previous representation. Indeed, the representation of $G(L)$ is obtained from the representation of $G(\mathbb{Q}_p)$ by holomorphic base change (see Proposition 3.17). Denote by $I_{\mathbb{Q}_p}(\chi)$ and $I_L(\chi)$, respectively, the two *globally analytic* representations (the character χ is defined by the parameters $(c_1, \ldots, c_n)$, we agree to identify the characters for the two fields). Then we have:

Theorem 3.18. *For a given embedding $L \hookrightarrow K$, with μ as in (3-21), if*

$$-\mu(H_\alpha) + i - j \notin \{1, 2, 3, \ldots\} \quad \textit{for all } \alpha = (i, j) \in \Phi^-,$$

then $I_L(\chi)$ is an admissible, irreducible (under both $G(L)$ and $G(\mathbb{Q}_p)$) globally analytic representation and it is the holomorphic base change of $I_{\mathbb{Q}_p}(\chi)$.

$I_L(\chi)$ is admissible, as holomorphic base change respects admissibility [Clozel 2018, Proposition 3.1]. With the notation of Section 2B, define the full (Langlands) base change of $I_{\mathbb{Q}_p}$ to be the representation of $\mathrm{Res}_{L/\mathbb{Q}_p} G(\mathbb{Q}_p)$ on $\widehat{\otimes}_\sigma (I_L(\chi))^\sigma := I(\chi \circ N_{L/\mathbb{Q}_p})$, where $N_{L/\mathbb{Q}_p}$ is the norm map from L to $\mathbb{Q}_p$ and $\widehat{\otimes}$ is the completed tensor product (see also [Clozel 2018, Definition 3.8]) and $\sigma \in \mathrm{Gal}(L/\mathbb{Q}_p)$. Note that, for each factor, the embedding $i : L \to K$ must be replaced by $i \circ \sigma$. Finally, we then have:

Theorem 3.19. *Let μ be as in (3-21). Assume $-\mu(H_\alpha)+i-j \notin \{1,2,3,\dots\}$ for all $\alpha=(i,j)\in\Phi^-$. Then the completed tensor product $\widehat{\otimes}_\sigma(I_L(\chi))^\sigma$ is irreducible, and is the representation of $G(L)$ on the space of globally analytic vectors, induced from $\chi\circ N_{L/\mathbb{Q}_p}$.*

Proof. Notice that by assumption, each factor in the completed tensor product is irreducible and admits the same description as in Theorem 3.18. The space of the representation $I(\chi\circ N_{L/\mathbb{Q}_p})$ is $\widehat{\otimes}_\sigma\mathcal{A}(U,K)=\mathcal{A}(\mathrm{Res}_{L/\mathbb{Q}_p}U,K)$, which is a space of globally analytic vectors (by Theorem 3.18) in the locally analytic representation $I_{\mathrm{loc}}(\chi\circ N_{L/\mathbb{Q}_p})$ of $\mathrm{Res}_{L/\mathbb{Q}_p}(G)$. The proof of irreducibility of $\widehat{\otimes}_\sigma(I_L(\chi))^\sigma$ follows from Theorem 3.9 using a natural generalization of the argument in [Clozel 2018]. □

4. Analyticity for the induction from the Weyl orbits of the upper triangular Borel subgroup of B

In this section we treat the global analyticity of the principal series induced from Weyl orbits of the Borel subgroup (Theorem 4.3). Then we base change our globally analytic representation to L.

Denote by $\mathbb{P}$ the Borel subgroup of the upper triangular matrices in $\mathrm{GL}_n(\mathbb{Q}_p)$, by $\mathbb{T}$ the maximal torus of $\mathrm{GL}_n(\mathbb{Q}_p)$, by P^+ the Borel subgroup of the upper triangular matrices in $\mathrm{GL}_n(\mathbb{Z}_p)$, and by W the ordinary Weyl group of $\mathrm{GL}_n(\mathbb{Q}_p)$ with respect to $\mathbb{T}$ which is isomorphic to the group of $n\times n$ permutation matrices. Write $P_w^+=B\cap wP^+w^{-1}$. Here B is the Iwahori subgroup in Section 3A. Denote by $\mathrm{ind}_{\mathbb{P}}^{\mathrm{GL}_n(\mathbb{Q}_p)}(\chi)_{\mathrm{loc}}$ the locally analytic induction, that is,

$$\begin{aligned}\mathrm{ind}_{\mathbb{P}}^{\mathrm{GL}_n(\mathbb{Q}_p)}(\chi)_{\mathrm{loc}}=\{f\in\mathcal{A}_{\mathrm{loc}}(\mathrm{GL}_n(\mathbb{Q}_p),K): f(gb)\\ =\chi(b^{-1})f(g),\ g\in\mathrm{GL}_n(\mathbb{Q}_p),\ b\in\mathbb{P}\}.\end{aligned}$$

The Iwasawa decomposition [Orlik and Strauch 2010, Section 3.2.2] gives

$$\mathrm{ind}_{\mathbb{P}}^{\mathrm{GL}_n(\mathbb{Q}_p)}(\chi)_{\mathrm{loc}}\cong\mathrm{ind}_{P^+}^{\mathrm{GL}_n(\mathbb{Z}_p)}(\chi)_{\mathrm{loc}}$$

as a $\mathrm{GL}_n(\mathbb{Z}_p)$-equivariant topological isomorphism. By the Bruhat–Tits decomposition [Orlik and Strauch 2010, Section 3.2.2; Cartier 1979, Section 3.5],

$$\mathrm{GL}_n(\mathbb{Z}_p)=\bigsqcup_{w\in W}BwP^+,$$

we obtain the decomposition

$$\mathrm{ind}_{P^+}^{\mathrm{GL}_n(\mathbb{Z}_p)}(\chi)_{\mathrm{loc}}\cong\bigoplus_{w\in W}\mathrm{ind}_{P_w^+}^{B}(\chi^w)_{\mathrm{loc}},$$

a B-equivariant decomposition of topological vector spaces, where the action of χ^w is given by $\chi^w(h)=\chi(w^{-1}hw)$. Let $\mathrm{ind}_{P_w^+}^{B}(\chi^w)$ be the space of globally analytic functions of $\operatorname{ind}^{B}_{P_w^+}(\chi^w)_{\mathrm{loc}}$. Our goal is to show that for all $w \in W$, $\operatorname{ind}^{B}_{P_w^+}(\chi^w)$ is a globally analytic representation of G. We have already showed, in Section 3, that for $w = \mathrm{Id}$ and χ analytic, the induction $\operatorname{ind}^{B}_{P_0}(\chi)$ is a globally analytic representation of G. (Note that $B \cap P^+ = P_0$.) Recall that U is the lower triangular unipotent subgroup of $\mathrm{GL}_n(\mathbb{Z}_p)$. Consider the decomposition (see Lemma 3.3.2 of [Orlik and Strauch 2010])

$$\begin{aligned} B &= (wUw^{-1} \cap B)(wP^+w^{-1} \cap B) \\ &= (wUw^{-1} \cap B)(P_w^+). \end{aligned}$$

For GL_3, and

$$w = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

the above decomposition is

$$B = \begin{pmatrix} \mathbb{Z}_p^\times & p\mathbb{Z}_p & p\mathbb{Z}_p \\ \mathbb{Z}_p & \mathbb{Z}_p^\times & p\mathbb{Z}_p \\ \mathbb{Z}_p & \mathbb{Z}_p & \mathbb{Z}_p^\times \end{pmatrix} = \begin{pmatrix} 1 & p\mathbb{Z}_p & p\mathbb{Z}_p \\ 0 & 1 & 0 \\ 0 & \mathbb{Z}_p & 1 \end{pmatrix} \begin{pmatrix} \mathbb{Z}_p^\times & 0 & 0 \\ \mathbb{Z}_p & \mathbb{Z}_p^\times & p\mathbb{Z}_p \\ \mathbb{Z}_p & 0 & \mathbb{Z}_p^\times \end{pmatrix}.$$

We extend a character χ of $\mathbb{T} \cap \mathrm{GL}_n(\mathbb{Z}_p)$ to a character of P_w^+ by acting trivially on the nondiagonal elements of P_w^+. By definition,

$$\operatorname{ind}^{B}_{P_w^+}(\chi)_{\mathrm{loc}} = \{ f \in \mathcal{A}_{\mathrm{loc}}(B, K) : f(gb) = \chi(b^{-1}) f(g),\ b \in P_w^+,\ g \in B \}.$$

With the decomposition $B = (wUw^{-1} \cap B)(P_w^+)$, the vector space of locally analytic functions $\operatorname{ind}^{B}_{P_w^+}(\chi)_{\mathrm{loc}}$ is the same as $\mathcal{A}_{\mathrm{loc}}(wUw^{-1} \cap B, K)$. Let $\mathcal{A}(wUw^{-1} \cap B, K)$ be the subspace of globally analytic functions of $\mathcal{A}_{\mathrm{loc}}(wUw^{-1} \cap B, K)$. With $i \neq j$ fixed, $y \in \mathbb{Z}_p$ if $i > j$ and $y \in p\mathbb{Z}_p$ if $i < j$, recall that the action of the one-parameter subgroup on

$$f \in \mathcal{A}(wUw^{-1} \cap B, K)$$

is given by

$$\begin{aligned} (1 + yE_{i,j}) f(C) &= f((1 + yE_{i,j})^{-1} C) \qquad \text{(with } C \in wUw^{-1} \cap B) \\ &= f((1 - yE_{i,j}) C) \\ &= f((1 - yE_{i,j}) wAw^{-1}) \quad \text{(with } C = wAw^{-1} \text{ for } A \in U). \end{aligned} \tag{4-1}$$

Our goal is to show that this action is globally analytic.

Since $w^{-1} \in W$, write w^{-1} in the form of a permutation matrix, i.e.,

$$w^{-1} = \sum_{r=1}^{n} E_{r, j_r}$$

with $j_r \neq j_s$ for $r \neq s$. Then,

$$w^{-1}(1 - yE_{i,j}) = \Big(\sum_{r=1}^{n} E_{r,j_r}\Big)(1 - yE_{i,j}) = \Big(\sum_{r=1}^{n} E_{r,j_r}\Big) - yE_{k,j},$$

where k is such that $j_k = i$. As the inverse of a permutation matrix is its transpose, we obtain

$$w^{-1}(1 - yE_{i,j})w = \bigg(\Big(\sum_{r=1}^{n} E_{r,j_r}\Big) - yE_{k,j}\bigg)\Big(\sum_{s=1}^{n} E_{j_s,s}\Big) = 1 - yE_{k,l},$$

where l is such that $j_l = j$. So we have deduced that

$$(1 - yE_{i,j})w = w(1 - yE_{k,l}) \qquad (k, l \text{ such that } j_k = i,\ j_l = j). \tag{4-2}$$

Inserting (4-2) into (4-1), we obtain

$$(1 + yE_{i,j})f(C) = f(w(1 - yE_{k,l})Aw^{-1}). \tag{4-3}$$

Now, the globally analytic function f on $w(1 - yE_{k,l})Aw^{-1}$ equals some globally analytic function g on $(1 - yE_{k,l})A$, because the conjugacy action of w on the matrix $(1 - yE_{k,l})A$ is just permuting the entries of $(1 - yE_{k,l})A$. So, (4-3) is

$$\begin{aligned} f(w(1 - yE_{k,l})Aw^{-1}) &= g((1 - yE_{k,l})A) \\ &= (1 + yE_{k,l})g(A) \qquad (\text{recall } A \in U) \end{aligned}$$

and we know from Lemmas 3.3 and 3.7 that the action of $(1 + yE_{k,l})$ on $g(A)$ is globally analytic. Thus, we have shown that:

Lemma 4.1. *The action of the lower and the upper unipotent one-parameter subgroups of G of the form $(1+yE_{i,j})$ on $f \in \mathcal{A}(wUw^{-1} \cap B, K)$ is a globally analytic action.*

A similar argument also shows that the action of the diagonal subgroup of G on $\mathcal{A}(wUw^{-1} \cap B, K)$ is globally analytic. More precisely, we write

$$w^{-1}\,\mathrm{Diag}(t_1, \ldots, t_n)w = \mathrm{Diag}(t'_1, \ldots, t'_n)$$

with $(t'_1, \ldots, t'_n)$ a permutation of $(t_1, \ldots, t_n)$. Then, with $C \in wUw^{-1} \cap B$,

$$\begin{aligned} \mathrm{Diag}(t_1^{-1}, \ldots, t_n^{-1})f(C) &= f(\mathrm{Diag}(t_1, \ldots, t_n)wAw^{-1}) \qquad (C = wAw^{-1}) \\ &= f(w[\mathrm{Diag}(t'_1, \ldots, t'_n)]Aw^{-1}) \\ &= g(\mathrm{Diag}(t'_1, \ldots, t'_n)A) \qquad (\text{for some analytic } g) \\ &= \mathrm{Diag}(t_1^{-1}, \ldots, t_n^{-1})g(A) \end{aligned}$$

and by Lemmas 3.2 and 3.1, the action of the diagonal subgroup of G on $g(A)$ is a globally analytic action. Therefore, we have shown:

Lemma 4.2. *The action of the diagonal subgroup of G on $\mathcal{A}(wUw^{-1} \cap B, K)$ is globally analytic.*

Recall that the vector space $\mathcal{A}(wUw^{-1} \cap B, K)$ is isomorphic to $\mathrm{ind}_{P_w^+}^B(\chi^w)$. Thus, Lemmas 4.1 and 4.2 together give:

Theorem 4.3. *Assume $p > n + 1$. Then, for all $w \in W$, the action of the pro-p Iwahori group G on $\mathrm{ind}_{P_w^+}^B(\chi^w)$ is globally analytic.*

Following the notation of Section 3B, we fix L a finite unramified extension of $\mathbb{Q}_p$ inside K. For each $w \in W$, consider the globally analytic admissible representation $I_{w,\mathbb{Q}_p}(\chi) := \mathcal{A}(wUw^{-1} \cap B, K)$ of $G(\mathbb{Q}_p)$. By Section 2B, $\mathcal{A}(wUw^{-1} \cap B, K)$ extends naturally to a globally analytic admissible representation of $G(L)$ called the "holomorphic base change" which we denote by $I_{w,L}(\chi)$. With the notation of Section 2B, define the full Langlands base change to be the representation of $\mathrm{Res}_{L/\mathbb{Q}_p} G(\mathbb{Q}_p)$ on $\oplus_{w \in W}(\widehat{\otimes}_\sigma I_{w,L}(\chi)^\sigma)$ (see [Clozel 2018, Section 3.5]). Finally, as in Theorem 3.19, we will then have:

Theorem 4.4. *The Langlands base change*

$$\oplus_{w \in W}(\widehat{\otimes}_\sigma I_{w,L}(\chi^w)^\sigma)$$

is a globally analytic admissible representation of $G(L)$.

In conclusion, for $p > n + 1$, we have shown that for all $w \in W$, $\mathrm{ind}_{P_w^+}^B(\chi^w)$ is a globally analytic representation of the pro-p Iwahori G under the analyticity assumption on the character χ. Furthermore we have treated the case of irreducibility of the principal series when $w = \mathrm{Id}$. We hope that it is possible to adapt and generalize the argument of our irreducibility proof to treat the case when $w \neq \mathrm{Id}$. Also it is an interesting future project to determine the globally analytic vectors of more general p-adic representations of $\mathrm{GL}(2, \mathbb{Q}_p)$, for example the "trianguline" representation of [2008] (see also [Colmez 2014]), which corresponds to a quotient of principal series. Also one can explore the connection with the globally analytic vectors of p-adic representations (under the pro-p Iwahori or a suitable rigid analytic subgroup of GL(2)) and (φ, Γ)-modules [Colmez 2010], similar to the existing correspondence for locally analytic representations [Colmez and Dospinescu 2014, Section VI.3].

There are other interesting questions that our work leads to. The most interesting of them is to show Schur's lemma for globally analytic representations. Schur's lemma for locally analytic representations is known by the works of Gabriel Dospinescu and Benjamin Schraen. For our case of topologically irreducible globally analytic principal series, Schur's lemma easily follows from our proof of irreducibility and Proposition 7.1.8(iv) of [Dixmier 1977]. The interesting question is to show Schur's lemma for general globally analytic representations.

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References

[Bertapelle 2000] A. Bertapelle, "Formal Néron models and Weil restriction", *Math. Ann.* **316**:3 (2000), 437–463. MR Zbl

[Bosch 2014] S. Bosch, *Lectures on formal and rigid geometry*, Lecture Notes in Math. **2105**, Springer, 2014. MR Zbl

[Cartier 1979] P. Cartier, "Representations of p-adic groups: a survey", pp. 111–155 in *Automorphic forms, representations and L-functions, I* (Corvallis, OR, 1977), edited by A. Borel and W. Casselman, Proc. Sympos. Pure Math. **33**, Amer. Math. Soc., Providence, RI, 1979. MR Zbl

[Clozel 2017] L. Clozel, "Globally analytic p-adic representations of the pro-p-Iwahori subgroup of GL(2) and base change, I: Iwasawa algebras and a base change map", *Bull. Iranian Math. Soc.* **43**:4 (2017), 55–76. MR Zbl

[Clozel 2018] L. Clozel, "Globally analytic p-adic representations of the pro-p Iwahori subgroup of GL(2) and base change, II: A Steinberg tensor product theorem", pp. 1–33 in *Cohomology of arithmetic groups* (Bonn, Germany, 2016), edited by J. W. Cogdell et al., Springer Proc. Math. Stat. **245**, Springer, 2018. MR Zbl

[Colmez 2008] P. Colmez, "Représentations triangulines de dimension 2", pp. 213–258 in *Représentations p-adiques de groupes p-adiques, I: Représentations galoisiennes et* (φ, Γ)*-modules*, edited by L. Berger et al., Astérisque **319**, Soc. Math. France, Paris, 2008. MR Zbl

[Colmez 2010] P. Colmez, "Représentations de $\mathrm{GL}_2(\mathbb{Q}_p)$ et (φ, Γ)-modules", pp. 281–509 in *Représentations p-adiques de groupes p-adiques, II: Représentations de* $\mathrm{GL}_2(\mathbb{Q}_p)$ *et* (φ, Γ)*-modules*, edited by L. Berger et al., Astérisque **330**, Soc. Math. France, Paris, 2010. MR Zbl

[Colmez 2014] P. Colmez, "La série principale unitaire de $\mathrm{GL}_2(\mathbb{Q}_p)$: vecteurs localement analytiques", pp. 286–358 in *Automorphic forms and Galois representations, I* (Durham, UK, 2011), edited by F. Diamond et al., Lond. Math. Soc. Lect. Note Ser. **414**, Cambridge Univ. Press, 2014. MR Zbl

[Colmez and Dospinescu 2014] P. Colmez and G. Dospinescu, "Complétés universels de représentations de $\mathrm{GL}_2(\mathbb{Q}_p)$", *Algebra Number Theory* **8**:6 (2014), 1447–1519. MR Zbl

[Dixmier 1977] J. Dixmier, *Enveloping algebras*, North-Holland Math. Library **14**, North-Holland, Amsterdam, 1977. MR Zbl

[Emerton 2017] M. Emerton, *Locally analytic vectors in representations of locally p-adic analytic groups*, Mem. Amer. Math. Soc. **1175**, Amer. Math. Soc., Providence, RI, 2017. MR Zbl

[Lazard 1965] M. Lazard, "Groupes analytiques p-adiques", *Inst. Hautes Études Sci. Publ. Math.* **26** (1965), 389–603. MR Zbl

[Orlik and Strauch 2010] S. Orlik and M. Strauch, "On the irreducibility of locally analytic principal series representations", *Represent. Theory* **14** (2010), 713–746. MR Zbl

[Ray 2020] J. Ray, "Explicit presentation of an Iwasawa algebra: the case of pro-p Iwahori subgroup of $\mathrm{SL}_n(\mathbb{Z}_p)$", *Forum Math.* **32**:2 (2020), 319–338. MR Zbl

[Robert 1984] A. Robert, "Représentations p-adiques irréductibles de sous-groupes ouverts de $\mathrm{SL}_2(\mathbf{Z}_p)$", *C. R. Acad. Sci. Paris Sér. I Math.* **298**:11 (1984), 237–240. MR

[Robert 1985] A. Robert, “Représentations p-adiques de dimension infinie de sous-groupes ouverts de $\mathrm{SL}_2(\mathbf{Q}_p)$”, exposé 23, pp. 1–14 in *Groupe d’étude d’analyse ultramétrique. 12e année: 1984/85. Fasc. 2*, edited by Y. Amice et al., Secrétariat Math., Paris, 1985. MR

[Schneider 2011] P. Schneider, *p-adic Lie groups*, Grundlehren der Math. Wissenschaften **344**, Springer, 2011. MR Zbl

[Schneider and Teitelbaum 2002] P. Schneider and J. Teitelbaum, “Locally analytic distributions and p-adic representation theory, with applications to GL_2”, *J. Amer. Math. Soc.* **15**:2 (2002), 443–468. MR Zbl

[Steinberg 1963] R. Steinberg, “Representations of algebraic groups”, *Nagoya Math. J.* **22** (1963), 33–56. MR Zbl

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